MATHEMATICAL
ELEMENTS
Henry Fletcher
OF
Natural Philoſophy,
CONFIRM'D BY
EXPERIMENTS:
OR, AN
INTRODUCTION
TO
Sir ISAAC NEWTON's Philoſophy.

VOL. I.

Written in LATIN
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R
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BRITISH MUSEUM

TO

Sir *Iſaac Newton*, Knt.

PRESIDENT

OF THE

ROYAL SOCIETY, &c.

SIR,

IN dedicating this Tranſlation to You, I do no more than what my learned Friend, the ingenious Author, would have done, if Cuſtom and Gratitude had not obliged him to offer the firſt Philoſophical Work he has publiſhed ſince his being Profeſſor, to the Governors of the Univerſity that gave him the Chair. And as there are more Admirers of your wonderful Diſcoveries, than there are Mathematicians able to underſtand the two firſt Books of your *Principia*; ſo I hope You will not be diſpleaſed, that both my Author and myſelf have, by Experiments, endeavoured to explain ſome of thoſe Propoſitions which were implicitly believed by many of your Readers; at the ſame Time that the greateſt Part of your third Book, and ſeveral of the *Corollaries* and *Scholia* in the other two, gave them the higheſt Satisfaction that an inquiſitive Mind is capable of receiving. Mathematicians of the firſt Rank, who want no ſuch Helps in reading your incomparable Works, take a freſh Pleaſure in ſeeing thoſe Experiments performed which you have made yourſelf: And though ſome of ours may not always prove, but ſometimes only illuſtrate a Propoſition; yet, ſuch Mathematicians as are of a communicative Temper, will be glad to uſe them, as a new Set of Words, to give Beginners ſo clear a Notion of the Syſtem of the World, as to encourage them to the Study of the higher Geometry; whereby they may know how to value your Solutions of the moſt difficult *Phænomena*, and learn from You, that a whole Science may be contained in a ſingle Propoſition.

For my own Part, ſince I cannot enough acknowledge the Advantages which I have had by being admitted to your Converſation, and your generous Way of gratifying me for ſuch Experiments as I have made by your Direction; therefore I ſhall here forbear to pay that Tribute which is due to You from all Lovers of Knowledge; and rather chooſe to be thought ſingular, than, by praiſing, to offend You. I am,

SIR,

Your moſt Obedient,

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and very Humble Servant,

J. T. DESAGULIERS.

THE TRANSLATOR TO THE READER.

THIS Book will ſufficiently recommend it ſelf; therefore I need not ſay any thing in Praiſe of it, either on Account of the uſeful Subject that it treats of, or the excellent Method and familiar Way in which our Author has handled it; only I thought proper to obſerve to the Reader, that the Numbers in the Margin expreſs ſo many Propoſitions, which are referred to, as you go forward in the Book, to avoid Repetitions and Tautology. If what is printed in Italic *Characters be read by it ſelf, it will appear to be a Compendium of the whole*

whole Book; or the Doctrinal Part of it, without the Experiments and Demonſtrations. I have endeavoured to anſwer this End in my Engliſh *Tranſlation, where you will find, that whatever is in* Italic *Characters, makes up the Senſe of the reſt of the Book; which alſo readily makes Senſe by it ſelf, though taken from Places where it ſeems irregularly diſperſed.*

The firſt Edition of this Tranſlation had ſome Errors of the Preſs, and Faults in the Plates, which were occaſioned by the Haſte in which it was printed off, to prevent the Injury that muſt have been done to Dr. 's Graveſande, *by a Tranſlation that ſome Bookſellers endeavoured to get out before mine; which was ſo ill done, that no body that had read the* Latin *Book would be able to know it again in their* Engliſh.

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I have, therefore, in this Fifth Edition, carefully review'd and corrected every Error, both in the Book and Plates.

THE PREFACE.

If we compare the Writings of different Philoſophers concerning **Physics,** *we may eaſily ſee that they call different Sciences by the ſame Name, tho' they all profeſs to explain the true Cauſe of Natural* * Phænomena. *And no wonder if they diſagree among themſelves, ſince even Mathematicians, who deal in Certainties, can hardly be kept from wrangling.*

But that Diverſity of Opinions ſhould not deter us from ſearching after Truth; ſince Labour and Study will find it out; and the more we are in love with it, the leſs we are liable to Errors, excepting ſuch as human Frailty renders unavoidable.

We muſt proceed cautiouſly in Phyſics, *ſince that Science conſiders the Works of the Supreme Wiſdom, and ſets forth,*

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*Appearances.

*What Laws JEHOVAH to himſelf preſcrib'd,
And of his Work the firm Foundation made,
When He of Things the firſt Deſign ſurvey'd.

How the whole Univerſe is govern'd by thoſe Laws, and how the ſame Laws run thro' all the Works of Nature, and are conſtantly obſerv'd with a wonderful Regularity.

We muſt take care not to admit Fiction for Truth; for by that Means we ſhut out all further Examination. No true Explanation of Phænomena can ſpring out of a falſe Principle: And what a vaſt Difference there is betwixt learning the Fictions of whimſical Men, and examining the Works of the moſt wiſe God! Since an Enquiry into Divine Wiſdom, and the Veneration inſeparable from it, is to be the Scope of a Philoſopher, we need not enlarge upon the Vanity of reaſoning upon fictitious Hypotheſes.

Nature herſelf is therefore attentively and inceſſantly to be examined with indefatigable Pains. That Way indeed our Progreſs will be but ſlow, but then our Diſcoveries will be certain; and oftentimes we ſhall even be able to determine the Limits of Human Underſtanding.

What has led moſt People into Errors, is an immoderate Deſire of Knowledge, and the Shame of confeſſing our Ignorance. But Reaſon ſhould get the better of that ill-grounded Shame; ſince there is a learned Ignorance that is the Fruit of Knowledge, and which is much preferable to an ignorant Learning.

Natural Philoſophy is placed among thoſe Parts of Mathematicks, whoſe Object is Quantity in general.

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* ——— quas dum primordia rerum
Pangeret, omniparens leges violare Creator
Noluit, æternique operis fundamina fixit.

Mathematics are divided into Pure and Mixed. Pure Mathematics enquire into the General Properties of Figures and abſtracted Ideas. Mixed Mathematics examine Things themſelves, and will have our Notions and Deductions to agree both with Reaſon and Experience.

Phyſics belong to mixed Mathematics. The Properties of Bodies, and the Laws of Nature, are the Foundations of Mathematical Reaſoning, as all that have examined the Scope of this Science will freely confeſs. But Philoſophers do not equally agree upon what is to paſs for a Law of Nature, and what Method is to be followed in Queſt of thoſe Laws. I have therefore thought fit in this Preface to make good the Newtonian *Method, which I have followed in this Work. What that Method is, I have briefly ſet down in the firſt Chapter.*

Phyſics do not meddle with the firſt Foundation of Things. That the World was created by God, *is a Poſition wherein Reaſon ſo perfectly agrees with Scripture, that the leaſt Examination of Nature will ſhew plain Footſteps of Supreme Wiſdom. It is confounding and overſetting all our cleareſt Notions, to aſſert that the World may have taken its Riſe from ſome general Laws of Motion, and that* it imports not *what is imagined concerning the firſt Diviſion of Matter.* And that there can hardly be any thing ſuppoſed, from which the ſame Effect may not be deduced by the ſame Laws of Nature; *and that for this Reaſon: That* ſince Matter ſucceſſively aſſumes all the Forms it is capable of by means of thoſe Laws, if we conſider all thoſe Forms in Order, we muſt at laſt come to that Form wherein this preſent World is framed; ſo that we have no Reaſon in this Caſe to fear any Error from a wrong Suppoſition. *This Aſſertion, I ſay, overthrows all our cleareſt Notions, as has been fully proved by many Learned Men; and is indeed ſo* *unreaſonable, and ſo injurious to the Deity, that it will ſeem unworthy of an Anſwer to any one that does not know that it has been maintained by many ancient and modern Philoſophers, and ſome of them of the firſt Rank, and far removed from any Suſpicion of Atheiſm.*

Then firſt laying it down as an undoubted Truth, that GOD *has created all Things, we muſt afterwards explain by what Laws every thing is governed; and to mention only the Moon, we muſt explain, why*

* The Silver Moon runs with unequal Pace,
Which yet Aſtronomers could never trace,
Or fix in Numbers her uncertain Place:
What Force her *Apſides* has forward driven,
And made her *Nodes* recede i'th' Starry Heaven:
What is her Pow'r to agitate the Sea,
Whoſe various Tides her Preſence ſtill obey;
When th'*Ocean* ſwells, its topmoſt Banks to lave,
Or ebbs from weedy Shores with broken Wave,
Leaving the Sands, the Sailor's Terror, bare:

In order to explain more fully which Way we trace out the Laws of Nature, we muſt begin by ſome previous and preparatory Reflections.

What Subſtances are, is one of the Things hidden from us. We know for Inſtance, ſome of the Pro-

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——————* Quâ cauſa argentea Phœbe
Paſſibus haud æquis graditur; cur ſubdita nulli
Hactenus Aſtronomo numerorum fræna recuſet.
Cur remeant Nodi, curque Auges progrediuntur.
——————quantis refluum vaga Cynthia pontum
Viribus impellit, dum fractis fluctibus ulvam
Deſerit, ac nautis ſuſpectas nudat arenas;
Alternis vicibus ſuprema ad littora pulſans,

Properties of Matter; but we are abſolutely ignorant, what Subject they are inherent in.

Who dares affirm that there are not in Body *many other Properties, which we have no Notions of? And who ever could certainly know, that, beſides the Properties of* Body *which flow from the Eſſence of Matter, there are not others depending upon the free Power of* God, *and that extended and ſolid Subſtance (for thus we define Body) is endowed with ſome Properties without which it could exiſt? We are not, I own, to affirm or deny any thing concerning what we do not know. But this* Rule *is not followed by thoſe, who reaſon in Phyſical Matters, as if they had a compleat Knowledge of whatever belongs to Body, and who do not ſcruple to affirm, that the few Properties of Body, which they are acquainted with, conſtitute the very Eſſence of Body.*

What do they mean by ſaying, that the Properties of Subſtance conſtitute the very Subſtance?

Can thoſe Things ſubſiſt when joined together, that can ſubſiſt ſeparately? Can Extenſion, Impenetrability, Motion, &c. *be conceived without a Subject to which they belong? And have we any Notion of that Subject?*

We muſt give up as uncertain what we find to be ſo, and not be aſhamed to confeſs our Ignorance. Tho' we need not fear being too bold in affirming, that a Subject altogether unknown to us may perhaps be endowed with ſome unknown Properties. And thoſe Men who at the ſame Time that they ſay, conformably to this Axiom, That we muſt not reaſon about Things unknown, lay it down as a Foundation of their Reaſonings, *that nothing relating to Body is unknown to us, are beholden to meer Chance, if they are not miſtaken.*

The Properties of Body cannot be known à priori: *We muſt therefore examine Body itſelf, and nicely* *conſider all its Properties; that we may be able to determine what natural Effects* do *flow from thoſe Properties.*

Upon a farther Examination of Body, we find there are ſome general Laws according to which Bodies are moved. It is paſt Doubt, for Inſtance, That a Body once moved continues in Motion; that Reaction is always equal and contrary to Action. *And ſeveral other ſuch Laws concerning Body have been diſcovered; which can no Way be deduced from thoſe Properties that are ſaid to conſtitute Body; and ſince thoſe Laws always hold good and upon all Occaſions, they are to be looked upon as general Laws of Nature. But then we are at a Loſs to know, whether they flow from the Eſſence of Matter, or whether they are deducible from Properties, given by* GOD *to the Bodies the World conſiſts of, but no Way eſſential to Body; or whatever finally thoſe Effects, which paſs for Laws of Nature depend upon external Cauſes, which even our Ideas cannot attain to.*

Who dares affirm any thing upon this Point concerning all, or any Laws of Nature, without incurring the Guilt of Raſhneſs? Beſides, whoever examines the Phænomena of Nature will be fully perſuaded, that many of its Laws are not yet diſcovered, and that many Particulars are wanting towards the compleat Knowledge of others.

The Study of Natural Philoſophy is not however to be contemned, as built upon an unknown Foundation. The Sphere of human Knowledge is bounded within a narrow Compaſs; and he that denies his Aſſent to every thing but Evidence, wavers in Doubt every Minute, and looks upon many Things as unknown, which the Generality of People never ſo much as call in Queſtion. But rightly to diſtinguiſh Things known from Things unknown, is a Perfection above the Level of human Mind. Though many

Things in Nature are hidden from us; yet what is ſet down in Phyſics, as a Science, is undoubted. From a few general Principles numberleſs particular Phænomena or Effects are explained, and deduced by Mathematical Demonſtration. For, the comparing of Motion, or in other Words, of Quantities, is the continual Theme; and whoever will go about that Work any other Way than by Mathematical Demonſtrations, will be ſure to fall into Uncertainties at leaſt, if not into Errors.

How much ſoever then may be unknown in Natural Philoſophy, it ſtill remains a vaſt, certain, and very uſeful Science: It corrects an infinite Number of Prejudices concerning natural Things, and Divine Wiſdom; and, as we examine the Works of GOD *continually, ſets that Wiſdom before our Eyes; and there is a wide Difference betwixt knowing the divine Power and Wiſdom by a Metaphyſical Argument, and beholding them with our Eyes every Minute in their Effects. It appears then ſufficiently, what is the End of Phyſics, from what Laws of Nature the Phænomena are to be deduced, and wherefore; when we are once come to the general Laws, we cannot penetrate any further into the Knowledge of Cauſes. There remains only to diſcourſe of the Method of ſearching after thoſe Laws; and to prove that the three* Newtonian *Laws, delivered in the firſt Chapter of this Work, ought to be followed.*

The firſt is, That we ought not to admit any more Cauſes of Natural Things, than what are true and ſufficient to explain their Phænomena. *The firſt Part of this Rule plainly follows from what has been ſaid above. The other cannot be called in Queſtion by any that owns the Wiſdom of the Creator. If one Cauſe ſuffices, it is needleſs to ſuperadd another; eſpecially, if it be conſidered, that an Effect from a double Cauſe is never exactly* *ly the ſame with an Effect from a ſingle one. Therefore we are not to multiply Cauſes, 'till it appears one ſingle Cauſe will not do the Buſineſs.*

In order to prove the following Rules, we muſt promiſe ſome general Reflections.

We have already ſaid, that Mathematical Demonſtrations have no Standard to be be judged by, but their Conformity with our Ideas; and when the Queſtion is about Natural Things, the firſt Requiſite is, that our Ideas agree with thoſe Things which cannot be proved by any Mathematical Demonſtration. And yet as we have Occaſion to reaſon of Things themſelves every Moment, and of thoſe Things nothing can be preſent to our Minds beſides our Ideas, upon which our Reaſonings immediately turn; it follows, that GOD *has eſtabliſhed ſome Rules, by which we may judge of the Agreement of our Ideas with the Things themſelves.*

All Mathematical Reaſonings turn upon the Compariſon of Quantities, and their Truth is evidenced by implying a Contradiction in a contrary Propoſition. A rectilineal Triangle, for Inſtance, whoſe three Angles are not equal to two right ones, is a Thing impoſſible. When the Queſtion is not about the Compariſon of Quantities, a contrary Propoſition is not always impoſſible. It is certain, for Inſtance, that Peter *is living, though it is as certain that he might have died Yeſterday. Now there being numberleſs Caſes of that Kind, where one may affirm or deny with equal Certainty; there follows, that there are many Reaſonings very certain, tho' altogether different from the Mathematical Ones. And they evidently follow from the Eſtabliſhment of Things, and therefore from the pre-determined Will of* GOD. *For by forcing Men upon the Neceſſity of pronouncing concerning the Truth or Falſhood of a Propoſition; he plainly ſhews they muſt aſſent to Agreements, which their Judgments neceſſarily*

ceſſarily acquieſce in ; and whoever reaſons otherwiſe, does not think worthily of God.

To return to Phyſics : *We are in this Science to judge by our Senſes, of the Agreement that there is betwixt Things and our Ideas. The Extenſion and Solidity of Matter, for Inſtance, aſſerted upon that Ground, are paſt all Doubt. Here we examine the Thing in general, without taking notice of the Fallacy of our Senſes upon ſome Occaſions, and which Way Error is then to be avoided.*

We cannot immediately judge of all Phyſical Matters by our Senſes. We have then Recourſe to another juſt Way of Reaſoning, though not Mathematical. It depends upon this Axiom ; (viz.) We muſt look upon as true, whatever being denied would deſtroy civil Society, and deprive us of the Means of Living. *From which Propoſition the ſecond and third Rules of the* Newtonian *Method moſt evidently follow.*

For who could live a Minute's Time in Tranquillity, if a Man was to doubt the Truth of what paſſes for certain, whatever Experiments have been made about it ; and if he did not depend upon ſeeing the like Effects produced by the ſame Cauſe ?

The following Reaſonings, for Example, are daily taken for granted as undoubtedly, without any previous Examination ; becauſe every Body ſees that they cannot be called in Queſtion without deſtroying the preſent Oeconomy of Nature.

A Building, this Day firm in all its Parts, will not of its ſelf run to Ruin to Morrow. *Thus, by a Parity of Reaſon, the Coheſion of Gravity of the Parts of Bodies, which I never ſaw altered, nor heard of having been altered, without ſome intervening external Cauſe, will not be altered to Night, becauſe the Cauſe of Coheſion and Gravity will be the ſame to Morrow as it is to Day. Who does not ſee, that the Certainty of this Reaſoning de-*

pends only upon the Truth of the fore-mentioned Principle?

The Timber and Stones of any Country, which are fit for a Building, if brought over here, will ſerve in this Place, except what Changes may ariſe from an external Cauſe; and I ſhall no more fear the Fall of my Building, than the Inhabitants of the Country, from whence thoſe Materials were brought, would do, if they had built a Houſe with them. *Thus the Power which cauſes the Coheſion of Parts, and that which gives Weight to Bodies, are the ſame in all Countries.*

I have uſed ſuch Kind of Food for ſo many Years, therefore I will uſe it again to Day without Fear.

When I ſee Hemlock, I conclude it to be poiſonous, tho' I never made an Experiment of that very Hemlock I ſee before my Eyes.

All theſe Reaſonings are grounded upon Analogy; and there is no Doubt, but our Creator has in many Caſes left us no other Way of Reaſoning, and therefore it is a right Way.

Which being once proved, we may afterwards make uſe of the ſame Method in other Matters where no abſolute Neceſſity forces us to reaſon at all. When an Argument is good in one Caſe, there is no Reaſon why we ſhould refuſe our Aſſent to it in another. For who can conceive, that Things proved the ſame Way are not equally certain? Beſides, tho' we conclude in general, that this Method of Reaſoning is right, from the Neceſſity of uſing it; yet it does not follow that particular Reaſonings depend upon that Neceſſity. I conclude from Analogy, that Food is not poiſonous; but is that Argument only good, when I am hungry.

In Phyſics then we are to diſcover the Laws of Nature by the Phænomena, then by Induction prove them to be general Laws; all the reſt is to be handled

dled mathematically. Whoever will ſeriouſly examine what Foundation this Method of Phyſics is built upon, will eaſily diſcover this to be the only true one, and that all Hypotheſes are to be laid aſide.

So much for the Method of Philoſophiſing. I have now a Word to ſay of the Work itſelf, of which this is the firſt Tome.

The whole Work is divided into four Books. The firſt treats of Body in general, and the Motion of Solid Bodies. The Second of Fluids. What belongs to Light is handled in the Third. The fourth explains the Motions of Celeſtial Bodies, and what has a Relation to them upon Earth. The two firſt Books are contained in this Tome.

In order to render the Study of Natural Philoſophy as eaſy and agreeable as poſſible, I have thought fit to illuſtrate every thing by Experiments, and to ſet the very Mathematical Concluſions before the Reader's Eyes by this Method.

He that ſets forth the Elements of a Science, does not promiſe the learned World any thing new in the main. Therefore I thought it needleſs, to point out where what is here contained is to be found. I have made my Property of whatever ſerved my Purpoſe; and I thought giving Notice of it once for all, was ſufficient to avoid the Suſpicion of Theft. I had rather loſe the Honour of a few Diſcoveries, diſperſed here and there in this Treatiſe, than rob any one of theirs. Let who will then take to himſelf what he thinks his own: I lay claim to nothing.

As to Machines which ſerve for making the Experiments, I have taken Care to imitate ſeveral from other Authors, have altered and improved others, and added many new ones of my own Invention. And no wonder I ſhould be forced to that Neceſſity, having made Experiments upon many

Things never tried perhaps by any one before. For Mathematicians think Experiments ſuperfluous, where Mathematical Demonſtrations will take Place: But as all Mathematical Demonſtrations are abſtracted, I do not queſtion their becoming eaſier, when Experiments ſet forth the Concluſions before our Eyes; following therein the Example of the Engliſh, *whoſe Way of teaching Natural Philoſophy gave me occaſion to think of the Method I have followed in this Work. I ſhall always glory in treading in their Footſteps, who with the Prince of Philoſophers for their Guide, have firſt opened the Way to the Diſcovery of Truth in Philoſophical Matters.*

As to the Machines, I will ſay thus much more by Way of Advertiſement, That moſt of them have been made by a very ingenious Artiſt of this Town, and no unskilful Philoſopher, whoſe Name is John van Muſſchenbroek, *and who has a perfect Knowledge of every thing that is here explained. Which Advertiſement, I ſuppoſe, will not be diſpleaſing to thoſe who may have a Fancy to get ſome of thoſe Machines made for themſelves.*

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BOOK

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MATHEMATICAL ELEMENTS OF *Natural Philoſophy*, Confirmed by EXPERIMENTS.

BOOK I.

PART I. *Of BODY in General.*

CHAP. I.

Of the Scope of Natural Philoſophy, and the Rules of Philoſophizing.

NATURAL PHILOSOPHY treats of *Natural Things*, and their *Phænomena*.

DEFINITION I, and II.

Natural Things *are all Bodies: And the Aſſemblage or Syſtem of them all, is called the* Univerſe.

Definition III.

2 Natural Phænomena are *all Situations and all Motions of Natural Bodies, not immediately depending upon the Action of an intelligent Being*; *and which may be obſerved by our Senſes.*

We do not exclude out of the Number of Natural Phænomena, thoſe which happen in our Bodies by our Will; for they are produced by the Motion of our Muſcles, and their Action depends upon another Motion: In theſe, there is only that Motion which ariſes from the immediate Action of the Mind, and is entirely unknown to us, which is not a Natural Phænomenon.

All theſe Motions are performed by certain Rules, and always ſubject to the ſame Laws.

The Sun riſes and ſets daily; and the Time of his Riſing and Setting may always be determined, according to the Time of the Year, and Latitude of the Place. Plants of the ſame Kind, under the ſame Circumſtances, are always produced and grow in the ſame Manner: And ſo on in other Caſes. Nay, even in thoſe things which appear to be wholly fortuitous and uncertain, certain Rules are without doubt obſerved.

3 Natural Philoſophy *explains Natural Phænomena*; that is, gives an Account of their Cauſes.

In enquiring after thoſe Cauſes, BODY in general is to be examined; and then the Rules which the Creator has eſtabliſhed, according to which, Motions are to be perform'd: Theſe *Rules* are called *Laws of Nature.*

Definition IV.

4 A Law of Nature then is, *the Rule and Law, according to which God reſolved that certain Motions ſhould always, that is, in all Caſes, be performed.*

Every Law does immediately depend upon the Will of God.

Alſo in reſpect to us, we call a Law of Nature, every Effect which in all Occaſions is produced after the ſame Manner, although its Cauſe is unknown to us, and we do not ſee that it flows from any Law known to us.

For we make no Difference between a thing which immediately depends upon the Will of God, and what it produces by the Intermediation of a Cauſe of which we have no Idea.

The Laws of Nature are only deduced from an Examination of Natural Phænomena.

By Help of the Laws, thus diſcovered, other Phænomena muſt be explained.

In order to find out the Laws of Nature, Sir *Iſaac Newton*'s following Rules are to be obſerved.

RULE I.

We are not to admit more Cauſes of Natural Things than ſuch as are true, and ſufficient for explaining their Phænomena. 5

RULE II.

Natural Effects of the ſame Kind have the ſame Cauſes. 6

Such Qualities of Bodies, whoſe Virtue cannot be increaſed and diminiſhed, and which belong to all Bodies upon which Experiments may be made, muſt be looked upon as Qualities of all Bodies. 7

CHAP. II.

Of BODY in General.

8 WHAT we firſt conſider in Body is *Extenſion.*

What is meant by Extenſion, no Body is ignorant of. Its Idea is moſt ſimple, and almoſt always obvious to our Mind; from whence it is very intelligible, tho' we want Words to deſcribe it.

Every Body has Extenſion; without Extenſion there is no ſuch thing as Body. And yet all that has Extenſion is not always a Body; although it is impoſſible to determine, how Body differs from mere Space, till the other Properties of Body ſhall firſt be aſcertained.

9 The Second Thing to be examined in Body is *Solidity.* Body, having no Power to remove itſelf, will conſequently exclude every other Body from the Place poſſeſſed by it; and the moſt fluid, as well as the hardeſt Bodies, have this Property.

10 The Third Property of Body is *Diviſibility*; becauſe if a Body be extended, it is alſo diviſible; for you may always conceive one Extenſion leſs than another. From whence we ſee, that there are Parts in all Extenſions; which Parts in a Body may be ſeparated from each other; Becauſe,

11 Body hath a Fourth Property, that is, that it may be carried from one Place to another; whence it is ſaid to be *Moveable.*

All Obſtacles being removed, a Body yields to the leaſt Blow: Nevertheleſs there is a greater Force required to move a Body with a greater Celerity than with a leſs, as alſo to move a greater than a ſmaller Body, allowing their Velocity to be equal. There is alſo a greater Force required in the ſame Caſe to ſtop the Velocity of different Bodies in Motion. Hence it is, that Bodies at Reſt, and Bodies in Motion, endeavour to continue in their State.

This ariſes from the Inactivity of Matter (*Inertia*) which in all Bodies is ever proportionable to their Quantity of Matter, becauſe it equally belongs to all the Particles of Matter.

All Bodies have ſome Figure; whence *Figurability* (that is, to be of ſome Shape or Figure) is commonly eſteemed one of the eſſential Properties of Body, though it ſeems rather to be derived from other Properties. 12

If a Body be divided on every Side, and thoſe Parts removed; what remains in the Middle is terminated on every Part, and conſequently has a certain Figure. The ſame Body is capable of having different Figures; becauſe it may be divided into Parts, and thoſe Parts placed in different Order in reſpect to each other. Neither does it imply a Contradiction to ſay, that a Body, that ſhould have no Figure, would be an infinite Body.

CHAP. III.

Of Extenſion, Solidity, and Vacuity.

HERE the Queſtion (ſo often handled by the Learned) concerning a *Vacuum*, is to be conſidered; namely, Whether there be an Extenſion void of all Matter; for this Extenſion is called a *Vacuum*, an *Emptineſs*, or *mere Space*. 13

That there is really a Vacuum, is proved from Phænomena: This Propoſition therefore ſhall be hereafter more fully treated of.

The Poſſibility of a Vacuum appears from the bare Examination of Ideas: For whatever we conceive to be poſſible may exiſt.

The Queſtion therefore amounts to this, *viz.* Whether we have an Idea of an Extenſion that is not ſolid?

We acquire an Idea of Solidity by the Touch: We feel that ſome Bodies reſiſt us; and indeed thoſe Bodies reſiſt us every Moment, that hinder us from deſcending to the loweſt Places: From which Reſiſtance it appears, that a Body excludes every other Body from the Place which itſelf takes up; that is, it appears that a Body is ſolid; which Idea of Solidity we transfer to thoſe more ſubtile Bodies, which, by reaſon of the Smallneſs of their Parts, eſcape our Senſes; and we find by Experience, that even thoſe reſiſt other Bodies, as well as the hardeſt.

14 *Experiment.*] The Air, in which we live, does almoſt always eſcape our Sight and Touch; yet in a Syringe, that is cloſe ſhut at the End, it reſiſts the Piſton, ſo that it can be puſh'd to the Bottom of the Syringe by no Force.

The Idea of Solidity is not indeed contained in the Idea of Extenſion; that only follows from Contact, but this may be had without it; for if a Man had never touch'd a Body, he would have no Notion of Solidity.

Let any one obſerve an Image projected in the Air, or repreſented between a Concave Mirror and the Object; ſuch an Image does not reſiſt, and yet it ſeems to be a Body as denſe as the Object itſelf; for the Colours may appear more vivid in the Image than in the Object itſelf: If a Man had never ſeen any thing elſe but ſuch Images, and his own Body was like ſuch an Image, could he have any Idea of Solidity? It does not appear that he could; and yet he would certainly have an Idea of Extenſion.

As we are here diſputing of Ideas, we ſhall not conſider what the above-mentioned Image is; it is enough that there is ſuch a Thing.

15 All the Difference between *Space* and *Body* does not conſiſt in a Privation of Solidity.

That Space is infinite, and can be contained by no Limits, is plain to any one that attentively conſiders it.

We plainly ſee that Space has Parts, but they cannot be ſeparated from one another, being immoveable as Space itſelf.

The Idea of Space is very ſimple; that of Body is more complex; it may be moved, its Parts are ſeparated, and what is finite is eaſily conceived.

16 *Solidity* is by ſome call'd *Impenetrability*; and they endeavour to deduce it from the Nature of Extenſion: For Example, one cannot add one cubic Foot of Extenſion to another cubic Foot of Extenſion, without having two cubic Feet; for each of them has all that is required to conſtitute that Magnitude: therefore one Part of Space excludes all others, and cannot admit them.

Anſwer. This is all true, becauſe the Parts of Space are immoveable; but it would be falſe, if it was not that it would imply a Contradiction, to ſuppoſe one Part of Space conveyed to another Place: And the Conſequence follows only from the Immobility, not from the Impenetrability or Solidity of the Parts of Space.

CHAP. IV.

Of the Diviſibility of Body in infinitum; *and of the Subtilty of the Particles of Matter.*

17 THE Extenſion of a Body implies its Diviſiblity; that is, one may conſider Parts in it.

Yet the Diviſibility of Body differs from the Diviſibility of Extenſion; for its Parts may be ſeparated from one another. But as this Property depends upon Extenſion, it muſt be examined under the Conſideration of Extenſion. And then we may eaſily transfer to Body what is demonſtrated.

18 Body is diviſible *in infinitum*; that is, you cannot conceive any Part of its Extenſion ever ſo ſmall, but ſtill there may be a ſmaller.

Let there be a Line A D perpendicular to B F, (*Plate* I. *Fig.* 1.) and another as G H at a ſmall Diſtance from A, alſo perpendicular to the ſame Line, with the Centers C, C, C, *&c.* and Diſtances C A, C A, *&c.* deſcribe Circles cutting the Line G H in the Points *e*, *e*, *&c.* The greater the Radius A C is, the leſs is the Part *e* G; the Radius may be augmented *in infinitum*, and therefore the Part *e* G may be diminiſhed in the ſame Manner; and yet it can never be reduced to Nothing, becauſe the Circle can never coincide with the Right Line B F.

Therefore the Parts of any Magnitude may be diminiſh'd *in infinitum*, and there is no End of ſuch a Diviſion.

The ſame Thing may be proved by a great many other Mathematical Demonſtrations.

19 The chief Objections are,—That an Infinite cannot be contained by a Finite:——That it follows from a Diviſibility *in infinitum*, that all Bodies are equal, or, that one Infinite is greater than another.

But theſe are eaſily anſwer'd; for to an Infinite may be attributed the Properties of a finite and determined Quantity. Who has ever proved that there could not be an infinite Number of Parts infinitely ſmall in a finite Quantity; or that all Infinites are equal? The contrary is demon-

ſtrated

ſtrated by Mathematicians in innumerable Inſtances.

There are alſo other Objections propoſed, ſuppoſing that we affirm an actual Diviſion of a Body into an Infinite Number of Parts ſeparated from one another. We neither defend nor conceive ſuch a Diviſion: We have demonſtrated, that however ſmall a Body is, it may ſtill be farther divided; and, upon that Account, we believe that we may call that a Diviſion *in infinitum*, becauſe what has no Limits is call'd *infinite*.

There are no ſuch Things as Parts infinitely ſmall; but yet the Subtilty of the Particles of ſeveral Bodies is ſuch, that they very much ſurpaſs our Conception; and there are innumerable Inſtances in Nature of ſuch Parts that are actually ſeparated from one another. 20

Mr. *Boyle* has proved it by ſeveral Arguments. He ſpeaks of a ſilken Thread 300 Yards long; that weighed but two Grains and a Half. 21

He meaſured Leaf-Gold, and found by weighing it, that 50 ſquare Inches weighed but one Grain. If the Length of an Inch be divided into 200 Parts, the Eye may diſtinguiſh them all; therefore there are in one ſquare Inch 40000 viſible Parts; and in one Grain of Gold there are two Millions of ſuch Parts; which viſible Parts no one will deny to be farther diviſible. 22

A whole-Ounce of Silver may be gilt with eight Grains of Gold, which is afterwards drawn into a Wire thirteen thouſand Foot long. 23

In odoriferous Bodies we can ſtill perceive a greater Subtilty of Parts, and which are ſeparated from one another; ſeveral Bodies ſcarce loſe any ſenſible Part of their Weight in a long Time, and yet continually fill a very large Space with odoriferous Particles. Whoever will be at the Pains to make Calculations concerning thoſe ſubtile 24 tile *Effluvia*, will find the Number of Parts to be amazing.

25 By Help of Microſcopes, ſuch Objects as would otherwiſe eſcape our Sight, appear very large. There are ſome ſmall Animals ſcarce viſible with the beſt Microſcopes; and yet theſe have all the Parts neceſſary for Life, Blood, and other Liquors: How wonderful muſt the Subtilty of thoſe Particles be, which make up ſuch Fluids!

We cannot end this Chapter more aptly than by the following *Theorem*, which is eaſily deduced from what has been ſaid of the Subtilty of Matter.

THEOREM.

26 ANY Particle of Matter, how ſmall ſoever, and any finite Space, how large ſoever, being given; it is poſſible that that ſmall Sand, or Particle of Matter, ſhall be diffuſed thro' all that great Space, and ſhall fill it in ſuch Manner, that there ſhall be no Pore in it, whoſe Diameter ſhall exceed a given Line.

CHAP. V.

Concerning the Coheſion of Parts; where we ſhall treat of Hardneſs, Softneſs, Fluidity, and Elaſticity.

27 ALL Bodies, that are perceived by our Senſes, conſiſt of very ſmall Parts, no one of which is indiviſible in itſelf; but all of them are in reſpect to us: For all the Diviſion we can make, is only a Separation of Parts.

When a great Force is required to make ſuch a Diviſion, a Body is ſaid to be *Hard*.

If the Parts yield more eaſily, and fall in by being preſs'd, ſuch a Body is ſaid to be *Soft*.

But this great and leſſer Force, in the common Signification, determine nothing; for a Body that is Hard, in reſpect to one Man, ſeems Soft to another.

DEFINITION I.

A Body is ſaid to be *Hard*, in a Philoſophical Senſe, *when its Parts mutually cohere, and do not at all yield inwards, ſo as not to be ſubject to any Motion in reſpect to each other, without breaking the Body.* 28

DEFINITION II.

A Body is ſaid to be *Soft*, in a Philoſophical Senſe, *when its Parts yield inwards, and ſlip in upon one another, even tho' it may require a Blow with a Hammer to do it.* 29

DEFINITION III.

A Body whoſe Parts yield to any Impreſſion, and by yielding are eaſily moved, in reſpect to each other, is call'd a Fluid. 30

All theſe Things depend upon the Coheſion of Parts; the cloſer a Body is, the nearer it approaches to perfect Hardneſs.

But the Hardneſs of the ſmalleſt Parts does not differ from their Solidity; it is an eſſential Property of a Body, which is no more to be explained, than why a Body is extended, or a Mind thinks.

I do not know whether all Bodies conſiſt of Parts that are equal and alike: And there are alſo ſeveral Things very difficult, in Relation to the Cauſe of the Coheſion of the ſmall Parts of Bodies.

The Laws of Nature, which are admitted here, are deduced from *Phænomena.*

It is a particular Law of Coheſion, that all the Parts have an *attractive Force.* 31

DEFINITION IV.

By the Word *Attraction* I underſtand, *any Force by which two Bodies tend toward each other*; tho' perhaps it may happen by Impulſe.

But that *Attraction* is ſubject to theſe Laws; *That it is very great, in the very Contact of the Parts; and that it ſuddenly decreaſes, inſomuch that it acts no more at the leaſt ſenſible Diſtance; nay, at a greater Diſtance it is changed into a* repellent Force, *by which the Particles fly from each other.*

By Help of this Law, ſeveral Phænomena are very eaſily explained; and that Attraction and Repulſion is fully proved by a vaſt Number of chymical Experiments. That there is ſuch a Thing, appears alſo from the following Experiments.

32 *Experiment* 1.] WE ſee that in all Liquors all the Parts attract one another, from the Spherical Figure that the Drops always have; and alſo becauſe there is no Liquor whoſe Parts are not ſticking to one another, which is evidently true even in *Mercury* itſelf.

33 *Experiment* 2.] BUT this mutual Attraction of Particles is much better proved; becauſe in all Liquids, two Drops, as A and B (*Plate* I. *Fig.* 2.) as ſoon as they touch one another ever ſo little, they immediately run into one larger Drop, as F. All which Things, as they alſo happen in liquified Metals, it follows, that the Parts of which they are compounded do attract one another, even when they are disjoined by the Motion of the Fire.

Theſe Appearances do not depend upon the Preſſure of the Air, becauſe they alſo happen where there is no Air; neither do they depend upon the Preſſure of any other Matter equally from all Sides; for though ſuch a Preſſure is able to keep the

Drops to their Spherical Figure, it can by no means bring them to it at firſt.

In the Oval Drop *a c b d*, (*Plate* I. *Fig.* 3.) the Preſſures upon the Surfaces *a d* and *c b* are leſs than the Preſſures upon the Surfaces *a c*, *db*; for the Drop is ſuppos'd to be preſs'd equally from all Parts; therefore the Preſſure is leſs in a leſs Space: Yet the Drop can never become round, till thoſe leſſer Preſſures overcome the greater, which is abſurd. 34

On the contrary, in Attraction, the greater the Number is of the Particles which attract one another between two Particles, the greater is the Force with which they are carried towards one another; which produces a Motion in the Drop, till the Diſtances between the oppoſite Points in the Surface become every where equal; which can only happen in a Spherical Figure.

Several Bodies act upon other extraneous Bodies by this Attraction. I ſhall give a few Examples, in which the Effects of it are moſt remarkable.

Experiment 3.] Immerge in Water the Ends of ſmall Glaſs Tubes open at both Ends, in the Manner repreſented in *Plate* I. *Fig.* 4. The Water will ſpontaneouſly aſcend in them, and ſo much the higher as the Diameter is leſs. It is not required that the Tubes be extremely ſmall; for the Experiments will ſucceed in Tubes whoſe Bore is the ſixth Part of an Inch. That this is not to be attributed to the Preſſure of the Air, appears from the following Experiment. 35

Experiment 4.] Having fixed the ſmall Tubes A to a Piece of Cork, and ſuſpended them with the Braſs Wire AB, (*Plate* I. *Fig.* 5.) pump out the Air from the Recipient R, which ſtands upon the Braſs Plate of the Air-Pump; then by moving the Wire AB, 36

AB, the Tubes may be immerſed in the Water which is contained in the Glaſs CD, and the Water in this Caſe will riſe up into the Tube juſt as it did in the foregoing Experiment. How the Wire may be moved, without letting the Air into the Recipient, will be explained hereafter.

37 *Experiment* 5.] ABCD are two Glaſs Plates, or Planes, (*Plate* I. *Fig.* 6.) touching one another at AB, but a little ſeparated at CD, by thruſting a thin Plate of any Kind between them: they are ſuſtained by the wooden Frame HILM, in ſuch Manner, that the Side DC is always at the ſame Height; the Planes may be brought to make any Angle with the Horizon, by raiſing the End AB where they are joined, the Cylinder NO likewiſe ſuſtaining the Plane in any Poſition. The Screw P makes faſt the Cylinder at any Height.

A Drop of Water or Oil, G, is put between the Planes, ſo as to touch both the Planes, which muſt before-hand be made wet with the ſame Liquor: this Drop is attracted by both Planes, but the Attraction has a greater Effect upon the Drop, where their Diſtance is the leſs: that is, a greater at *e* than at *f*; therefore the Drop is moved towards *e*; that is, aſcends, and moves upwards the faſter, in Proportion as it is higher, the Surfaces in which the Drop touches the Glaſſes growing very much where the Diſtances between the Planes is diminiſhed. The Angle of Inclination of the Planes may be ſo increaſed, that the Gravity of the Drop ſhall ballance the Attraction, and then the Drop is at reſt; and in that Caſe, if you raiſe the End AB of the Planes ſtill higher, the Drop will deſcend by its Gravity, which will then overcome the Attraction.

Experiment 6.] Let two Glaſs Planes, ABCD, (*Plate* I. *Fig.* 7.) touch one another at AB, and at CD let them be a little ſeparated by the Interpoſition of ſome thin Plate; and then let their Ends be immers'd into Water tinged with ſome Colour, in ſuch Manner, that the Sides AB and CD may be in a vertical Poſition, the Planes having been moiſtened with the ſame Liquor beforehand: The Water will riſe between thoſe Planes by their Attraction, and riſe higheſt where the Planes are neareſt together; and as their Diſtance continually decreaſes from CD to AB, the Water riſes up to different Heights in every Place, and makes the Curve Line *efg*. 38

Experiment 7.] Quickſilver unites itſelf to Tin and Gold; alſo Water and Oil ſtick to Wood and clean Glaſs. 39

We have Inſtances of Repulſion between Water and Oil, and generally between Water and all unctuous Bodies; between *Mercury* and Iron; as alſo between the Particles of any Duſt. 40

Experiment 8.] If any greaſed Body, lighter than Water, is laid upon the Surface of Water, or Piece of Iron upon *Mercury*, the Surface of the Fluid will be depreſſed about the Bodies laid upon it, as it appears about the Ball A (*Plate* I. *Fig.* 8.) And after the ſame Manner, where the Attraction obtains, the Surface of the Liquor is higher about the floating Bodies, as about the Ball B, and does not run to a Level by its Gravity: So here where the repellant Force exerts it Action, Liquors, notwithſtanding their Gravity, do not run down to fill up the Cavities which are made round the floating Bodies. 41

Upon this depend all the Phænomena of very light Glaſs Bubbles (*Plate* I. *Fig* 18.) which ſwim upon 42 upon Water; when they are clean, the Water riſes about them all round, as at B; but, when they are made greaſy, the Water makes an Hollow all round them, as at A: in the Glaſs Veſſel where the Experiments are made, the Water alſo ſtands higher all round next to the Glaſs, as at C and D: but when the Glaſs is ſo fill'd that the Water runs down from all Parts, then, by the mutual Attraction of the Parts of the Water, it ſtands higher in the Middle than at the Sides, and forms the convex Surface ABC (*Plate* I. *Fig.* 9.): From theſe Principles only can the following Experiments be explain'd.

Experiment 9, 10, 11, 12 and 13.] When a Glaſs is not quite full of Water, a clean Glaſs Bubble always runs to the Side, and there ſticks, provided it be not laid on too far from it. The Bubble is preſs'd every way by the Water, when it comes to the Side of the Veſſel; the ſame Force, by which the Water is raiſed there, does in part take off that Preſſure; ſo the Preſſure on the other Side overcomes, and the Bubble is moved towards the Side of the Glaſs.

When the Glaſs is ſo full as to be ready to run over, the Bubble goes off it ſelf from the Side to the Middle of the Glaſs, for the ſame Reaſon; becauſe the Force, by which the Water is raiſed in the Middle, does alſo diminiſh the Preſſure upon the Bubble towards the Middle.

Juſt the Reverſe happens when the Bubble is greaſy; becauſe that Force, by which the Water and the Bubble repel one another, is greateſt where the Water is higheſt.

Two clean Bubbles, or two greaſy ones, run towards each other. As to clean Bubbles, we have juſt given the Reaſon: when they are made greaſy, there is a Cavity round each of them;

and where the Cavities join, the Preſſure is diminiſh'd, and the Bubbles run that Way.

If one Bubble be clean, and the other greaſy, they fly from one another, for the Reaſons before given.

43 The Particles of any Salts attract one onother with a very great Force, as appears by ſeveral Experiments: The following will be ſufficient to prove, that that Attraction exerts it ſelf at a very ſmall Diſtance, and the repellent Force at a greater.

Experiment 14.] Diſſolve Salt in Water, and, when that Water is reduced into Vapour, the ſmall ſaline Particles will unite together and form greater Lumps; which proves the Attraction.

Theſe Particles are all equal, and of the ſame Figure: Whence it follows, that the leaſt Parts, of which they are form'd, had every where the ſame Situation in reſpect to each other; that is, were every where diffuſed in the Water at equal Diſtances; which cannot be, unleſs they all repel one another with equal Forces.

44 *The Elaſticity of Bodies*, namely, that Property *whereby they return to their former Figure, when it has been alter'd by any Force, is eaſily deduced from what has been ſaid:* For if a compact Body be dented in without the Parts falling into that Dent, the Body will return to its former Figure, from the mutual Attraction of its Parts.

45 We ſhall alſo in its proper Place ſhew, that that Property of the Air, which is call'd its *Elaſticity*, ariſes from the Force whereby its Parts repel one another.

46 And leſt any one ſhould imagine, becauſe we don't give the Cauſe of the ſaid Attraction and

Repulſion, that they muſt be look'd upon as *occult Qualities* : We ſay here with Sir *Iſaac Newton*, ‘That we conſider thoſe Principles not as occult Qualities, which are ſuppoſed to ariſe from the ſpecifick Forms of things ; but as univerſal Laws of Nature, by which the things themſelves are form'd : For the Phænomena of Nature ſhew that there are really ſuch Principles, tho' it has not been yet explain'd what their Cauſes are. To affirm that the ſeveral Species of Things have occult ſpecifick Qualities, by which they act with a certain Force, is juſt ſaying nothing. But from two or three Phænomena of Nature to deduce general Principles of Motion, and then explain in what Manner the Properties and Actions of all things follow from thoſe Principles, would be a great Progreſs made in Philoſophy, tho' the Cauſes of thoſe Principles ſhould not yet be known.

PART II.

Of the Motion of Solid Bodies.

CHAP. VI.

Of Motion *in general; where we ſhall ſpeak of* Place *and* Time.

THE Subject we are now entring upon has a large Scope in Phyſicks: All that happens in natural Bodies belongs to Motion; and even what has been ſaid of the Coheſion of Parts, has a Relation to it: For though the Parts are not moved in their Coheſion itſelf, yet that Coheſion cannot be explain'd, nor can what is ſaid about it be confirm'd by Experiments, without Motion.

Motion *is a Tranſlation from one Place to another Place, or a continual Change of Place:* Every Body has an Idea of it; and Philoſophers have in vain laboured to find a Definition of the ſimple Idea, and proved with a great deal of Pains, that one may come to be ignorant of a Thing, which otherwiſe every Body knows. 47

Place *is the Space taken up by a Body*; of which may be ſaid, what has juſt been ſaid concerning *Motion*. 48

It is Twofold; *True* or *Abſolute*, and *Relative*.

DEFINITION I.

True Place is that Part of immoveable Space, which a Body takes up. 49

DEFINITION II.

50 Relative Place, which only can be diſtinguiſh'd by our Senſes, *is the Situation of a Body in reſpect of other Bodies.*

True Place is often changed, whilſt relative Place is not, and ſo reciprocally.

Whence ariſes a *True* and *Abſolute* Motion, and another Sort call'd a *Relative* Motion.

Whilſt a Body moves, Time goes on.

51 Time alſo is Twofold; *True* or *Abſolute*, and *Relative.*

True Time has no Relation to the Motion of Bodies, nor to the Succeſſion of Ideas in an Intelligent Being, but by its Nature it always flows equally.

DEFINITION III.

52 Relative Time *is Part of the true Time meaſured by the Motion of Bodies*; and this is the only Time that our Senſes perceive.

All Motion may become ſwifter, as likewiſe a Body may move ſlower than it did before; and it is very likely that there is no Motion of Bodies wholly equable; whence it follows, that *relative* Time differs from *true* Time, which never flows faſter nor ſlower.

DEFINITION IV.

53 *That Affection of Motion, by which a Body runs thro' a certain Space in a certain Time, is call'd* Celerity *or* Velocity; which is greater or leſs, according to the *Bigneſs* of that *Space, to which it is always proportionable.*

DEFINITION V.

54 The greater Force is impreſs'd upon a Body to make it change its Place, the greater is its

Motion; and that Force is called the *Quantity* or *Momentum of Motion.*

DEFINITION VI.

The Direction of Motion *is in a Right Line, which we ſuppoſe drawn towards the Place where the moving Body tends.* 55

DEFINITION VII.

A Power *is any Force acting upon a Body to move it.* 56

DEFINITION VIII.

The Greatneſs of that Force is call'd, the *Intenſity of the Power.* 57

CHAP VII.

Of Motions compared together.

AXIOMS.

I.

THE Quantity, *or* Momentum of Motion, *follows the Proportion of the Cauſe producing the Motion.* 58

II.

The whole Effects of Motions, produced at the ſame Time, have the ſame Relation to each other, as the Momenta *of thoſe Motions.* 59

If two equal Motions act in a contrary Direction, they deſtroy each other; and the one can never overcome the other. 60

Bodies in Motion may differ in two Reſpects, either in Reſpect to the Quantity of Matter in each, or in Reſpect of the Space gone thro' in the ſame Time, that is, in Reſpect of the Velocity, * and there is no other Difference: Theſe two 61 *53.

two Things therefore, and only theſe two, are to be conſidered in comparing of Motions.

When the Velocities are equal, nothing is to be conſidered but the Quantities of Matter; and if it be double in one Body, the Quantity of Motion in that Body will alſo be double; becauſe ſuch a Body is made up of two Bodies, each equal to the leaſt Body, and moved with the ſame Celerity as the little one. The ſame may be ſaid of all other Relations between two Bodies; whence we deduce this general Rule:

62 *In equal Bodies, that move with the ſame Velocity, the Quantity of Motion is as the Quantity of Matter in each.*

When the Quantities of Matter are equal, the Velocities only are to be conſidered: And then,

63 *In equal Bodies, the* Momenta *are as the Velocities:* That is, as the Spaces gone thro' in the ſame Time*. For the going thro' thoſe Spaces are the whole Effects of the Motions produced in that Time, and are to one another as thoſe Spaces; therefore the *Momenta* alſo are in the ſame Proportion*.

*53.

*59.

In order to determine the Relation between two

64 Motions, *when the Velocities are unequal, and the Bodies different in Quantity of Matter; you muſt find two Quantities that are to one another as the Maſſes and as the Velocities. Multiplying the Velocities of each Body by its Maſs or Quantity of Matter, the Products will be to each other in the ſaid Proportion.*

When, for Example, the Velocity is double, and the Maſs triple, a double Quantity of Motion muſt be tripled; therefore it will be ſextuple: This is the Caſe when in one Body the Velocity is two, and four in another; and the Maſs of the firſt Five, the Maſs of the other being Fifteen; multiplying each Maſs by its Velocity, the Products are 10 and 60, the laſt of which is ſix times the firſt.

This is called a Ratio, compounded of the Ratio of the Maſſes and the Celerities.

A greater Body may move more ſlowly than a leſs, in ſuch a Manner, that the leſſer Body may have an equal Quantity of Motion with, or a greater than the other.

When the Velocity in the leſſer Body is to the Velocity in the greater, as the Maſs of the greater to the Maſs of the leſſer; the Quantities of Motion are equal in the two Bodies. 65

As much as the Quantity of Motion in the leſſer Body is leſs, in reſpect of its Maſs, ſo much is it greater, in reſpect of the Velocity: Whence an Equality ariſes. Likewiſe in that Caſe, the Products of the Maſs of each Body by its Velocity are equal; and the Celerities are ſaid to be in an inverſe Ratio of the Maſſes, or reciprocally as the Maſſes.

When ſuch Momenta *of Motion act contrariwiſe, they deſtroy each other**. 66 *60.

CHAP. VIII.

How to compare the Actions of Powers.

THE Actions of Powers, acting upon Bodies, may be compared together, in the ſame Manner as the Quantities of Motion; and the ſame Rules ſerve for both.

We ſhall hereafter ſhew, that a Body once in Motion will continue in that Motion, tho' the Cauſe that firſt gave it, ceaſeth: So that if a Body ſhould be continually acted upon by any Power, the Motion would become ſwifter every Moment.

We do not here conſider ſuch an Action of Powers; but we take notice only of Powers that act againſt an Obſtacle in ſuch Manner, that by the Reſiſtance of the Obſtacle the Action of the Power is continually deſtroyed, which is to be obſerved; for in another Caſe, the following Demonſtrations do not obtain. When therefore we ſpeak here of an Obſtacle to be removed by any Power, we ſpeak of the *greateſt Obſtacle* that can be moved by that Power; for otherwiſe the Obſtacle would not deſtroy the whole Action of the Power.

The Actions of Powers may differ from one another, both in reſpect of the Greatneſs of the Obſtacles, and in reſpect of the Spaces run thro' by the Obſtacles; (that is, by the Points to which the Powers are applied:) Theſe two Things alone are to be conſidered in comparing of Powers.

The Obſtacles, which may be removed by Powers, are to one another as the Intenſities of
57. the Powers,

67 From whence it follows, *that the Actions of Powers, equal in their Intenſities, are to one another as the Spaces run thro'*. For they only differ in that reſpect, becauſe the Obſtacles are equal.

68 *When the Spaces run thro' are equal, thoſe Actions are as the Intenſities.*

69 *When both the Spaces run thro', and the Intenſities, are different, the Actions of the Powers are to one another, in a Ratio compounded of the Intenſities and the Spaces gone through.*

70 When the Spaces gone thro' are in an inverſe Ratio of the Intenſities, the Actions are equal.

DEFINITION.

71 We call the *whole Force of Power* its *Action, in Reſpect to Time*; and therefore the whole Forces

of Powers are to one another, as the Actions produced in the ſame Time.

Theſe Things may be demonſtrated in the ſame Manner, as what has been ſaid in the foregoing Chapter.

CHAP. IX.

General Things concerning Gravity.

PHENOMENON I.

ALL Bodies near the Earth, if hinder'd by no Obſtacle, are carried towards the Earth. 72

DEFINITION I.

The Force, by which Bodies are carried towards the Earth, is called Gravity. 73

DEFINITION II.

That Force in Reſpect to a Body acted upon by it, is call'd the Weight *of a Body.* 74

PHENOMENON II.

The Force of Gravity acts equally, and every Moment of Time, near the Earth's Surface. 75

There is indeed a ſmall Difference of Gravity in different Countries, which we ſhall take Notice of hereafter; but it is too ſmall to be conſidered here, eſpecially becauſe it is wholly inſenſible in neighbouring Countries.

76 When the Deſcent of a Body is hinder'd by an Obſtacle, it continually preſſes that Obſtacle equally, tending towards the Earth's Center; therefore it may be look'd upon as a Power acting upon an Obſtacle; and therefore what has in the foregoing Chapter been demonſtrated, concerning Powers, does obtain here alſo.

PHENO-

PHENOMENON III.

77 *Bodies which deſcend by the Force of Gravity (if all Reſiſtance be taken off) fall with the ſame Velocity.* Which is prov'd by an

Experiment.] Pump out the Air from the tall Recipient A B (*Plate* II. *Fig.* 1.) which is made up of two Glaſſes, and is about three Foot high: Then from the Top of the Glaſs within, by moving the Handle C D, let fall a Piece of Gold and a very light Feather juſt at the ſame time, and they will always come down to the Braſs Plate of the Pump upon which the Receiver ſtands, at the ſame Inſtant of Time.

78 For making the Experiment readily, the Top of the Recipient is cover'd with a Braſs Plate laid upon it. A thin Plate, bent into the Figure E, is fixed to the covering Plate at *e*, by Help of two Screws H, that go thro' two leſſer Plates, one of which you ſee at *g f*, and are joined to the other Plate E.

The Ends of the Plate ſpring together, and ſo hold the Feather and the Gold, whilſt the Receiver is exhauſting.

A Braſs Wire runs thro' the Cover, which, by Means of the Handle C D, may be turned round without admitting the external Air; which we ſhall explain, when we come to ſpeak of the Air-Pump.

The Wire goes thro' an Hole in the upper Part of the Plate *e*, and the End of the Wire, which comes down between the ſpringing Plate, may be ſeen at L: It is ſquare and hollow, that the Oval Plate I may be joined to it.

You muſt obſerve, that the ſmall Diameter of the Oval be ſmall enough for this little Plate to be contained between the Springs E, when their Ends come together.

Now when the Braſs Wire, and by it the Plate I*i* is turned round; by reaſon of the Difference of Diameters in the Oval, the Ends of the Springs open; and then the Bodies, that are ſuſpended, are let go at the ſame Moment of Time.

The ſame Phænomenon is alſo deduced from another Experiment, which we ſhall mention hereafter*. *160.

79 Hence it follows, *that Gravity in all Bodies*, that is, their Weight, *is proportionable to their Quantity of Matter**. *62.

80 Therefore all Bodies conſiſt of Matter that is equally heavy; and the Reaſon that Bodies do not appear equally heavy, is becauſe ſome have more Matter than others under the ſame Bulk; that is, in the ſame Space.

81 When Weight is conſidered as a Power, the Intenſity of the Power is proportional to the Quantity of Matter in the heavy Body; and the Direction of the Power is towards the Center of the Earth.

CHAP. X.

Of the ſingle Pulley, Balance, and of the Center of Gravity.

DEFINITION I.

82 A Single Pulley *is a little Wheel moveable about its Axis, over which goes a drawing* or *running Rope*, d c e (*Plate* I. *Fig.* 10.)

By this Engine, the Direction of the Power is changed, neither is it of any other Uſe when fix'd; for in that Caſe, 83 *if the Force of Power apply'd to the drawing Rope, as* M, *be equal to the Obſtacle* P, *it balances that Obſtacle**; *for in that Caſe the Power can't move*, *71.

move, unleſs the Obſtacle does at the ſame Time go through an equal Space.

*79. Weights are found, that is, the Quantities of Matter in Bodies are compared together, * by a Balance, or a Pair of Scales, which is a well known Inſtrument.

Definition II.

84 The Axis of a Balance *is that Line about which the Balance moves, or rather turns round.*

Definition III.

85 When we conſider the Length of the *Brachia*, or of the Beam, then the Axis is to be looked upon as a Point, and called *the Center of the Balance.*

Definition IV.

86 We call Points of Suſpenſion, or Application, *thoſe Points where the Weights really are, or from which they hang freely; or the Scales in which the Weights are placed.*

Concerning this Machine, we are to obſerve,

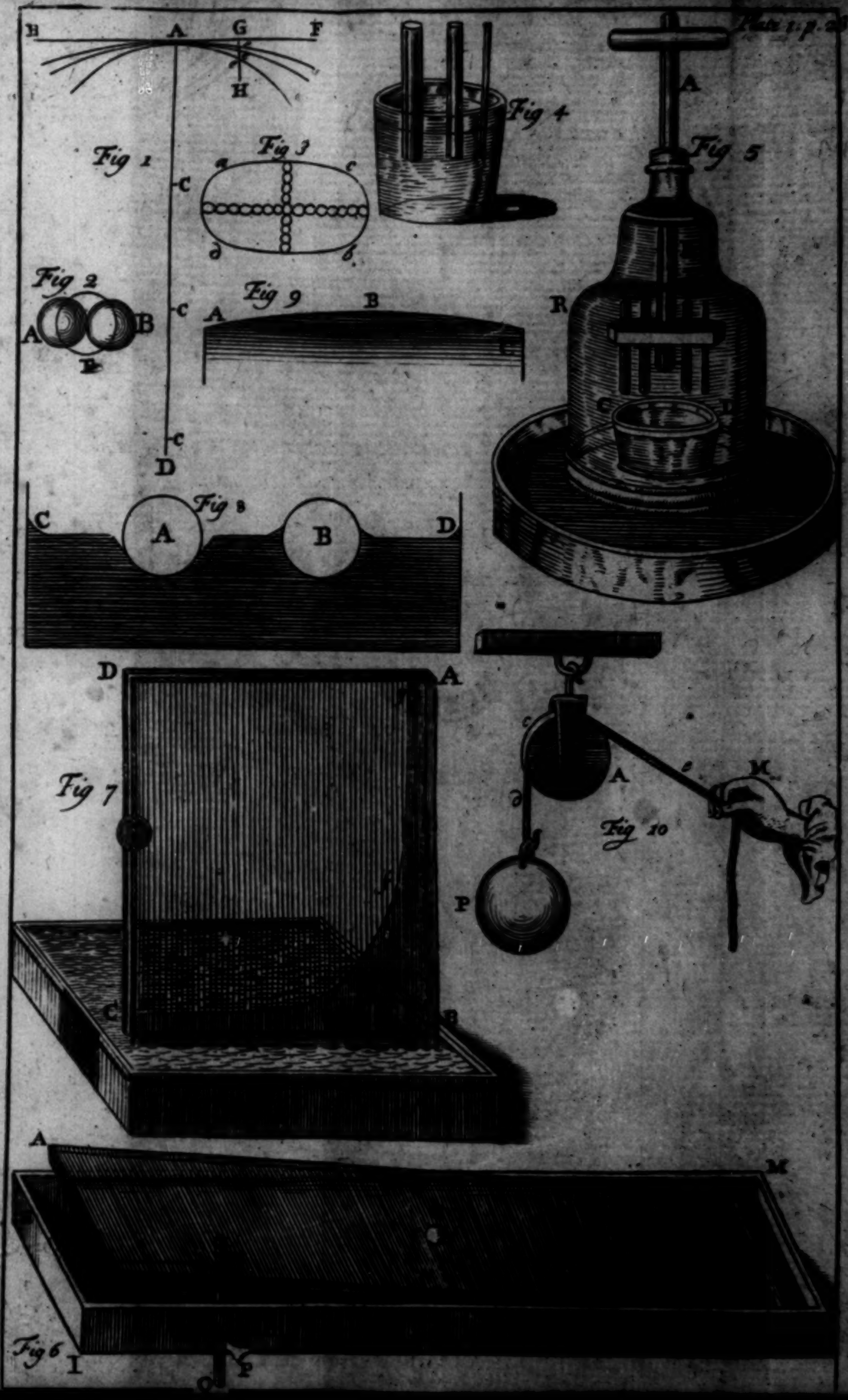
87 *That the Weight does equally preſs the Point of Suſpenſion, at whatever Height it hangs from it, and in the ſame Manner as if it was fixed at that very Point.*

74.75 For the Weight, at all Heights, equally ſtretches the Rope by which it hangs. This is alſo proved by

Experiment 1.] In the Balance A B, the Weight P, by Means of the Rope B D (*Plate* II. *Fig.* 2.) is ſuſpended at different Heights: And the Poſition of the Balance is not changed by it.

The

Plate 1. p. 28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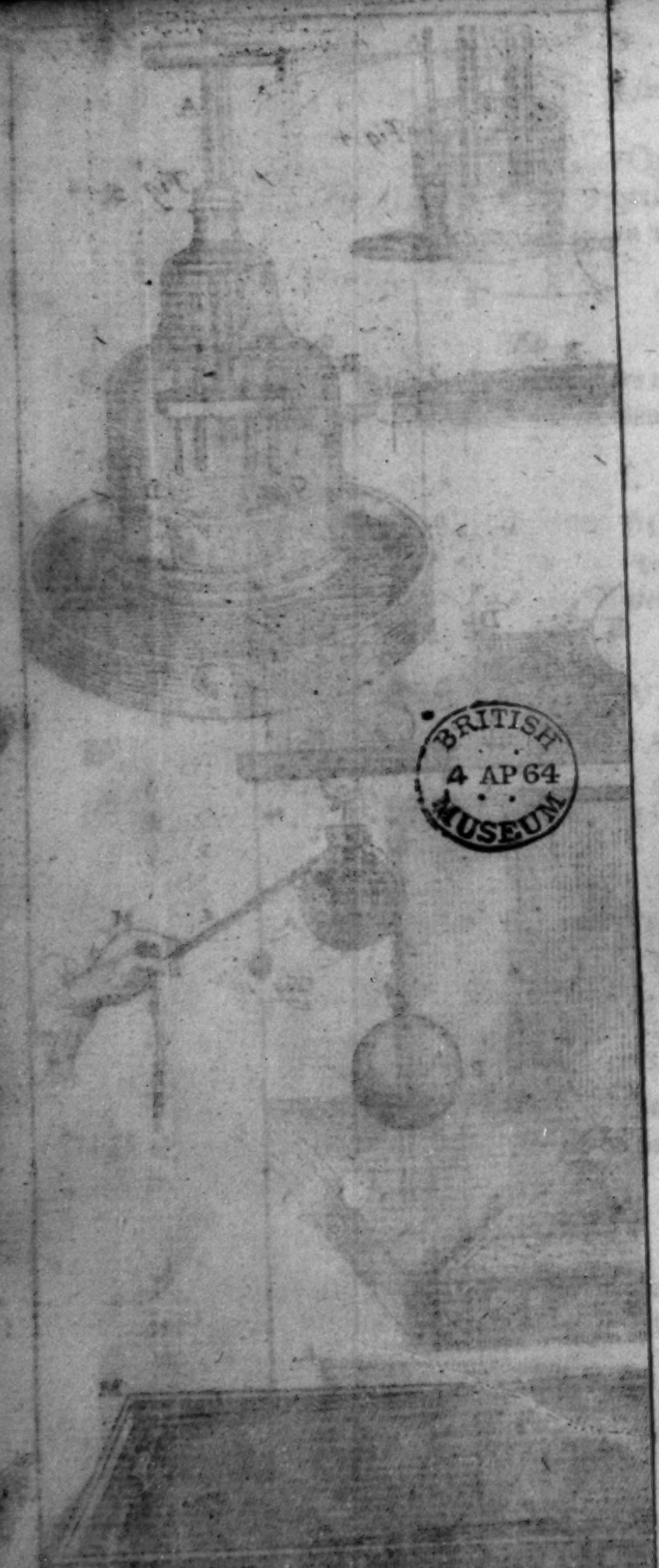

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88 *The Action of a Weight to move a Balance is by ſo much greater, as the Point preſſed by the Weight is more diſtant from the Center of the Balance; and that Action follows the Proportion of the Diſtance of the ſaid Point from that Center.*

When the Balance moves about its Center, the Point B deſcribes the Arc B*b* (*Plate* II. *Fig* 3.) whilſt the Point A deſcribes the Arc A*a*, which is the biggeſt of the two; therefore, in that Motion of the Balance, the Action of the ſame Weight is different, according to the Point to which it is applied, and it follows the Proportion of the Space*gone through by that Point: At A therefore it is as A*a*, and at B as B*b*; but thoſe Arcs are to one another, as CA, to CB. *67, 76

Experiment 2.] The *Brachia* of the Balance AB (*Plate* II. *Fig*. 4.) are divided into equal Parts; and one Ounce applied to the ninth Diviſion from the Center, is as powerful as three Ounces at the third; and two Ounces at the ſixth Diviſion act as ſtrongly as three at the fourth, *&c.*

The Conſtruction of a Balance, for this and ſome other following Experiments, is plain enough from the Figure, adding to it what is ſaid at *Numb.* 102. Hence it follows, that the Action of a Power to move a Balance is in a Ratio compounded of the Power itſelf, and its Diſtance from the Center*; for that Diſtance is as the Space gone through in the Motion of the Balance. *69

DEFINITION. V.

89 *A Balance is ſaid to be in* Æquilibrio, *when the Actions of the Weights upon each* Brachium, *to move the Balance, are equal; ſo that they mutually deſtroy each other*; as appears by the foregoing Experiment.

DEFINITION VI.

When a Balance is in Æquilibrio, the Weights on each Side are ſaid to *æquiponderate.*

90 *Unequal Weights can equiponderate.* For this, it is requiſite, that the Diſtances from the Center be reciprocally as the Weights*. In that Caſe, if each Weight be multiplied by its Diſtance, the Products will be equal. *89, 70

Experiment 3.] In the above-mentioned Balance (*Plate* II. *Fig.* 4.) one Ounce at the Diſtance of the ninth Diviſion from the Center, æquiponderates with three Ounces at the third Diviſion.

91 The Steel-yard, or *Statera Romana*, which weighs every Thing with one Weight, is made upon this Principle.

Experiment 4.] The Steel-yard AB (*Plate* II. *Fig.* 5.) has two Brachia very unequal; a Scale hangs at the ſhorteſt; the longeſt is divided into unequal Parts: Apply ſuch a Weight to it, that, at the firſt Diviſion, it ſhall æquiponderate with one Ounce laid in the Scale; then the Body to be weighed is put into the Scale, and the abovementioned Weight is to be moved along the longeſt Brachium, till you find the Æquilibrium; the Number of Diviſions between the Body and the Center, ſhew the Number of Ounces that the Body weighs, and the Subdiviſions the Parts of on Ounce.

92 Upon this Principle alſo is founded the deceitful Balance, which cheats by the Inequality of the Brachia.

Experiment 5.] Take two Scales of unequal Weights, in the Proportion of 9 to 10 (*Plate* III. *Fig.* 1.) and hang one of them at the tenth

Diviſion of the Balance above deſcribed, and the other at the ninth Diviſion, ſo that there may be an Æquilibrium: If then you take any Weights, which are to one another as 9 to 10, and put the firſt in the firſt Scale, and the other in the other Scale, they will æquiponderate.

93 *Several Weights, hanging at ſeveral Diſtances on one Side, may æquiponderate with a ſingle Weight on the other Side.* To do this, it is required, that the Product of that Weight, by its Diſtance from the Center, be equal to the Sum of the Products of all the other Weights, each being multiplied by its Diſtance from the Center.

Experiment 6.] Hang three Weights of an Ounce each, at the ſecond, third and fifth Diviſions from the Center, and they will æquiponderate with the Weight of one ſingle Ounce applied at the tenth Diviſion of the other Brachium (*Plate* II. *Fig.* 6.) And the Weight of one Ounce at the ſixth Diviſion, and another of three Ounces at the fourth Diviſion, will æquiponderate with a Weight of two Ounces on the other Side at the ninth Diviſion.

Several Weights, unequal in Number, on either Side, may æquiponderate. In that Caſe, if each of them be multiplied by its Diſtance from the Center, the Sums of the Products on either Side will be equal; and if thoſe Sums are equal, there will be an Æquilibrium.

Experiment 7.] Hang on a Weight of two Ounces (*Plate* II. *Fig* 7.) at the fifth Diviſion, and two others, each of one Ounce, at the ſecond and ſeventh; and on the other Side hang two Weights, each alſo of one Ounce, at the ninth and tenth Diviſions; and theſe two will æquiponderate with thoſe three.

DEFINITION VII.

The Center of Gravity *is a Point in a Body, about which all the Parts of the Body* (*whatever Poſition it is in*) *are* in æquilibrio.

98 When two or more Bodies are joined, whether they are contiguous or ſeparated, they have a common Center of Gravity.

When the Center of Gravity is ſuſtained, the Body remains at reſt.

Experiment 8.] The Body A (*Plate* III. *Fig*. 2.) is ſuſtained and at reſt, becauſe its Center of Gravity is ſuſtained by the Prop F.

When the Center of Gravity is not ſuſtained, the Body moves till that Center comes to be ſuſtained.

Experiment 9.] The Body A, laid upon the Table, will fall, and the Body B will not remain in its Poſition, becauſe their Centers of Gravity are not ſuſtained.

99 Hence may be known, why ſome Bodies, laid upon inclined Planes, will roll off, whilſt ſome only ſlide off.

Experiment 10.] The Body A ſlides, becauſe its Center of Gravity is ſuſtained by an inclined Plane; (*Plate* III. *Fig* 4.) that is, the Vertical Line, which goes through that Center, *c*, cuts the inclined Plane within the Body. But the Body B rolls, becauſe the Vertical Line, thro' its Center of Gravity, cuts the inclined Plane without the Body.

100 From what has been ſaid it follows alſo, that a Body deſcends, when its Center of Gravity deſcends, that is, is moved towards the Center of the Earth.

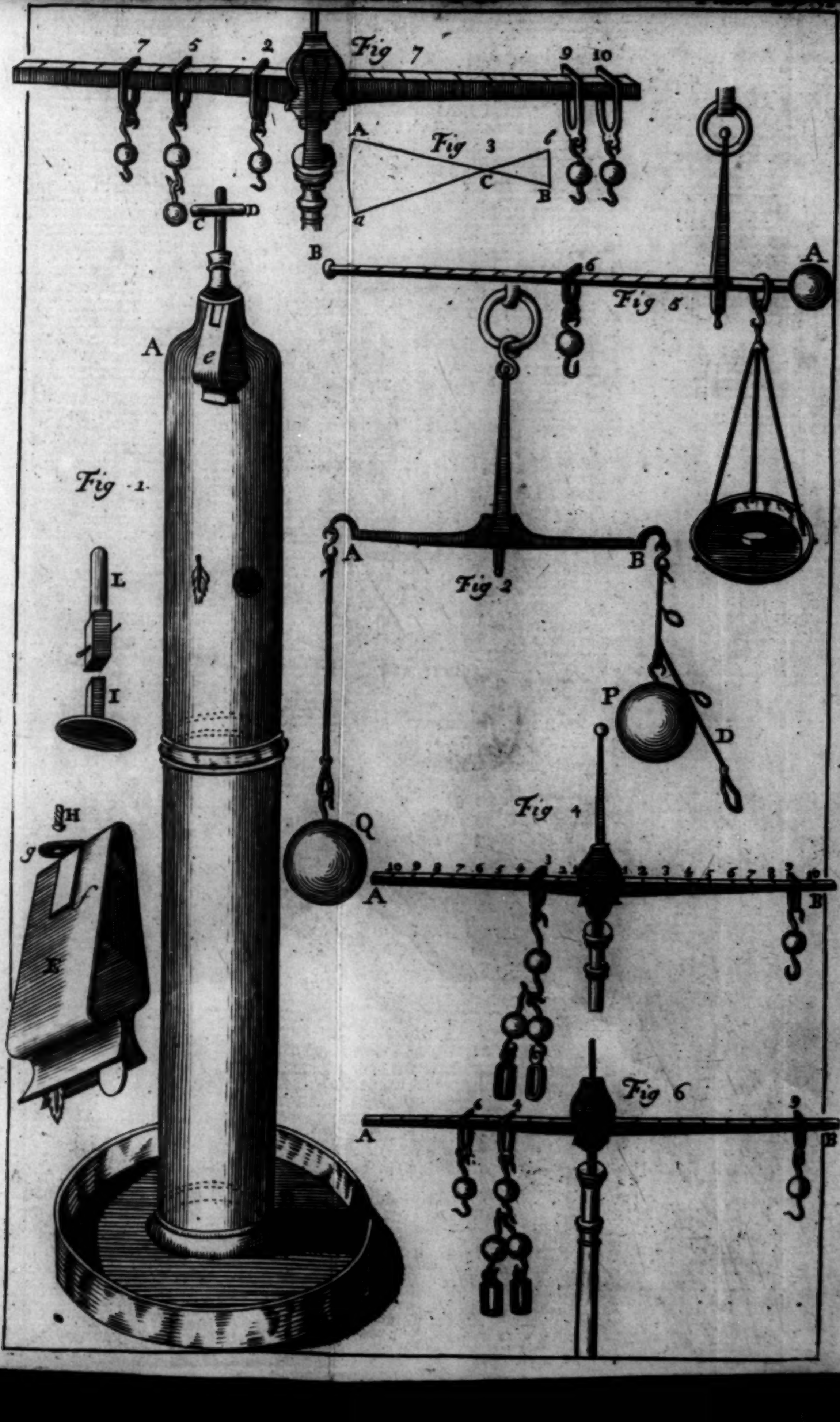
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Fig 3
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Fig 4
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A
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Fig 6
6
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B

Sometimes in that Caſe a Body ſeems to aſcend, and oftentimes it does really aſcend; but as all Bodies deſcend by Gravity, that is, their Centers of Gravity deſcend; it follows, that a Body ſeems to aſcend by its Gravity, and can really aſcend.

Experiment 11.] The Wheel A (*Plate* III. *Fig.* 6.) whoſe Axis is made of two Cones, the Baſes of which join to the Wheel, when put between two Planes, whoſe Sides D G, F H continued, make the Angle F C D, which has the Baſe (ſuppoſing a Triangle made) higher than the Vertex, will from H G, the lower Part of the Planes, roll towards FD, the higheſt Part of the Planes.

By reaſon of the greater Diſtance between the Planes at FD, the Wheel A, whoſe Axis is a Cone both Ways, deſcends more between the Planes, when it moves towards that Part, and is ſo carried thither by its Gravity, provided that Deſcent be greater than the Aſcent from the Inclination of the Angle F C D with the Horizon.

Experiment 12.] The Wooden Cylinder A (*Plate* III. *Fig.* 5.) has within it, near the Side, a Leaden Cylinder; their common Center of Gravity is in a Section parallel to the Baſe, which divides the Cylinder into two equal Parts, and in a Point anſwering to the Point *c* of the Baſe.

Whatever Poſition this Cylinder be laid in, it will move until the above-mentioned Center of Gravity be in the loweſt Place which it can come to.

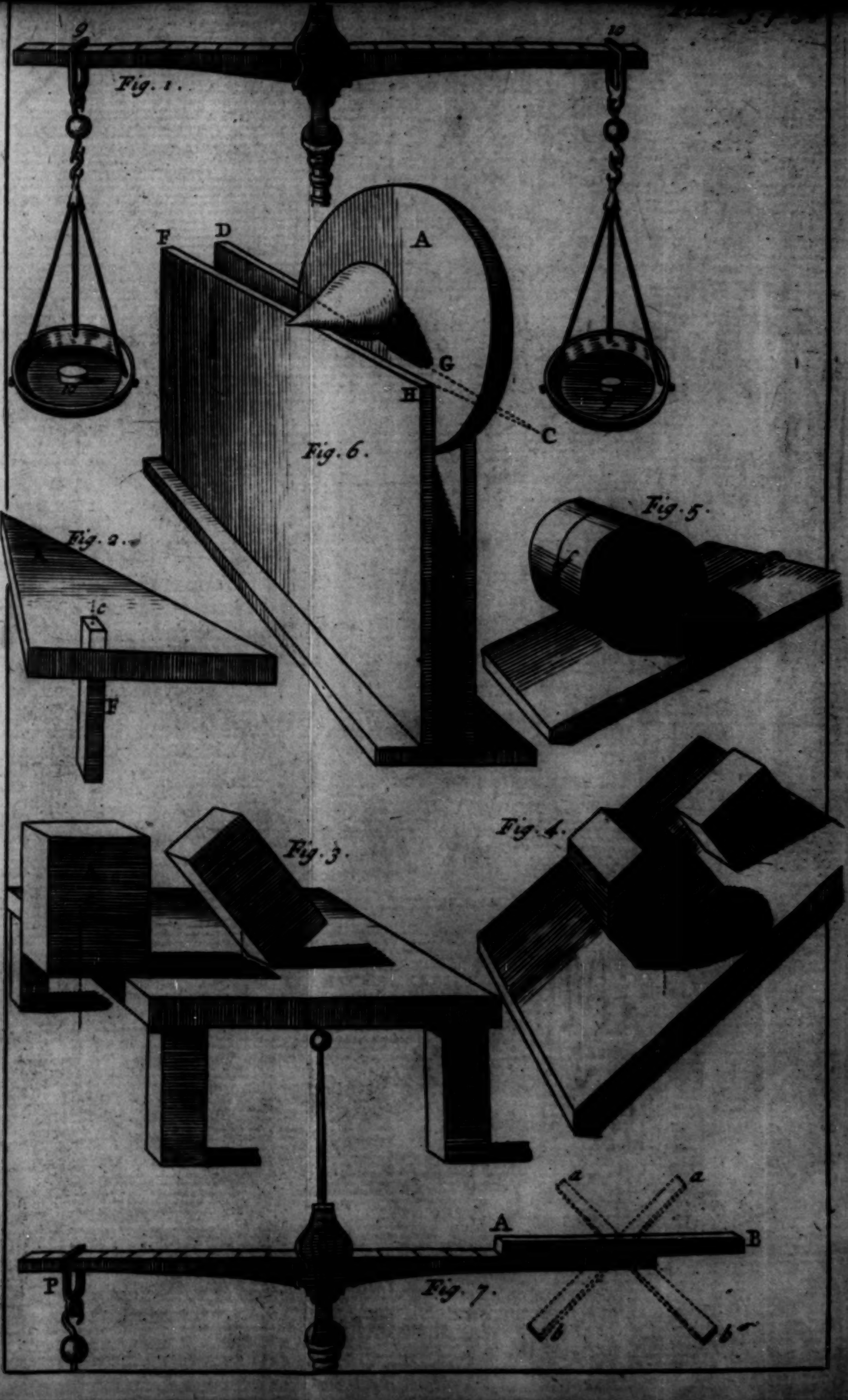
If it be laid upon an inclined Plane, in the Poſition deſcribed in the Figure, the Center of Gravity will deſcend whilſt the Body riſes along the Plane, if it be inclined in a fit manner.

The Body aſcends by rolling towards the upper Part of the Plane, but care muſt be taken, that whilſt it is endeavouring to roll up, it does not ſlide down along the Plane; and therefore you muſt uſe a Rope, which goes in part round the Cylinder; one End of which is joined to the Cylinder at *f*, and the other is fixed to the Plane at *d*.

101 From what has been ſaid of the Center of Gravity, it is farther deduced, that whatever Point of a Body or Machine ſuſtains the Center of Gravity of any Weight, that Point ſuſtains the whole Weight: So that the whole Force, by which any Body tends towards the Earth, is, as it were, collected to that Center.

Experiment 13.] If the Body AB (*Plate* III. *Fig.* 7.) whoſe Center of Gravity is laid upon the Brachium of a Ballance, does in any Poſition æquiponderate with any Weight P; it will in any other Poſition, as *ab*, *ab*, æquiponderate with it, provided the Center of Gravity be ſtill at C.

102 That a Ballance may be perfect, it is required, 1. That the Points of Suſpenſion of the Scales or Weights be exactly in the ſame Line as the Center of the Ballance. 2. That they be preciſely equidiſtant from that Point on either Side. 3. That the Brachia of the Ballance be as long as they conveniently can. 4. That there be as little Friction as poſſible in the Motion of the Beam and Scales. 5. And, laſtly, that the Center of Gravity of the Beam be placed a little below the Center of Motion.


9
10
Fig. 1.
F
D
A
G
H
C
Fig. 6.
Fig. 5.
Fig. 2.
c
F
Fig. 3.
Fig. 4.
a
a
A
B
P
Fig. 7.
b
b

CHAP. XI.

Of the LEVER.

DEFINITION I.

A Lever is called by Mathematicians, *an inflexible Right Line, made uſe of to raiſe Weights, either weighing nothing itſelf, or of ſuch Weight as may be ballanced.* (*Plate* IV. *Fig.* 1.) 103

It is the firſt of thoſe that we call Simple Machines (or Mechanical Powers) as being the moſt ſimple of all; and it ſerves, when Weights are to be raiſed but to a ſmall Height.

There are four other Simple Machines, which we ſhall treat of in the three following Chapters.

Concerning the Lever, three things are to be conſidered.

1. The Weight to be raiſed or ſuſtained, as P.
2. The Power, by which it is to be raiſed or ſuſtained, which here is repreſented by the Weight M, tho' commonly it is the Action of a Man.
3. The *Fulcrum*, or Prop, by which the Lever is ſuſtained, or rather on which it moves round, whilſt the ſaid Point F remains fixed.

The Lever is three-fold.

1. Sometimes the Fulcrum is placed between the Weight and the Power. (*Plate* IV. *Fig.* 1.) 104
2. Sometimes the Weight is between the Fulcrum and the Power. (*Plate* IV. *Fig.* 2.)
3. And often alſo the Power acts between the Weight and the Fulcrum. (*Plate* IV. *Fig.* 3.)

The ſame Rules ſerve in all theſe Caſes, which follow from what has been ſaid of the Ballance*: *88 And this ſhews the Analogy between the Lever and the Ballance. The Lever of the firſt Kind is, as it were, a Steel-yard to raiſe Weights.

The Action of a Power, and the Reſiſtance of the Weight increaſe, in proportion to their Diſtance from the 105

*88 *the Fulcrum** ; and therefore, *that a Power may be able to ſuſtain a Weight, it is required, that the Diſtance of the Point in the Lever, to which it is applied, be to the Diſtance of the Weight, as the* *90 *Weight to the Intenſity of the Power** ; *which, if it be ever ſo little increaſed, will raiſe the Weight.*

Experim. 1, 2, and 3.] This Rule is confirm'd by Experiments, in reſpect of the three Levers, as it appears from the firſt, ſecond and third Figures of the fourth Plate; for there is an Equilibrium, when the Weights P, and the Weights M, which repreſent the Powers, and alſo the Diſtances from the *Fulcra*, bear thoſe Proportions to each other, as the Numbers written in the Figures expreſs. A Sight of the Figures ſo plainly ſhews the Conſtruction of the Machines wherewith the Experiments are made, that a farther Explanation would be needleſs.

Workmen alſo make uſe of a Lever to carry Weights; and there are ſeveral remarkable Caſes of thoſe Levers, the Demonſtration of which may be deduced from what has been ſaid.

106 In all Caſes this is generally to be obſerved, that *the Intenſity of the Power, or the Intenſities of the Powers, taken all together (when there are more than one) muſt act as ſtrongly as the Gravity of the Weights to be carried or ſuſtained.*

107 If a Weight is to be carried or ſuſtained by two Powers, it muſt be placed between the two Powers; and the Diſtances of the Powers on each Side of the Weight muſt be in an inverſe Ratio of the Intenſities of the Powers.

Experiment 4.] This Propoſition is confirm'd by the Experiment of *Fig.* 4. which requires no farther Explanation.

Experiment 5.] When two Weights are to be 108
ſuſtained by one Power, that Power muſt be placed between the Weights; and then what has been ſaid before of the Powers, muſt be applied to the Weights. See *Fig.* 1. *Plate* V.

Several Weights are often carried or ſuſtained by one or more Powers. In which Caſes it is to 109
be obſerved, that *all Weights, in whatever Poſition, have one common Center of Gravity*; which Center is ſuch, that if, on either Side, each Weight be multiplied by its Diſtance from that Point, the Sums of the Products on each Side will be equal*. *95,99

Let the Powers alſo be diſpoſed in any Poſition, they have a common Center of Gravity; for they may be repreſented by Weights*; and here the *76
Intenſity of each Power is to be multiplied by its Diſtance from the Center, and the Sums of the Products will then be equal on both Sides: *That the Powers may be able to ſuſtain the Weights, it is required that the Center of Gravity of the Powers and the Weights be the ſame.*

Experiment 6 and 7.] What has been ſaid, ſufficiently explains the Figures, (*Plate* V *Fig.* 2 and 3.) where C denotes the Center of Gravity common to the Weights and the Powers.

Experiment 8.] What has been ſaid is true, if 110
the Lever is drawn (*Plate* V. *Fig.* 4.) by Powers on each Side; which we ſee in the Lever of *Fig.* 4. which is drawn horizontally on each Side; where the Æquilibrium only depends upon what has been laid down in the Rules above-mentioned.

We may alſo make uſe of a compound Lever 111
for raiſing Weights: In which Caſe, inſtead of a Power, a ſecond Lever is applied to the firſt,

and

and a third to that, and ſo on as far as you will, and a Power is applied to the laſt Lever; and then *the Ratio of the Power to the Weight* (when it ſuſtains it) *is compounded of the Ratio's of the Power to the Weights in each Lever, when they are uſed ſeparately.*

Experiment 9.] The three Levers A, B, D, are ſo diſpoſed (*Plate* IV. *Fig.* 5.) that the Power M ſuſtains the Weight P. In the Lever A, if it were uſed ſingly, the Power would be to the Weight as 1 to 5; the Lever B, as 1 to 4; and in the Lever D, as 1 to 6. The Ratio, compounded of all theſe, is as 1 to 120: For one Ounce M does here ſuſtain the Weight P, of 120 Ounces. Obſerve, that in the Motion of this Engine, the Spaces gone thro' by the Weight and the Power are to one another, as 1 to 120; that is in the ſaid inverſe Ratio, which is requir'd

70 to make an Æquilibrium.

CHAP. XII.

Of the Axis in Peritrochio, and Wheels with Teeth.

THE Lever, as was ſaid in the Beginning of the foregoing Chapter, ſerves to raiſe Weights to a ſmall Height; when they muſt be raiſed higher, we uſe an *Axis in Peritrochio.*

DEFINITION.

112 We call Axis in Peritrochio, a *Wheel which turns together with its Axis.* (*Plate* V. *Fig.* 5.)

The Power in this Machine is applied to the Circumference of the Wheel, by whoſe Motion a Rope that is tied to the Weight, is wound about the Axis by which the Weight is raiſed.

Let

Plate 4. p. 38.

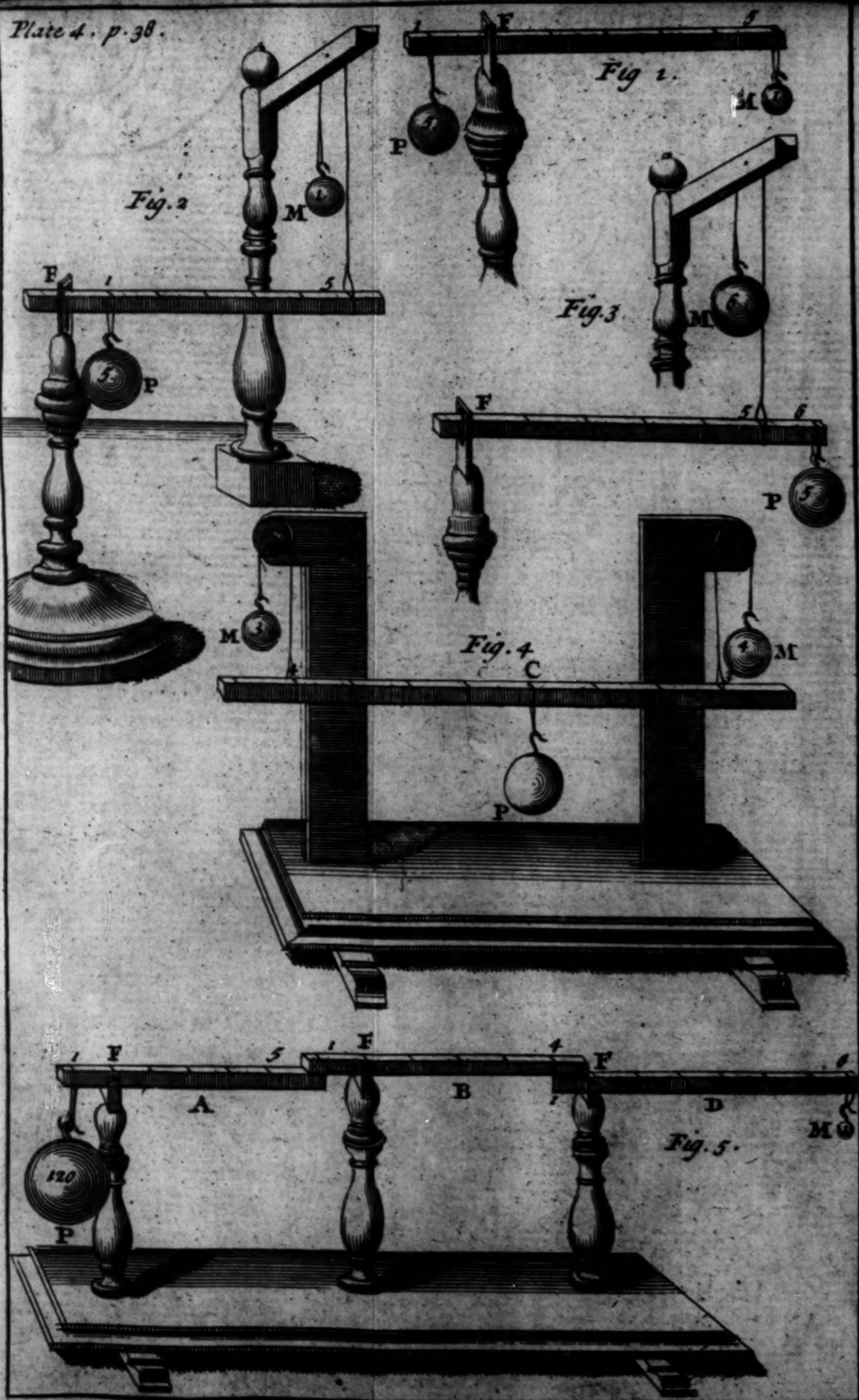


Fig 1.
1
F
5
P
5
M
1
Fig. 2
M
1
F
1
5
5
P
Fig. 3
M
6
F
5
6
P
5
M
3
M
4
Fig. 4
4
C
P
1
F
5
1
F
4
F
6
A
B
1
D
M
Fig. 5.
120
P

Let *a b* be the Wheel, (*Plate* V. *Fig.* 6.) *d e* the Axis, *p* the Weight to be raiſed, *m* the Power; as the Wheel is moved by the Power, the Points *b* and *d* deſcribe ſimilar Arcs, which are the Ways of the Power and the Weight, and are to each other, as *c b* to *c d*, that is, as the Diameter of the Wheel to the Diameter of the Axis; whence the following Rule is deduced.

The Power has the greater Force, the greater the 113 *Wheel is, and its Action increaſes in the ſame Ratio as the Wheel's Diameter. The Weight reſiſts ſo much the leſs as the Diameter of the Axis is leſs, and its Reſiſtance is diminiſhed in the ſame Ratio as the Diameter of the Axis. And that there may be an Æquilibrium between the Weight and the Power, it is always requiſite that the Diameter of the Wheel be, to the Diameter of the Axis, in an inverſe Ratio of the Power to the Weight*.* *70

It is to be obſerved, that you muſt add the Diameter of the Rope to that of the Axis.

Experiment 1.] This Rule is variouſly confirm'd (*Plate* V. *Fig.* 5.) by help of the Machine here repreſented. When the Axis is the twelfth Part of the Diameter of the Wheel, half a Pound ſuſtains ſix Pounds; and ſo on.

The Power alſo may be applied to an Handle or Spoke; as at D, and then the Diſtance of the Point to which it is applied, reckon'd from the Center, muſt be look'd upon as the Wheel's Semidiamiter.

The Wheels, that have Teeth, work in the ſame Manner as this Machine; they being, in reſpect of the Axis in Peritrochio, what the compound Lever is, in reſpect of the ſimple Lever.

If the Axis of the Wheel has Teeth alſo, it 114 ſerves to move a Wheel, whoſe Circumference has Teeth; and this Axis of the laſt Wheel may again communicate Motion to a third Wheel, and ſo on. In that Caſe, *that the Power may ſuſtain the Weight, its Ratio to the Weight is compounded of the Ratio of the Diameter of the Axis of the laſt Wheel, to the Diameter of the firſt; and the Ratio of the Revolutions of the laſt Wheel, to the Revolutions of the firſt in the ſame Time.*

The Demonſtration of which Rule is alſo deduced from the Compariſon of the Ways run thro' by the Weight and the Power. (*Plate* V. *Fig.* 7.)

Experiment 2.] Let the Power repreſented by the Weight M be applied to the Wheel A B, and the Weight P to the Axis of the Wheel F G; the Diameter of that Axis is the eighth Part of the Diameter of the Wheel A B; and this Wheel goes round five Times, whilſt the Wheel F G goes round once: Therefore the Ratio of the Power to the Weight is compounded of the Ratio's of one to 8; and one to 5: Therefore it is the Ratio of 1 to 40; half a Pound in this Caſe ſuſtaining 20 Pounds.

CHAP. XIII.

Of the PULLEY.

IN ſeveral Caſes, where the Axis in Peritrochio cannot conveniently be applied, Pullies muſt be made uſe of to raiſe Weights; a Machine made by combining ſeveral of them, lies in a little Compaſs, and is eaſily carried about.

*82 What a Pulley is, has been already explain'd *.

115 If the Weight be fixed to the Pulley, ſo that it may be drawn up along with it, each End of the drawing or running Rope ſuſtains half the Weight. *Therefore when one End is fixed, either to a Hook, or any other Way, the moving Force or Power applied to the other End, if it be equal to half the Weight, will keep the Weight in Æquilibrio.*

Plate 5. p. 46

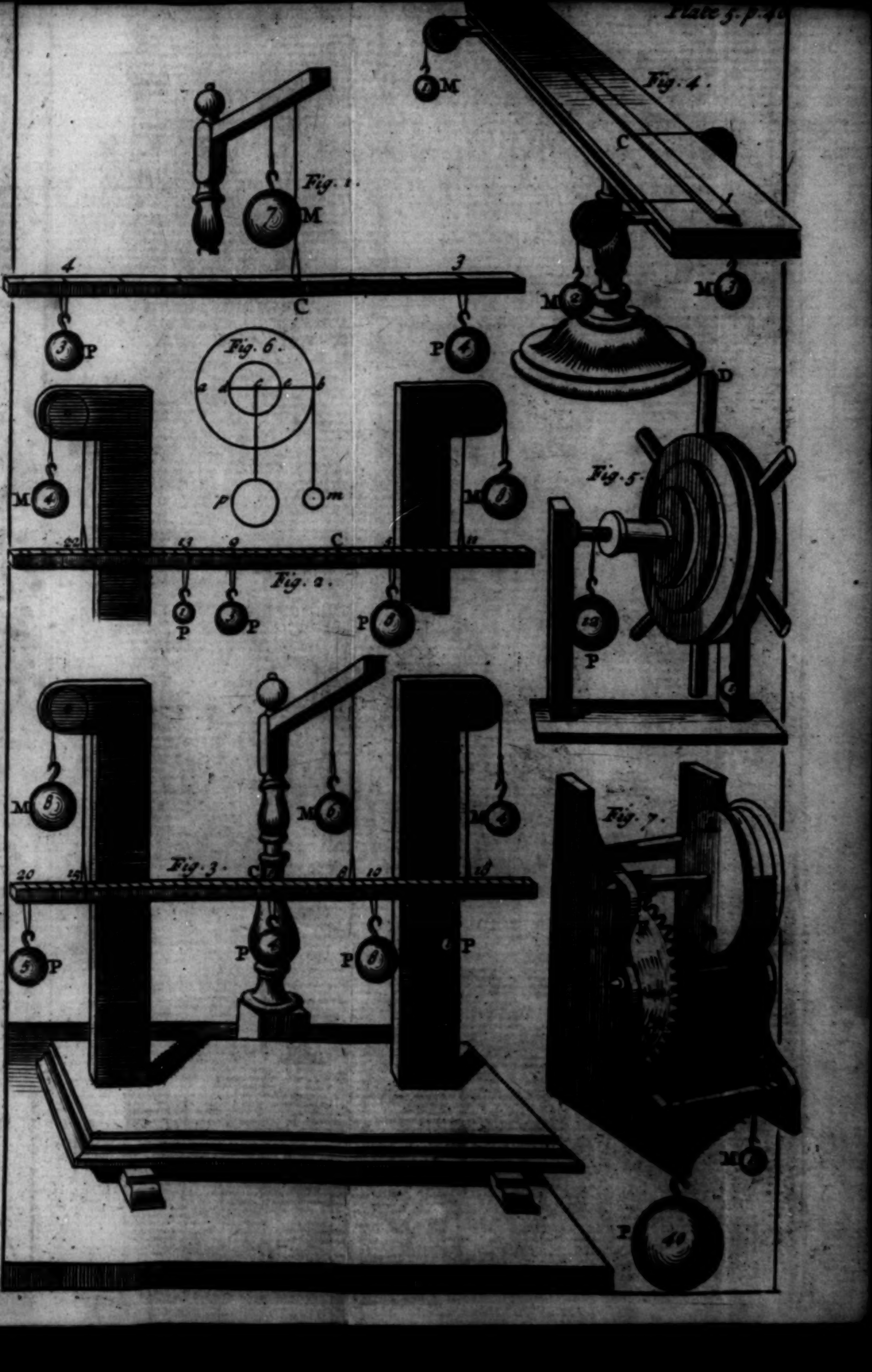


Fig. 1.
7 M
4
3
C
3 P
4 P
Fig. 6.
a d c e b
p
m
M 4
M 8
12
9
C
8
11
Fig. 2.
1 P
3 P
P 8
Fig. 4.
1 M
C
M 2
M 1
D
Fig. 5.
12 P
M 8
M 6
M 4
Fig. 3.
20
15
C
8
10
18
5 P
P 4
P 8
P
Fig. 7.
M 1
P 40

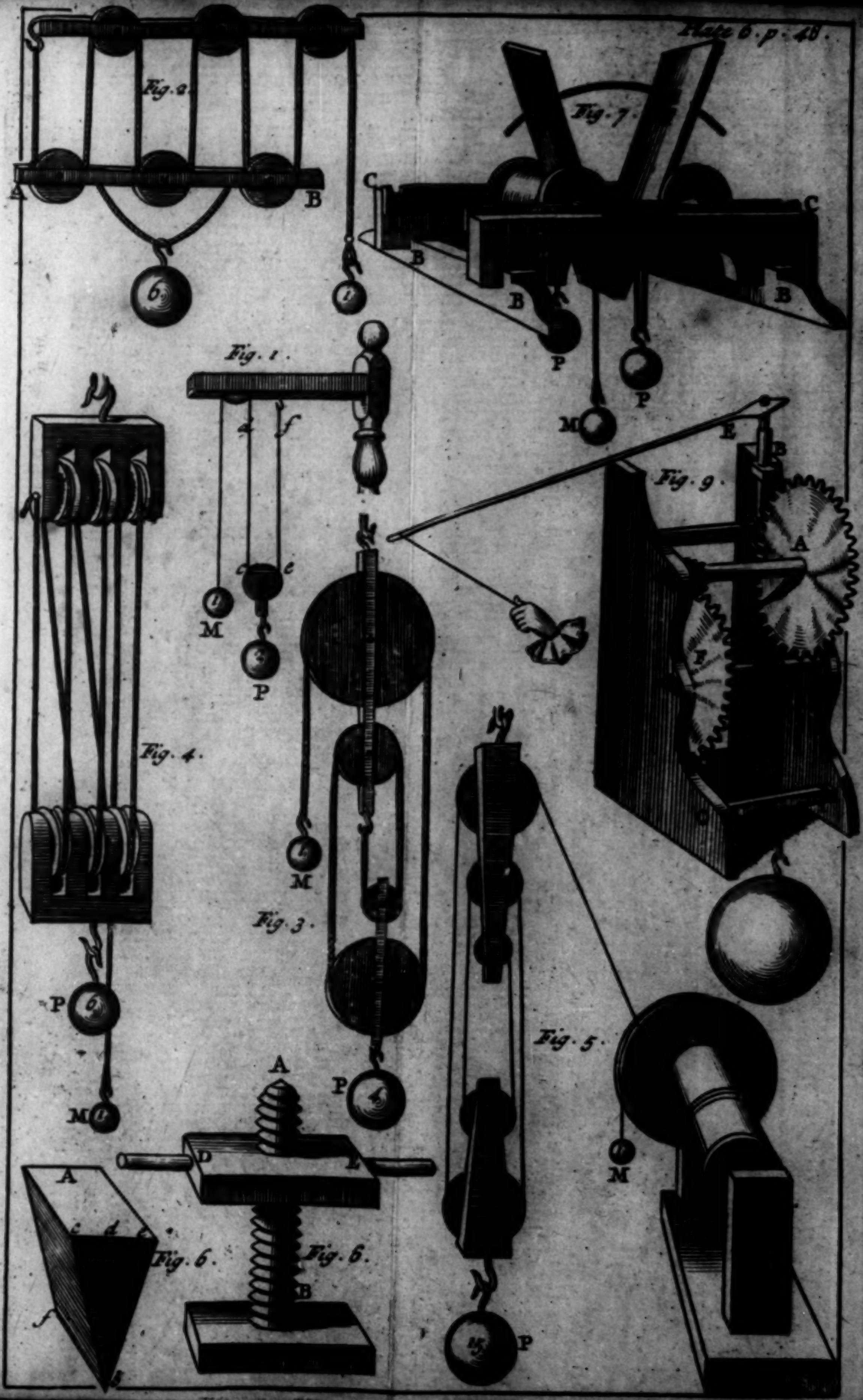


Fig. 1.
a
f
c
e
M
P
Fig. 2.
A
B
6
1
Fig. 3.
M
P
4
Fig. 4.
P
6
M
1
Fig. 5.
M
P
15
Fig. 6.
A
c
d
e
f
g
Fig. 6.
A
D
E
B
Fig. 7.
C
C
B
B
B
P
P
M
Fig. 9.
E
B
A
F
G

Experiment 1.] Make faſt the Weight P of two Pounds to a Pulley (*Fig.* 1. *Plate* VI.) yet ſo that the Wheel or Sheave may not be hindered from turning round; let one Part of the Rope *ef* be tied to a Hook, and the other End *cd* go round the fix'd Pulley, to change the Direction, * the Weight M of one Pound, fix'd to this End of the Rope, will ſuſtain the Weight P. *83

116 Several Sheaves may be joined in any manner, and the Weight be fix'd to them; then if one End of the Rope be fix'd, and the Rope goes round all thoſe Sheaves, and as many other fix'd ones as is neceſſary, a great Weight may be raiſed by a ſmall Power: In that Caſe *the greater the Number of Sheaves fix'd in a moveable Pulley, or of moveable Wheels,* (for the fix'd ones do not change the Action of the Power *) *ſo much may the Power be leſs, which ſuſtains the Weight, and a Power which is to the Weight, as the Number One to twice the Number of the Sheaves, will ſuſtain the Weight.* *85

The Reaſon is, that the Number laſt-mentioned is the Number of the Ropes that ſuſtain the Weight, and the Power is applied only to one Rope.

N. B. *The Workmen in* England *call a* Block, *the Box or Piece of Wood that has one or ſeveral Wheels in it; and thoſe Wheels,* Sheaves or Sheavers.

Experiment 2.] Hang the Weight P of 6 Pounds to the Piece A B (*Plate* VI. *Fig.* 2.) in which three Sheaves turn freely round. Let one End of the Rope to be faſten'd to an Hook, and let the Rope go round thoſe three Sheaves, and three other fix'd ones: One Pound, fix'd to the other End of the Rope, will make an Æquilibrium.

Experiment 3 and 4.] The different Make of the Pullies, or the different Way of joining the

Sheaves together, makes no Alteration; the laſt Sort is not very convenient for raiſing Weights, and therefore Workmen make uſe of unequal Sheaves, joined together in the manner repreſented in *Fig.* 3.; for the different Bigneſs of the Sheaves makes no Alteration. Oftentimes all the Sheaves move round the ſame Axis, as in the 4th *Fig.* and ſo the Pullies lie in the leaſt Room. Now in both theſe Caſes the Experiments anſwer as before.

When the End of the Rope, which in the foregoing Experiments was fix'd, is joined to the Weight, or to the moveable Wheels; then the Ratio of the Power to the Weight is no longer as one to twice the Number of the Sheaves jointo the Weight; but this double Number muſt be increaſed by 1; and then, where two Sheaves are joined to the Weight, the Ratio will be as 1 to 5; for there are juſt ſo many Ropes which ſuſtain the Weight. See *Plate* VI. *Fig.* 5.

117 *Experiment* 5.] If ſeveral moveable Pullies with one Wheel in each, and each having its own particular Rope, be diſpoſed in the manner repreſented in *Plate* VII. *Fig.* 1. the Action of the Power will be very much increaſed; for every Wheel doubles it, and therefore it is four times greater with two Wheels, and eight times greater with three, and ſo on.

The Rule above-mentioned (namely, that the Spaces gone thro' by the Power and Weight, when they ballance each other, are to one another inverſely as the Power to the Weight) may be applied in all the Caſes above-mentioned.

Here we always ſuppoſe the Ropes parallel: We ſhall hereafter ſhew what Difference is made by the Obliquity of the Ropes.

CHAP. XIV.

Of the WEDGE *and* SCREW.

FRom what has been ſaid, it plainly appears, how great a Weight may be ſuſtained, or even raiſed, by a little Power; but thoſe are not the only Ways of producing the ſame Effect. Mechanicks are not confined only to theſe Methods; the Actions of Powers may be increaſed in all Caſes: A very good Inſtance of it appears in the *Wedge*, which is contrived for cleaving Wood, and alſo uſeful in ſeveral other Caſes.

DEFINITION I.

118 A Wedge *is a Priſm of a ſmall Height, whoſe Baſes are æquicrural Triangles*, as A, *Plate* VI. *Fig.* 6.

DEFINITION II.

The Height of the Triangle is the Height of the Wedge; as *d b*.

DEFINITION III.

The Baſe of the Triangle is alſo called the Baſe of the Wedge; as *c e*.

DEFINITION IV.

The Edge of the Wedge *is a Right Line, which joins the Vertices of the Triangles*; as *b f*.

The Edge of the Wedge is applied for cleaving Wood, and the Power is the Blow of a Hammer or Mallet, which drives the Wedge into the Wood.

When the whole Wedge is driven in, the Space gone thro' by means of the Blow or Blows, is the Height of the Wedge, which therefore may be look'd upon as the Space gone through by the Power; and the Space which the Wood goes thro',

thro', as it yields on each Side, is half the Baſe of
119 the Wedge. Whence it follows, *That the Power is to the Reſiſtance of the Wood* (when its Action is equal to it) *as the half Baſe of the Wedge is to its Height.*

What is here ſaid of the Reſiſtance of the Wood, may be applied to all the other Uſes of the Wedge.

120 The two wooden Rules A A, A A, are kept up in a parallel and horizontal Situation by the Feet B B, B B. (*Plate* VI. *Fig.* 7.)

The Braſs Rulers C C, C C, are fix'd to them on the Inſide.

Between them are moved the two Barrels, or Wooden Cylinders E E, which turn upon ſmall Steel Axes that come out behind the Rulers, and have a ſmall Return at their Ends, or the Baſes are bigger than the Cylinders; each Return is a little Convex on the Outſide, that the Friction againſt the Rulers C C, C C, may be the leſs. In the Middle of each Ruler A A, there are two Pullies *d*, *d*, which almoſt touch one another, and whoſe upper Part is even with the Top of the Rulers C C.

The Rope, which in its Middle carries the Weight P, goes round the Pullies *d*, *d*, and each End of it is fixed to the Axis of one Cylinder E, by means of a ſmall Plate that has a Hole through which the Axis goes. The other Weight P hangs in the ſame Manner upon ſuch another Rope.

Therefore the Cylinders E E muſt be carried towards one another in an horizontal Motion (their Axis remaining parallel) by the Weight P, if they are equal.

Let there be a Wedge made of two wooden Planes F F, which make any Angle at Pleaſure by help of the Screw *g g*.

Experiment.] The Cylinders EE are ſeparated by letting down the Wedge FF between them, which is drawn down by the Weight M, and you have an Æquilibrium, when the Weight M, together with the Weight of the Wedge, is to the Weights P, P, as the half Baſe of the Wedge to its Height.

The Force with which the Cylinders are carried towards one another, and which muſt be overcome, to ſeparate them, is here inſtead of the Reſiſtance of the Wood, the Force with which the Wedge is driven or drawn between the Cylinders; that is, the Weight of the Wedge, together with the Weight M, is here taken for a Blow with a Mallet; and ſo the foregoing Rule is reduced to Experiment, and confirmed by it.

The *Screw* has a great Affinity with the *Wedge.* It conſiſts of two Parts.

DEFINITION V.

The firſt, which is called the Male Screw, or Outſide Screw, is *a Cylinder cut in, in a Helical Form,* as AB (*Plate* VI. *Fig.* 8.) 121

The ſecond, which is called the Female Screw, or Inſide Screw, and ſometimes the Nut, and is different according to the different Uſes of the Machine of which it is made a Part, is *a ſolid Body, that contains an hollow Cylinder, whoſe Concave Surface is cut in the ſame Manner as the Male Screw, ſo that the Prominent Part of the one may fit the Cavity of the other; as* DE.

N. B. The Prominent Helical Part is called the *Thread of the Screw.*

Theſe two Parts are to move one within another, when the Screw is applied to Uſe. It ſerves chiefly to preſs together ſuch Bodies as muſt be joined and firmly united; for in this Machine the ſmalleſt Power may produce a very great Preſſure. The Screw may alſo be uſed for raiſing Weights. In every Revolution of it, one Part remaining at reſt, the other is thruſt out as far as the Interval between two Threads. The Power which moves the Screw is applied to an Handle or *Hand-Spike*; and then *the Power*, which acts as ſtrongly as the Reſiſtance, *is to the Reſiſtance, as the* ſaid *Diſtance between two Threads to the Periphery of a Circle, run thro' by that Point of the Handle to which the Power is applied.* For the Way gone through by that Point or Plane, wherewith the Reſiſtance is overcome, has the ſame Ratio to the Way of the Power.

Here we muſt obſerve, that when the Power ballances the Weight in any Machine whatever, where no Friction is ſuppoſed; that, by increaſing the Power ever ſo little, it will over-ballance the Weight. But when there is any Friction, that Friction muſt alſo be overcome by the Power; and how much muſt be added to it, to produce that Effect, cannot be determined mathematically. In the Machine laſt-mentioned, this Friction is very ſenſible, and alſo of a great Uſe; for by it the Machine is kept in its Poſition, and cannot (either by the Action of the Bodies that are preſſed, or the Gravity of the Weights) receive a contrary Motion, ſo as to be puſhed back to its firſt Poſition.

CHAP. XV.

Of Compound Engines.

111 WE have already ſhewn, how a Machine †114 may be compounded of ſeveral Levers †, or ſeveral Wheels; and that in ſuch Machines *the Power is to the Reſiſtance* (when it counterballances it) *in a Ratio compounded of all the Ratio's, which the Powers in each ſimple Machine would have to the Reſiſtance, if they were ſeparately applied.* This Rule alſo obtains in all other Machines.

Not only ſimple Machines of the ſame Kind may be joined; but one may compound a Machine of ſeveral other Machines in different manner: This will be plain enough by two Examples.

Experiment 1.] Join the running Rope of the Pullies to the Axle of the Wheel (*Plate* VI. *Fig.* ſ.) and apply the Power to the Wheel: Now, as in this Caſe, the Action of the Power becomes five times greater by help of theſe Pullies, and the Diameter of the Axis is but the third Part of the Diameter of the Wheel; the Ratio of the Power to the Weight is compounded of the Ratio's of 1 to *ſ, and 1 to † 3; it is therefore as 1 to 1ſ; and therefore one Pound M ſuſtains the Weight P of 1ſ Pounds. 122 *116 †113

The Axis in Peritrochio may be moved by a Screw: For this Purpoſe the Wheel muſt have Teeth, and thoſe Teeth muſt ſtand a-skew, or be inclined, as you may ſee in the Wheel A, (*Fig.* 9.) which is carried round by the Screw B C. Such a Screw is called an *endleſs Screw*, and very much increaſes the Action of the Power; for there are ſo many Revolutions of the Screw, or of the Handle of it, required to turn the Wheel once about as the Wheel has Teeth. And if another Wheel with Teeth be added to the firſt, the Action of the Power will ſtill be much more increaſed. 123

Experiment 2.] The Machine of *Plate* VI. *Fig.* 9. conſiſts of an endleſs Screw, which is moved by the Handle DE. Here the Ratio of the Power to the Weight when it ballances it, is compounded of the Ratio of the Semidiamiter of the Axis of the laſt Wheel F, to the Length of the Handle D E, and the Ratio of the Revolutions of that Wheel to the Revolutions of the Handle or Screw. The firſt Ratio in this Machine is the Ratio of 1 to 30; the ſecond is gathered from the Number of Teeth; the laſt Wheel F has in its Circumference 35 Teeth, and the Axis of the firſt Wheel 7; therefore the firſt Wheel goes round five times while the ſecond Wheel goes round once: But this firſt contains 36 Teeth; therefore the Screw goes round ſo many times for one Turn of the Wheel*. The Ratio compounded of theſe two, is 1 to 180, which is the ſecond Ratio ſought; and the Ratio made up of that and the firſt (which is 1 to 30) is the Ratio of 1 to 5400, which would be the Ratio of the Power to the Weight, if there was no Friction; but as it is pretty great in all theſe Engines, the Power muſt be pretty much increaſed, to make it raiſe the Weight; though ſtill a very ſmall Power applied to it, will raiſe a prodigious Weight. The Handle E D may be twice or three times as long, or ſtill longer, which will double, or triple, or farther increaſe, the Action of the Power: And, in that Caſe, a ſmall Hair will overcome the Force of the ſtrongeſt Man. *123

A great number of other compound Machines may be made, whoſe Forces are in the ſame Manner determined, by Computation, by the Rule mentioned in the beginning of this Chapter; or alſo by comparing the Way gone through by the Power with that gone through by the Weight, or any other Obſtacle; for their Ratio will be the inverſe Ratio of the Power, and the Weight or Reſiſtance.

The Forces, which acting contrariwiſe ballance one another, are always equal: if therefore the Intenſity of a Power be leſs than that of the Reſiſtance, it muſt run thro' a greater Space in the ſame Time; and that muſt always be in proportion as its Intenſity is leſs; for the Forces can differ in no other Reſpect, neither can we compenſate any other way for the Difference of Intenſity.

CHAP. XVI.

Of Sir Iſaac Newton*'s Laws of Nature.*

IN what we have ſaid of Machines, we have conſider'd the Actions of Powers and Weights acting continually againſt Obſtacles and other Reſiſtance; now we ſhall conſider Bodies left to themſelves, and continuing in Motion, or freely falling: And here we muſt reaſon from Phænomena, (as one muſt do in all Natural Philoſophy) and from them deduce the Laws of Nature.

Sir *Iſaac Newton* has laid down three, by which we think that every thing that relates to Motion may be explained.

LAW I.

All Bodies continue in their State of Reſt, or Motion, 124 *uniformly in a Right Line, except ſo much as they are forc'd to change that State by Forces impreſs'd.*

We ſee that Bodies, by their Nature, are inactive, and incapable of moving themſelves; wherefore unleſs they be moved by ſome extrinſical Agent, they muſt neceſſarily remain for ever at reſt.

A Body alſo, being once in Motion, continues in Motion according to the ſame Direction, in the ſame Right Line, and with the ſame Velocity, as we ſee by daily Experience; for we never ſee any Change made in Motion, but from ſome Cauſe. But (ſince Motion is a continual Change of Place) how the Motion in the ſecond

Moment of Time ſhould flow from the Motion in the firſt, and what ſhould be the Cauſe of the Continuation of Motion, appears wholly unknown to me; but, as it is a certain Phænomenon, we muſt look upon it as a Law of Nature.

LAW II.

125 *The Change of Motion is always proportionable to the moving Force impreſs'd, and is always made according to the Right Line in which that Force is impreſs'd.*

If to a Body that is already in Motion, another Force be ſuperadded to move it in the ſame Direction, the Motion becomes quicker, and that in proportion to the new Force impreſs'd *. *58

When a new Force impreſs'd is contrary to the Body's Motion, the Retardation follows the Proportion of the Impreſſion; ſo that a Force which is double or triple, &c. produces a double or triple Retardation. And generally all Forces produce Changes in Motion, according to their Directions and Quantities; other Actions of Forces would imply a Contradiction: This will appear more clearly by ſuch Experiments made upon oblique Forces, as we ſhall mention in ſome of the following Chapters.

LAW III.

126 *Action is always equal, and contrary to Re-action; that is, the Actions of two Bodies upon each other are always equal, and in contrary Directions.*

Which Way ſoever one Body acts upon another, we ſee that Body always ſuffers an equal and contrary Re-action. If I preſs a Stone with my Finger, my Finger is equally preſs'd by the Stone. A Horſe that draws a Cart forward, is as much drawn backward by the Cart; for the Geers or Traces are equally ſtretched both Ways.

When a Body ſtrikes againſt another, whatever the Stroke be, both ſuffer it equally; but the Impreſſions are contrary. This is clearly confirmed by the Experiments of the Congreſs of Bodies.

The Loadſtone draws Iron, and is equally drawn by Iron.

Experiment.] Suſpend the Loadſtone M, (*Plate* VII. *Fig.* 2.) in ſuch manner that it may eaſily be moved: then, bringing a Piece of Iron within a ſmall Diſtance of it, the Loadſtone will come to the Iron: And if you pull back the Iron, before the Stone be come to it, the Stone will follow the Iron; juſt as the Iron goes towards the Stone and follows it, when the Iron is ſuſpended and moveable, and the Loadſtone brought to the Iron.

When a Man ſits in a Boat, and by a Rope pulls towards him another Boat, juſt as big and as much laden, both Boats will be equally moved, and meet in the Middle of the Diſtance of the Places in which they were at firſt. If one Boat be greater than the other, or more laden, the Velocities in each will be different, when they have different Quantities of Matter; but the Quantities of Motion on both Sides will be equal, abſtracting from the Reſiſtance of the Water.

And this Law takes place generally in all the Actions of Bodies upon one another.

CHAP. XVII.

Of the Acceleration and Retardation of heavy Bodies.

DEFINITION I.

AN accelerated Motion *is that Motion, whoſe Velocity becomes greater every Moment.* 127

DEFINITION II.

128 A retarded Motion *is that, whoſe Velocity is diminiſhed every Moment.*

The Force of Gravity acts continually upon all Bodies, in proportion to their Quantity of Matter.* When a Body falls freely, the Impreſſion made upon it the firſt Moment is not deſtroyed in the ſecond Moment; therefore there is ſuperadded to it the Impreſſion made in the ſecond Moment, and ſo on. *The Motion* then *of a Body that falls freely, is accelerated,* and that *equally in equal Times*; becauſe the Force of Gravity acts every Moment in the ſame Manner,* and therefore communicates an equal Velocity to Bodies in equal Times. Whence *that Celerity, which is acquired in the Fall, is always as the Time in which the Body has fallen.* For Example: The Velocity acquired in a certain Time will be double, if the Time be double; and triple, if the Time be triple, &c.

* 79

129

* 75

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Let that Time be expreſſed by the Line AB (*Plate* VII. *Fig.* 3.) and let the Beginning of the Time be A. In the Triangle ABE, the Lines 1 *f*, 2 *g*, 3 *h*, which, being parallel to the Baſe, are drawn through the Points, 1, 2, 3, are to one another as their Diſtances from A, A 1, A 2, A 3; that is, as the Times which are expreſſed by thoſe Diſtances, and expreſs the Velocities of a Body falling freely after thoſe Times. If, inſtead of Mathematical Lines, others be taken with a very ſmall Breadth equal to each of them, the Proportion will not be changed thereby; and thoſe ſmall Surfaces will in the ſame manner denote the above-ſaid Velocities. In the leaſt Time the Velocity may be looked upon as equable, and therefore the Space gone through in that Time is proportionable to the Velocity.* In each

* 53

each of thoſe ſmall Surfaces above-mentioned, if the Breadth of the Surface be called the Time, the Surface itſelf will be the Space gone thro'. The whole Time A B conſiſts of thoſe very ſmall Times; and the Area of the Triangle A B E, of the Sum of all thoſe very little Surfaces, anſwering to thoſe ſmall Parts of Times: Therefore that Area expreſſes the Space gone through in the Time A B. After the ſame Manner the Area of the Triangle A 1 *f* repreſents the Space gone thro' in the Time A 1; thoſe Triangles are ſimilar, and their Areas are to one another as the Squares of the Sides A B, A 1: That is, *the Spaces, gone through from the Beginning of the Fall, are* 131 *to one another, as the Squares of the Times during which the Body fell.*

This is confirmed by Experiments made on the following Machine.

The Ballance A B (*Plate* VII. *Fig.* 4.) which 132 has but one Scale, is exactly in Æquilibrio; when a Weight is put into the Scale, an Iron, made in the Form of a Gnomon, keeps faſt the Brachium A, and the Ballance is retained in a horizontal Poſition.

At *f* there is a thin Spring *f g* fixed to the Gnomon, and which, when extended, reaches to *i*, where the End *g* is retained by help of the little Plate *i*, which is made faſt to the Brachium A. Now by this Means the leaſt Motion of the Ballance becomes ſenſible; becauſe then the Spring *f g*, being free, flies out, and returns to the Figure *f g*.

At the End of the Brachium B, there is a Hole, thro' which the String faſtened to the Hook D paſſes freely, that String is kept in a vertical Situation by hanging on the Weight N.

The Weight M has a Hole thro' it for the above-mentioned String to paſs freely thro'; in making Experiments, the Weight M is raiſed up along the String, and, when you let it go, it falls upon the ſame Point of the Brachium B.

Experiment.] Put the one Pound Weight P into the Scale; then the Body M, falling from the Height of three Inches, will move the Ballance. When P is a two Pound Weight, let M fall from twelve Inches, and the Ballance will be moved. If you lay on three Pounds in the Scale, the Body M muſt be let fall from a Height of 27 Inches, to move the Ballance and raiſe P. And in all theſe Caſes, if the Height from which M falls be taken but a little leſs, the Ballance, with the Weight in the Scale, will not be moved.

In this Experiment, the Weight which is laid upon the Scale, and raiſed by the Blow of the falling Body, is proportionable to the Stroke; the Quantity of Motion in the Body follows the Proportion of the Stroke: And that Quantity (becauſe we make uſe of the ſame Body) is propor-
*63 tionable to the Celerity *; and, laſtly, the Celeri-
*130 ty here is proportioned to the Time of the Fall *:
Therefore the Weights above-mentioned follow the ſame Proportion of the Time. The Weights here are as the natural Numbers 1, 2, 3, and therefore the Times are in that Proportion: But the Spaces gone thro' in thoſe Times are as 3, 12, 27, or as 1, 4, 9, which Numbers are the Squares of the others.

133 Having divided the Time A B (*Plate* VII, *Fig.* 3.) into the equal Parts A 1, 1 2, 2 3, 3 B, thro' the Diviſions draw Lines parallel to the Baſe; *the Spaces gone thro'* in thoſe Parts of Time, that is, *in the firſt, ſecond, and third Moment*, &c. *ſuppoſing the Moments equal, are to one another* as the Areas A 1 *f*, 1 *f g* 2, 2 *g b* 3, 3 *b* E B; which Areas, as appears by the Figure, are to one another *as the odd Numbers* 1, 3, 5, 7.

If the Body, after it has fallen, during the Time A B, ſhould be no more accelerated, but with the Celerity B E, acquired by that Fall, ſhould uniformly continue its Motion, during the equal Time B C, the Space gone thro' by that Motion is expreſs'd by the Area B E D C, which is double the Triangle A B E. And therefore,

A Body falling freely from any Height, with that Velocity which it has acquired by that Fall, will in a Time equal to the Time of the Fall (by an equable Motion) run thro' a Space double the ſaid Height. 134

Which Propoſition we ſhall alſo confirm by an Experiment*. *211

The Motion of a Body thrown upwards is retarded in the ſame Manner, as the Motion of a falling Body is accelerated by the ſecond Law*: In this Caſe the Force of Gravity conſpires with the Motion acquired; and in that it acts contrary to it. But, as the Force of Gravity is equal every Moment, *the Motion of a Body thrown up is equally diminiſhed or retarded in equal Times.* *125 135

The ſame Force of Gravity generates Motion in the falling Body, and deſtroys it in the riſing Body; therefore the ſame Forces are generated and deſtroyed in equal Times. *A Body thrown up riſes till it has loſt all its Motion; and ſo goes up during the ſame Time that a Body falling could have acquired a Velocity equal to the Velocity with which the Body is thrown up.* 136

If a Body be thrown up with the ſame Velocity that it would acquire in falling down the Line B C (*Plate* VII. *Fig.* 5.) it would aſcend in a Time equal to the Time of the Fall*, (and with an equable Motion) ſo as to come up to the Height C A, the double of B C*; but as in the ſame Time, by the Force of Gravity, the Body goes thro' a Space equal to the Space A B, or B C: as theſe two Motions obtain here at the *136 *134

the ſame Time, and act contrariwiſe, the leſſer
muſt be ſubſtracted from the greater; therefore
the Body, after the End of the Aſcent, will be at
137 B. Whence it follows, that *a Body thrown up
will riſe to the ſame Height, from which falling, it*
138 *would acquire the Velocity with which it is thrown
up.* And therefore, *the Heights, which Bodies thrown
up with different Velocities can riſe to, are to one ano-*
*131 *ther as the Squares of thoſe Velocities* *.
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CHAP. XVIII.

Of the Deſcent of heavy Bodies upon inclined Planes.

Definition I.

139 WE *call an* inclined Plane *that which makes an oblique Angle with the Horizon.*

C B in *Plate* VII. *Fig.* 6. repreſent a Line parallel to the Horizon; A B makes with it the oblique Angle A B C, and repreſents an inclined Plane; and the Perpendicular A C is let fall from A, the upper Part of the Plane, to the Horizon.

Definition II.

140 *The Length* A B *is called* the Length of the Plane.

Definition III.

141 *The Line* A C *is called* the Height of the Plane.

Let two equal Bodies deſcend by the Force of Gravity from the Point A, the one along the Line A B, and the other along the Line A C; when they are come to the Points B and C, they have deſcended equally, that is, they will be got each equally near to the Earth's Center: Therefore the Forces with which they are impelled, as they are directed towards the Earth's Center,

are equal; but the *Intenſities* of equal Forces are reciprocally as the Spaces gone through*; and * 70
therefore here the Intenſity of the Force, by which the Body is impelled along an inclined Plane, is to the Intenſity of it, by which it is directly impelled towards the Center of the Earth, as A C to A B. Therefore, *A Body laid upon an inclined Plane, loſes Part of its Gravity; and the Weight* 142
required to ſuſtain it, is to the Weight of the Body, as the Height of the Plane to its Length.

The Plane N O Q L (*Plate* VII. *Fig.* 7.) is 143
placed in an horizontal Situation; the Plane A B I H moves upon Hinges; and may be fixed at any Height, by help of the Screw V, and Quadrant *t*.

The wooden Ruler E F has a Pully C faſtened at one end, and revolves about the other; the Head D, about whoſe Center this Ruler moves, may be fixed (in any Place of the Slit *p s*) to the Plane N O Q L, by a Screw under the Plane.

M is a wooden Cylinder, whoſe Axis is of Steel, and whoſe Baſes ſomewhat exceed the Cylinder; ſo that, as it turns round along the Plane A B I H, the Baſes only touch the Plane.

The Cylinder is ſuſtained by a String that goves over the Pulley G; which String is fixed to a thin Braſs Ruler, bent in ſuch Manner, that the Axes of the Cylinder go thro' its Ends, and turn in them.

In making Experiments, the Pulley is ſo plac'd by the inclining Ruler E F, and moving the Head D along the Slit *r s*, that the String by which the Cylinder is ſuſtained, is parallel to the inclined Plane A B I H.

Experiment 1.] Let the Plane A B I H be inclined in any Manner, the Weight of the Body M has the ſame Ratio to the Weight P, as the Length of the Plane A B to its Height A C; and the Body M, in what Part ſoever of the inclined Plane it be ſet, will be ſuſtained upon it by the Weight P.

As the Force, by which a Body is made to deſcend along an inclined Plane, ariſes from Gravity, it is of the ſame Nature with it; and therefore that Force every Moment, and in all Parts of the
75 Plane, is equal: For the ſame Reaſon *the Motion*
144 *of a Body, freely running down upon an inclined Plane*, is of the ſame Nature with the Motion of a Body freely falling; and what has been ſaid of the one, may alſo be affirmed of the other. It *is* therefore
*129 *a Motion equally accelerated in equal Times*; * and therefore the Propoſitions of Numb. 130, 131, 133, 134, 135, 136, 137 and 138, may be here applied, if we ſuppoſe a Motion upon an inclined Plane, inſtead of a direct Aſſent or Deſcent.

145 The Forces *by which two Bodies deſcend, one of which falls freely, and the other runs down an inclined Plane, if the ſaid Bodies begin to fall at the ſame Time, are* always to one another *in the ſame*
*129 *Ratio as in the Beginning of the Fall**; therefore
144 the Effect of thoſe Forces, that is, *the Spaces gone thro' in the ſame time, are in the* ſame *Ratio*; name-
*142 ly, that *of the Length of the Plane to its Height**.

In the Plane AB (*Plate* VII. *Fig.* 8.) the Space gone through by a Body, whilſt another falls freely down the Height of the Plane AC, is determined by drawing CG perpendicular to AB: for then the Length of the Plane AB is to its Height AC, as AC to AG. If a Circle be deſcribed with the Diameter AC, the Point G will be in the Periphery of the Circle; becauſe an Angle in a Semicircle, as AGC, is always a Right Angle; and therefore a Point taken, as G, in any Inclination of the Plane, will always be in the Periphery of the ſaid Circle: Whence it follows, that all the

Chords, as A G, are gone thro' by Bodies running along them, in the Time that a Body, falling freely, would run down the Diameter A C; and therefore the Time of the Falls thro' thoſe Chords are equal. Thro' the Point C there can be drawn no Chord, as H C, but what a Chord, as A *g*, may be drawn parallel to it, (that is, 147
equally inclined) and equal; therefore in a Semicircle, as A H C, *whether a Body falls freely along the Diameter* A C, *or whether it falls down along any Chord, as* H C, *it will in the ſame Time come to the loweſt Point of the Semicircle.*

The Time of the Fall along the whole Plane A B may be compared with the Time of the Deſcent along the Height A C; which for that Time is equal to the Time of the Fall along A G; and ſo the Squares of thoſe Times are to one another,
as A B to A G.* But A B is to A C, as A C* 145
to A G; therefore the Squares of the Lines A B 131
and A C are to one another, as A B to A G; and
therefore *thoſe Lines* A B *and* A C *are to one ano-* 148
ther as the Times of the Fall along A B *and* A G, *or* A C; *that is, the Times in that Caſe are, as the Spaces gone through.*

In the ſame Caſe, the Velocities, at the End of 149
the Deſcent, are equal; for after equal Times, when the Bodies are at G and C, the Velocities are in
the ſame Ratio as in the Beginning of the Fall*,* 129
that is, as the Forces by which the Bodies are 144
impell'd, or as A C to A B.* When the Bo-* 142
dy deſcends from G to B, the Velocity increaſes as the Time; and the Velocity at G is to the Ve-
locy at B, as A C to A B:* Therefore the* 148
Velocities at B and C have the ſame Ratio to the Velocity at G, and ſo are equal. Hence it appears, that *a Body acquires the ſame Velocity,*
in falling from a certain Height, whether it falls di- 150
rectly down, or along an inclined Plane; and ſince

the Angle of Inclination cauſes no Alteration, *a Body may run down ſeveral Planes differently inclined, and even along a Curve,* (which may be conſider'd as made up of an innumerable Number of Planes differently inclined) *and the Celerity acquired will always be the ſame, when the Height is equal.*

Experiment 2.] In this Experiment it is to be obſerv'd, that a Body hanging by a Thread, and deſcribing a Curve by its Fall, falls in the ſame manner, as if it was to run down ſuch a Curve cut hollow in a ſolid Body without any Friction.

Let the Body P, (*Plate* VII. *Fig.* 9.) ſuſpended by a Thread, fall from the Height A C, in the Curve B C, and in the Curve D C, and in the Curve E F G C, made up of Parts of different Circles, and in each Caſe it will, with the ſame Force, ſtrike againſt the Body Q, which is at reſt; and always drives it to the ſame Height.

151 *A Body that has acquired any Celerity in falling down along any Surface, whether Plane or Curve, will riſe up to the ſame Height along another ſimilar Surface, with the ſame Velocity, in the ſame Time.*

152 *A Body will, with the ſame Celerity that it has acquired in falling down from a certain Height, riſe up to the ſame Height in any Curve whatever.*

Experiment 3.] Let the Body P, (*Plate* VIII. *Fig.* 1.) hanging by a Thread, fall from the Height A C, along any Curve B C; with the Celerity which it has thereby acquired, it will aſcend to the ſame Height, on the other Side, in the Curves C D or C E, or C H G F.

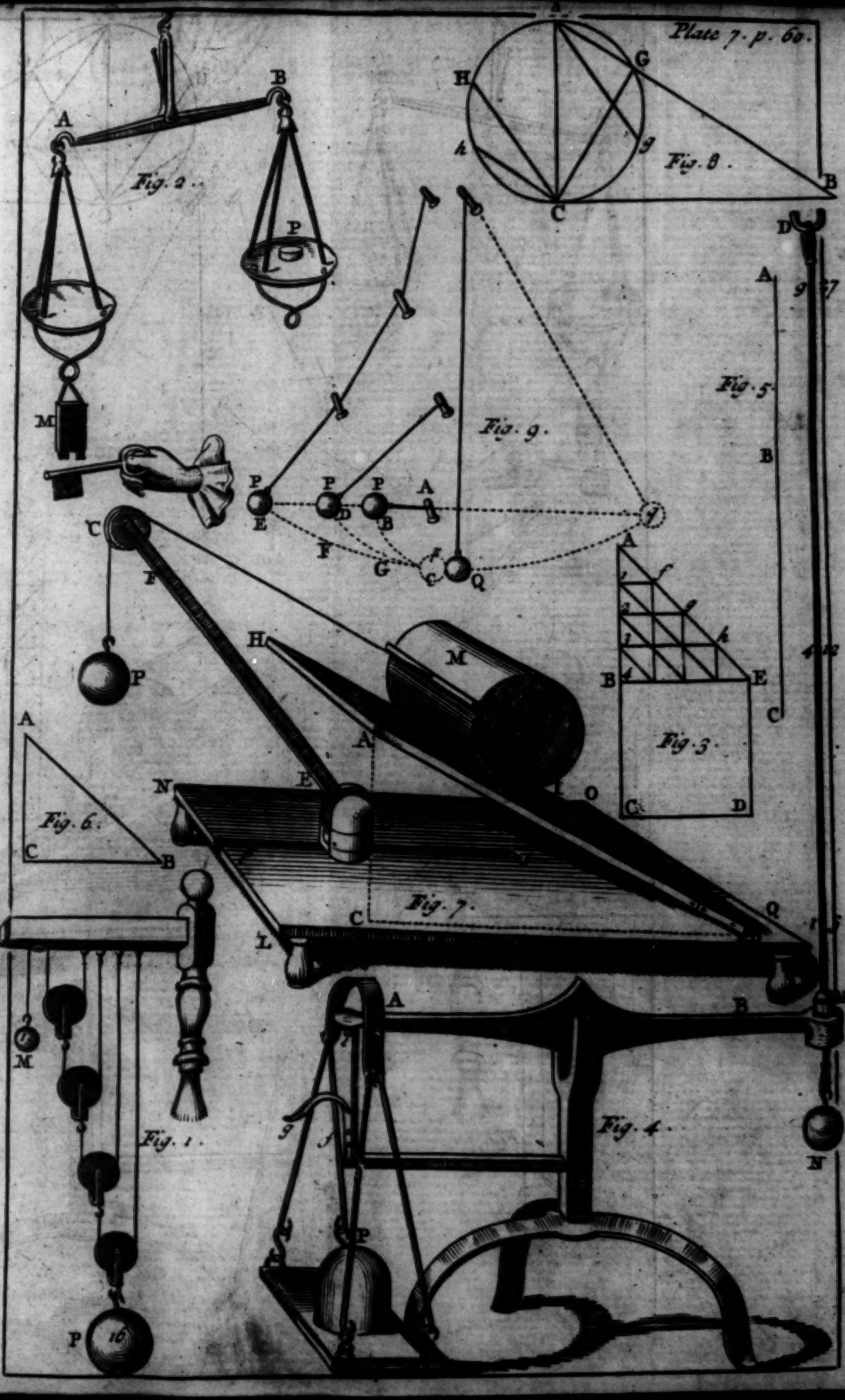
Plate 7 · p. 60.
Fig. 1
Fig. 2
Fig. 3
Fig. 4
Fig. 5
Fig. 6
Fig. 7
Fig. 8
Fig. 9
A
B
C
D
E
F
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CHAP. XIX.

Of the Oſcillation or Vibration of Pendulums.

DEFINITION I.

153 A *Heavy Body, hanging by a ſmall Thread, and moveable with the Thread about the Point to which the Thread is fix'd, is call'd* a Pendulum.

The Motion of a Pendulum is an oſcillatory or
vibratory Motion. When the Weight, the Thread
being extended, is raiſed upon one Side, it deſcends
by its Gravity, and, with the Celerity that
it has acquired, riſes up to the ſame Height on the
other Side *; and then it returns by its Gravity, * 151
and ſo continues in its Vibrations.

We here ſuppoſe the Motion about the Point of Suſpenſion to be perfectly free, and that there is no Reſiſtance of Air, which is very ſmall in great Pendulums.

The Body P (*Plate* VIII. *Fig.* 2.) does, in its 154
Motion, deſcribe the Arc P B F; if, inſtead of
that Motion, *a Body* ſhould deſcend along the
Chord P B, and again aſcend along the Chord B F,
and ſo *ſhould perform its Vibration in Chords*; the
Deſcent would be made in that Time in which
the Body by its Fall would go thro' the Length
of the Diameter A B *; that is, twice the Length * 147
of the Pendulum. In an equal Time, it aſcends
along the Chord B F *; therefore *in the Times of* * 151
one whole Vibration, the Body in falling might run
through four Diameters *; that is, *eight Times the* * 131
Length of the Pendulum. And as the Deſcent and
Aſcent in any Chord is performed in the ſame
Time, all the Vibrations in Chords, whether great
or ſmall, are likewiſe performed in the ſame Time.
In ſmall Vibrations, the Arcs of a Circle do not
ſenſibly

ſenſibly differ from the Chords; *and the Vibra-*
155 *tions of the ſame Pendulum, tho' unequal, are performed in the ſame Time, as far as our Senſes can diſtinguiſh.*

Experiment 1. *Plate* VIII. *Fig.* 4.] If the two equal Pendulums, CP and *cp*, are let fall from the Points P and *p* in the ſame Moment of Time, they will at the ſame Time come to B and *b*, and then to F and *f*; and ſo they will continue their Motion in the Arcs PBF and *pbf*, always in the ſame Time.

Here it is to be obſerved, that tho' the Propoſition 155 be true in all Pendulums, the Demonſtration given is only to be applied to ſhort Pendulums; in the longer Sort, the Time of the Deſcent along a Chord differs ſenſibly enough from its Deſcent along an Arc; but in ſmall Arcs the Differences are equal, tho' the Chords be of different Lengths.

Let the Circle FB (*Plate* VIII. *Fig.* 3.) roll along the Line AD, till the Point B comes to A in the ſame Line; by ſuch a Motion the Point B deſcribes a Portion of the Curve BPA. Such another Curve BD may be deſcribed in the ſame Manner, and the whole Curve ABD is call'd *a Cycloid.* Let it be divided into two equal Parts at B, and let the Parts BA and BD be ſo diſpoſed as to have the Points A and D fall in together at C; and let the Point B coincide with the Points A and D in the Line AD. Let two Plates of Metal be bent according to theſe Curves, ſo that the Thread of a Pendulum, ſuſpended at C, may on either Side apply itſelf to thoſe Plates, and fall in with their Curves as the Pendulum vibrates. Now if the Length of the Pendulum be CB, the Body P in its Vibrations will deſcribe the Cycloid ABD.

It

156 It is a Property of this Curve, that in whatever Point of it the Body P be placed, the Force, with which it is carried by its Weight along the Curve, is proportional to the Part of the Curve which is between that Point and the loweſt Point of the Curve B. Whence it follows, that if two Pendulums, as CP, be let fall in the ſame Moment from different Heights, the Velocities, with which they begin to fall, are to one another, as the Spaces to be run through before they come to B: If therefore they ſhould be acted upon by thoſe Forces alone with a Motion not accelerated, they would come to B at the ſame Moment of Time*: After the ſame Manner, by the Forces *53 which are acquired the ſecond Moment, they alſo come to B at the ſame Time. The ſame may be ſaid in relation to the following Moments; and the half Vibrations made up of all the Forces together, however unequal they are, as alſo the whole Vibrations are performed in the ſame Time.

157 If is moreover demonſtrated by Geometricians, that *the Time of each Vibration is to the Time of a vertical Fall, along the half Length of a Pendulum, as the Periphery of a Circle to its Diameter.* In this Curve the lower Part coincides with a ſmall Arc of a Circle, as to Senſe: And this is the true Reaſon, why in a Circle the Times of ſmall Vibrations (however unequal thoſe Vibrations be) are equal; and therefore alſo the Duration of thoſe Vibrations has the above-mentioned Ratio to the Time of a vertical Fall. 158 The Durations of the *Vibrations of unequal Pendulums* may be compared together: When the Arcs are ſimilar, the Deviations, in reſpect of the Chords, are alſo ſimilar; and the Times of the Vibrations in the Arcs are, as the Times of the Vibrations along the Chords; but they are, as the Times of the

Deſcent along Lengths, eight times greater than
* 154 the Length of Pendulums*; and ſo the *Squares
of the Durations are* as thoſe eightfold Lengths,
* 131 *or *as the Lengths of the Pendulums.*

Experiment 2.] Two Pendulums, CP, *cp*, (*Plate* VIII. *Fig.* 5.) whoſe Lengths are as 4 to 1, are let fall at the ſame Time from the Points P and *p*, ſo that in their Vibrations they deſcribe ſimilar Arcs; the longer Pendulum vibrates once, whilſt the ſhorteſt vibrates twice; and ſo the Squares of the Durations of the Vibrations are as 4 to 1, namely, as the Lengths of the Pendulums.

When the Vibrations are ſmall, this Ratio alſo holds, tho' the Pendulums ſhould not vibrate
* 155 in ſimilar Arcs*.

159 *The Velocities of Pendulums in the loweſt Point, when the Vibrations are unequal, are to one another, as the Subtenſes of thoſe Arcs, which the Body deſcribes in its Deſcent.* So the Velocity of the Body P, (*Plate* VIII. *Fig.* 2.) falling in the Arc PB, is to its Velocity when it falls along DB, as the Chord PB, to the Chord DB: For if you draw in a Circle the Lines P*f*, D*d* parallel to the Horizon, the Squares of the ſaid Chords are to one another, as the Lines *f*B, *d*B. The Squares of the ſaid Velocities are alſo, as thoſe Lines *f*B, *d*B;
* 150 therefore the Velocities are as the Chords*.

Concerning all that has hitherto been ſaid of
131
130 Pendulums, it is to be obſerved, that it is no
160 matter how big the Weight of the Pendulum is, or whether the Weights of two Pendulums be different in Magnitude, or different Sorts of Bodies; ſince Gravity is proportioned to the Quantity of Matter in all Bodies, all Bodies in the ſame Circumſtances are moved by Gravity with

with the ſame Velocity. Which is alſo confirm'd by the following Experiment.

Experiment 3.] Take two equal or unequal Balls, the one of Lead, and the other of Ivory; hang them up by Threads, that they may make Pendula of equal Lengths; let them vibrate, and their equal Vibrations (or even all their unequal ones, provided they be ſmall Vibrations) are perform'd in the ſame Time.

Oftentimes inſtead of a Thread, a ſmall, but 161 ſtiff, Iron-Rod is made uſe of, and ſometimes alſo two or more Weights are fix'd to it, and it is called a *Compound Pendulum:* in that Caſe the Rules above-mentioned are not applicable; but thoſe Pendulums are reduced to ſimple ones, by determining in them ſuch a Point, that, if all the Weights were united in it, the Vibrations would be of the ſame Duration as thoſe of the compound Pendulum. This Point is called the *Center of Oſcillation.*

The Center of *Percuſſion* in a compound Pen- 162 dulum is a Point, in which the whole Force of the Pendulum is as it were collected; ſo that if that Point ſtrikes againſt an Obſtacle, the Blow will be greater, than if any other Point of the Pendulum ſhould ſtrike againſt it.

In a Vacuum, or a Medium that does not reſiſt, theſe two Centers coincide. They alſo coincide in the Air, as to Senſe, by reaſon of the ſmall Reſiſtance.

A Body of any Figure may be ſuſpended, and vibrate about a Point, or rather an Axis; and in ſuch a Body one may alſo determine the Center of Oſcillation.

When a Right Line, ſuch as is an Iron Wire, 163 *vibrates about one End, the Center of Oſcillation is* *diſtant from the Point of Suſpenſion two third Parts of the Length of the Wire.*

Experiment 4.] The flat Iron AB (*Plate* VIII. *Fig.* 6.] muſt be ſo hung up, as to vibrate about the End A; let the ſimple Pendulum CP, whoſe Length is equal to two third Parts of AB, be ſuffered to deſcend at the ſame Time as the Iron; and the Vibrations of the Pendulum and the Iron will be perform'd at the ſame Time.

The Vibrations of Pendulums, as we have ſaid, * 155 tho' unequal, are perform'd in the ſame Time,* and this Property of Pendulums is of great Uſe in Clocks, to which an equable Motion is communicated by fixing on a Pendulum.

By carrying Clocks to different Places, it has appear'd that the Force of Gravity is not equal in all Parts of the Earth, becauſe the Vibrations of the ſame Pendulum, in divers Countries, have been found unequal, in reſpect to Time; and that Difference of Gravity is meaſured by Pendulums.

164 *Let there be two Pendulums, whoſe Lengths are to one another, as the Forces of Gravity by which they are actuated*; if they run out into ſimilar Arcs, in correſpondent Points, the Force will always have the ſame Ratio to one another, and indeed the Ratio of the Spaces to be gone thro', (becauſe ſimilar Arcs are as the Lengths of Pendulums,) which therefore will be run thro' in * 53 equal Times, * that is, *the Vibrations will be perform'd in the ſame Time.*

If they be reduced to the ſame Length by changing one Pendulum, the Square of the Time of the Vibration of the Pendulum that is changed, is to the Square of the Time of Vibration before the Change (that is, to the Square of the Time of the Vibration of the Pendulum that

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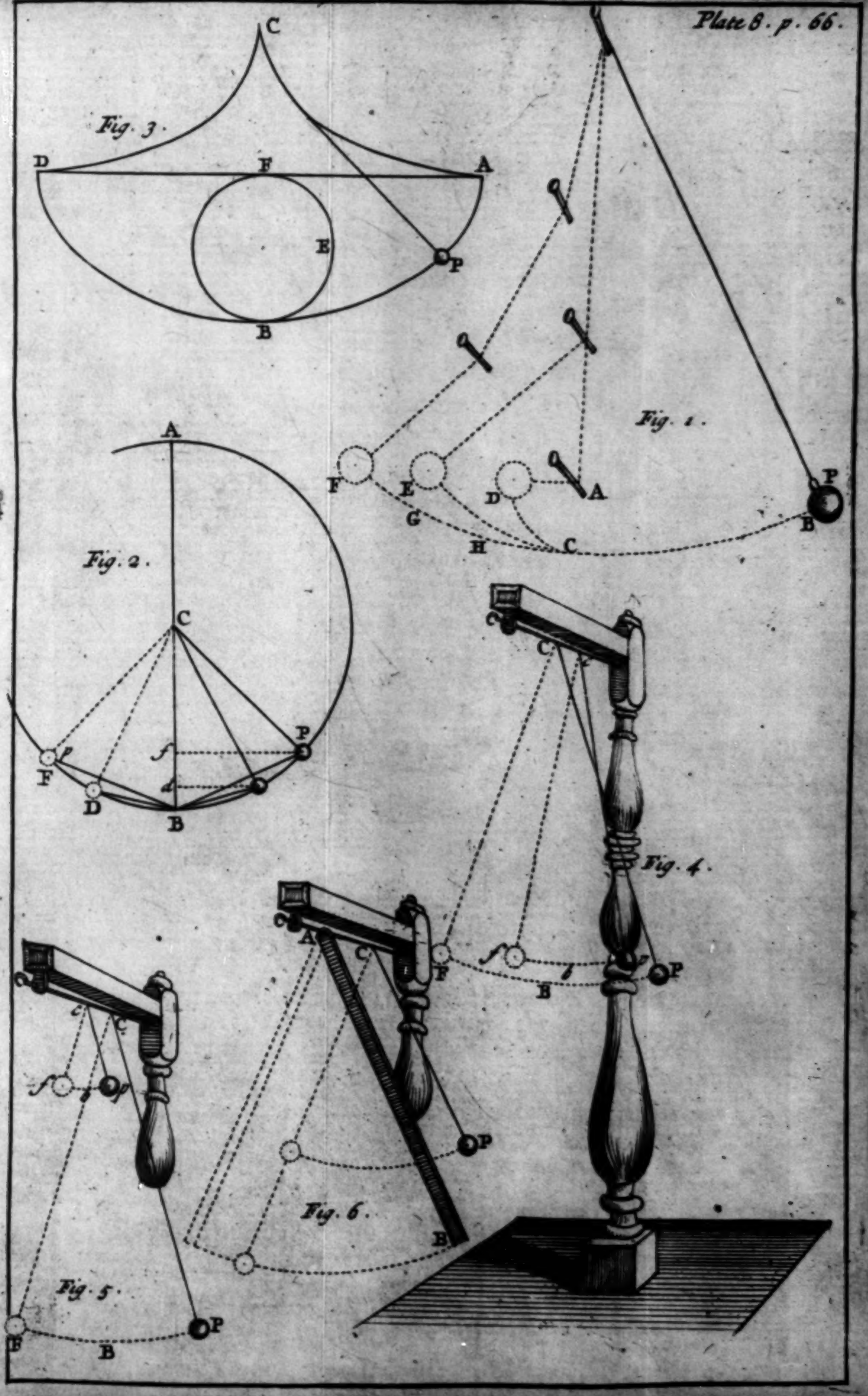


is not changed) as the Length of the Pendulum
after the Change to its firſt Length: * which * 158
Lengths are to one another, as the Force of Gravity in the Pendulum that is not changed, to the
Force of Gravity in the Pendulum that is changed
And therefore the Squares of the Times of the Vi- 165
brations *in equal Pendulums* are to one another,
inverſely, as *the Forces of Gravity with which the Pendula are acted upon:* which therefore *are to one another*, directly, *as the Squares of the Vibrations perform'd in the ſame Time.*

But whence this Difference of Gravity ariſes, ſhall be explain'd hereafter, when we ſpeak of the Figure of the Earth.

CHAP. XX.

Of Percuſſion, and the Communication of Motion.

EVery Body that is at Reſt, and hinder'd by
no Obſtacle, may be puſh'd forward by any
other Body in Motion; and, when once it is put
in Motion, it will continue in it, till it is hinder'd
by ſome external Cauſe.* That Cauſe is ſome- * 142
times a Stroke of another Body againſt it, or a
Stroke which itſelf gives another Body; or, laſtly,
a Stroke of both meeting.

The Laws to be obſerved in that Percuſſion, are here to be explain'd.

All Bodies, here taken notice of, are ſuppoſed ſpherical; becauſe the Laws of Motion ought to be examin'd in the moſt ſimple Caſe.

DEFINITION I.

A Body is ſaid to *impinge* directly againſt ano- 166
ther, or two Bodies to ſtrike or impinge againſt
one another, *when the Direction of the Motion, or
Motions, (if both are moved) goes thro' the Centers
of both Bodies.*

DEFINITION II.

167 *In all other Caſes the* Stroke *is ſaid to be* Oblique. When elaſtic Bodies impinge againſt one another, the Parts that are ſtruck yield inwards, and, by the Reſtitution of the Parts, the Bodies repel one another, and are ſeparated from one another.

168 *In Bodies that are perfectly ſoft, or perfectly hard,* there is no ſuch Action; and therefore, *in a direct Stroke they are not ſeparated after the Blow,* becauſe
169 *after their meeting, as well as before, they are moved in the ſame Line*; for nothing happens that can change the Direction.

I ſhall in this Chapter ſpeak of the Percuſſion of Bodies that are not elaſtic; and here, as alſo in the whole following Chapter, I ſhall ſpeak of direct Percuſſion; and confirm the whole by Experiments made with the following Machine.

170 ABC is a vertical Plane of Wood almoſt triangular, about 4 Foot and a half high, and 3 Foot wide at the Bottom. *Plate* IX.

In the upper Part there is a Slit *st* quite thro' it, which is horizontal, along which two ſquare Pins (and ſometimes more) are moved; theſe Pins, having a Shank that goes through the Plane, may be made faſt in any part of the Slit by Screws, which take the Shanks behind the Plane or flat Board, as may be ſeen from the Figure of the Pins at V.

A little ſquare Pipe of Iron X, ſlips upon each Pin, and may be faſten'd to it by a little Screw *e*, in the upper Part of any Place of the whole Length of the Pin. Theſe little Pipes have Hooks in the under Parts, thro' which ſmall Threads or Fiddle-ſtrings run, and ſuſtain ſuch Balls as P and Q. Thoſe Strings go round the wooden Keys *l*, *l*, by turning which, the Balls are rais'd or lower'd.

The Pin, from which any Ball hangs, is fix'd to ſuch a Part of the Slit *s t*, that its Center is diſtant from the Line A D (which divides the Machine into two Parts vertically) juſt one Semidiameter of the Ball; and that is to be done for all the Balls, by means of Marks in the Surface of the Board.

The little Pipe and Hook, from which the Ball hangs, is fix'd to ſuch a Part of the Pin, that the Thread hangs but a little farther from the Surface of the Board than a Semidiamiter of the Ball. There are Diviſions in the Pins, to determine the Place of the Iron Pipes upon the Pins, according to the Bigneſs of the Balls.

When you uſe two Balls, the Line A D ſeparates them, and in that Caſe (as alſo when ſeveral at once is made uſe of) if they are of different Bigneſs, the great Ball always determines the Diſtance of the little Ball from the Board; and the little Pipes are fix'd at ſuch Diviſions of the Pins, that the Centers of all the Balls may be equally diſtant from the Board. The Keys *l* bring all thoſe Centers to the ſame Heights; which is to be obſerv'd in all the Experiments.

There are two Braſs Rulers E G, E G, which ſlide horizontally in the Board, whoſe Surface is hollowed to receive them, ſo that their Surface may lie even with it. Behind each Ruler there is a Slit in the Board of about 5 Inches, to tranſmit a Screw coming from the Backſide of the Ruler, which is fix'd behind a Nut in any Part of the Slit. In making Experiments, the End G of each Ruler is diſtant from the Line A D, one Semidiameter of the Ball, which hangs on the ſame Side.

Theſe Rulers are ſo divided, as to ſhew equal Angles run thro' by the Threads which carry the Balls.

To meaſure thoſe Angles in making the Experiments, there are four Indices, two great ones M M, and two leſs N N.

Theſe Indices, ſliding in a Groove, are moved along the Slits *or*, *or*, and are faſtened behind the Board, where you pleaſe, in the ſame Line by Screws. The longer Indices reach to the Edge of the Board, tho' the Slits want about 3 Inches of it.

The ſeparated Figure M repreſents the greater Indices, in which *a b* is a Plate, which ſlides in the Groove of the Board; *c d* is the Index, perpendicular to that Plate, and about 3 Inches long.

The other ſeparated Figure N, repreſents one of the leſſer Indices, whoſe Length is equal to the Semidiameter of the ſmaller Balls, which are applied to the Machine, and whoſe Diameter may be about 1 Inch and a Half: Theſe Indices are put among the great ones, becauſe they don't hinder the Motion of the Balls: Sometimes the two ſmall ones are put in the ſame Slit, when three Angles are to be meaſured on one Side.

In that Caſe, the Ball Q is raiſed up, or riſes after its Fall towards the Side of the Board B. That the Index may be placed right for meaſuring that Angle, the end G of the Ruler E G, which is on the Side B, muſt be joined with the End G of the other Ruler, placed as above-mentioned.

The three Iron Screws F F F ſerve to ſet the Machine or Board truly vertical, ſo as to bring the Line A D perpendicular to the Horizon; which may be eaſily done by hanging on any one of the Balls, and putting on one of the great Indices; ſo that the Thread, cutting any Mark on the Index, may hang parallel to the Line A D.

For making Experiments on Bodies that have no Elaſticity, you muſt uſe Balls of ſoft Clay, made in the wooden Mould L.

This Mould conſiſts of five Parts, four of which may be ſeen at H, H, H, H; theſe being join'd, contain a ſpherical Cavity of an Inch and a Half Diameter; with a Hole in the lower Part: there is a Screw on the Outſide, by which they are preſs'd together by means of the Ring I, that has a Screw on the Inſide.

L repreſents all the Parts join'd together; there is a Hole in *v*, which has a Communication with the Inſide of the Mould: thro' this Hole muſt go a Thread, which lies irregularly in the Clay, almoſt thro' it. Before you put the Clay into the Mould, you muſt anoint the Inſide of it with Oil; then, when all the Parts have been join'd and preſs'd together by ſcrewing on the Ring, take them aſunder again, and you will find a ſmooth and round Clay Ball, to the Thread of which you may faſten another Thread, and immediately hang it upon the Machine.

The Experiments, relating to elaſtick Bodies, are made with Ivory Balls. You muſt have ſix ſmall ones, of an Inch and half Diameter. Beſides thoſe, one of double the Weight, another of three times the Weight, and a fourth of four times the Weight.

In the 11th Experiment of the following Chapter, the ſix equal Balls above-mentioned are hung on to the Machine at the ſame Time, ſo as to touch one another. And this is done (ſee *Fig.* Z) by means of the Plate *mn*, which is fix'd to the Machine by help of the Screws *qq*, which go thro' the Slit *st*. This Plate contains four Pins, *p*, *p*, *p*, *p*, in whoſe Ends are Holes, thro' which the Threads paſs that carry the Balls. The Threads are brought to a proper Length, and ſtaid by the

Keys *l*, *l*, *l*, *l*. The two other Balls hang from the two Pins V, already deſcribed.

To make the 13th Experiment of the next Chapter, there muſt be three ſuch Pins as V.

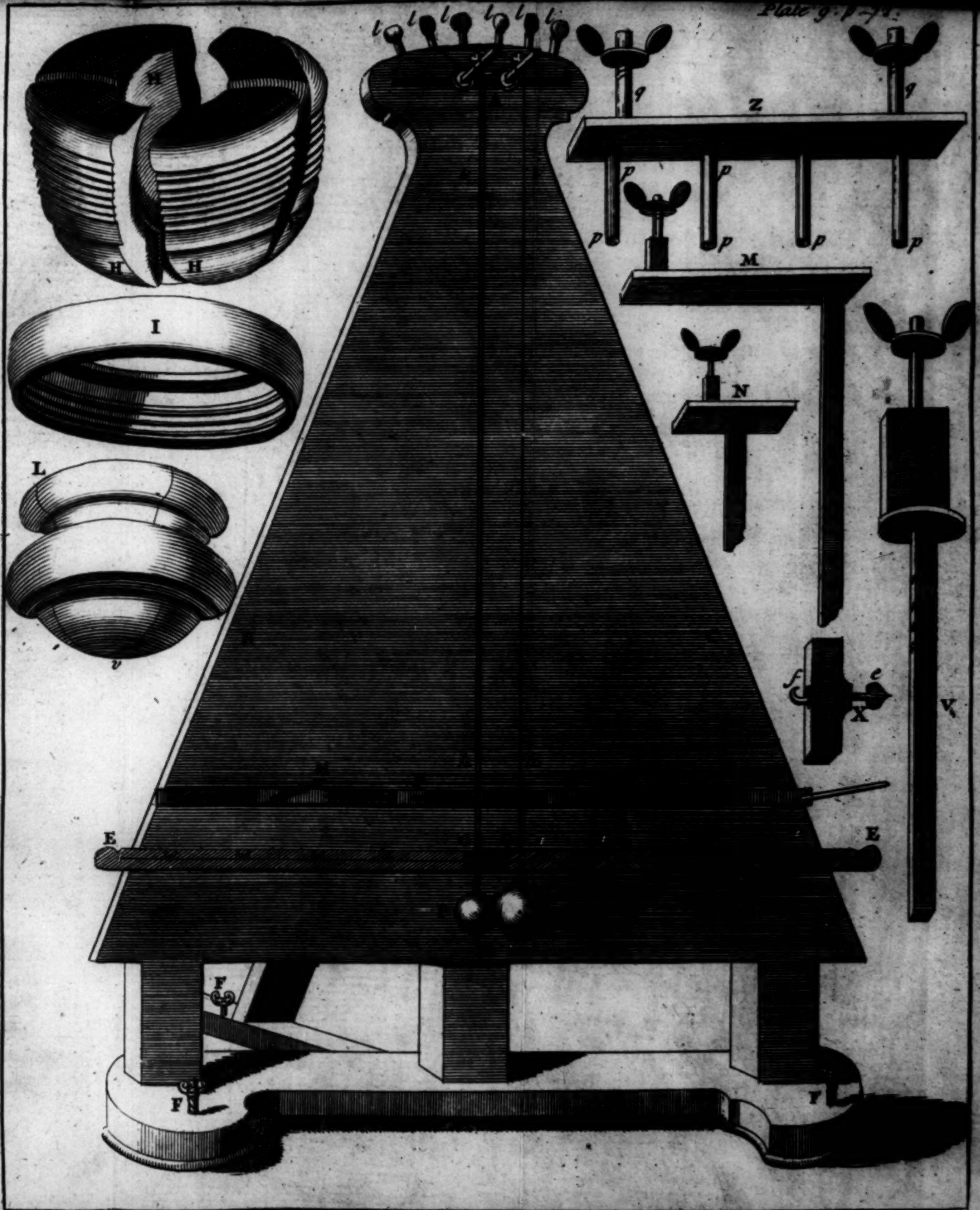
In this Machine the Percuſſion of the Balls, in the loweſt Part of it, is always direct; and the Balls (whether you let them go from different Heights, the ſame Way or contrary Ways) will *155 always come to the Bottom at the ſame Time*; and ſo in that Caſe the Percuſſion is always direct; the Celerities at the Bottom are mark'd by the *159 Diviſions of the Rulers EG, EG*; for in Arcs no greater than ſuch as the Balls deſcribe in this Machine, the Ratio between the Arcs and Chords does not ſenſibly differ. The Heights, from which the Balls are let fall, determine the Celerities before the Stroke; and the Heights, to which the Balls riſe, their Celerities after the Stroke.

All that relates to the Percuſſion of Bodies not elaſtick, may be referred to the four following Caſes.

171 Caſe 1.] *If one Body ſtrikes againſt another Body that is at reſt, both together will continue their Mo-* *168 *tion in the ſame Direction as the firſt Motion**; *and* 169 *the Quantity of Motion, in the two Bodies, will be the ſame after the Stroke, as in the ſingle one before it.*

For the Action of the Body in Motion, upon the other, communicates to it all the Motion that it acquires: Now the Re-action of this laſt in the firſt retards its Motion; and as Action *126 and Re-action are equal*, therefore the Quantity of Motion, acquir'd by one Body, is equal to the Quantity of Motion loſt by the other; and ſo the Quantity of Motion is not changed by the Stroke.

Plate 9. p. 71.



This Quantity of Motion is found by multiplying the Maſs of the firſt Body by its Velocity; * and dividing that Quantity by the Maſs of both Bodies, you will have the Velocity after the Stroke. * 64

For Example, take two equal Bodies, in each of which the Quantity of Matter may be expreſs'd by One; let the Velocity of the moving Body be Ten, the Quantity of Motion will alſo be Ten, which muſt be divided by Two, the Maſs of both Bodies, and Five, the Quotient of the Diviſion, will be the Celerity of the Bodies after the Stroke.

Experiment 1.] Take the two ſoft Clay Balls P and Q, and hang them upon the Machine of *Numb.* 170. See *Plate* X. *Fig.* 1.

Let fall the Ball P from the Height anſwerable to the tenth Diviſion of the Ruler E G, ſo that it may ſtrike againſt the Ball Q, which is at reſt; after the Stroke they will both move together, and riſe up on the other Side to the fifth Diviſion of the other Ruler E G: The reſt of the Experiments in this Chapter are made with the ſame ſort of Balls.

Caſe 2.] *If one Body ſtrikes another that moves the ſame Way, but ſlower, they will both continue their Motion in the ſame Direction as before; and the Quantity of Motion, after the Stroke, will be the ſame as before.* 172

The Reaſon of this Propoſition is the ſame as that of the foregoing.

In this Caſe the Celerity of the Bodies, after the Stroke, is determined by multiplying each Body by its Celerity; the Products of which Multiplications will give the Quantities of Motion in each Body; * by collecting them into one Sum, *64

Sum, you have the Quantity of the whole Motion; which if you divide by the Maſs of both Bodies, the Quotient will be the Celerity required.

Experiment 2.] Take the equal Bodies P and Q (*Plate* X. *Fig.* 2.) and let them go towards the ſame Side, P with the Velocity 10, and Q with the Velocity 6; as the Maſs of each Body is 1, the
* 63 Quantity of Motion in both together will be 16;* which if you divide by 2, the Maſs of both Bodies, the Quotient will be 8; and the Experiment will ſhew the Velocity to be anſwerable to this.

173 Caſe 3.] *When two Bodies, with equal Quantities of Motion, are carried towards contrary Sides, the whole Motion will be deſtroyed by their meeting, and the Bodies will be at reſt.*

* 168 The Bodies are not ſeparated after the Stroke,* and the Line in which they move cannot be
* 169 changed;* but that they may continue to move in the ſame Line, it is required that one Motion ſhould overcome the other, which implies a Con-
* 62 tradiction.*

Experiment 3. *Plate* X. *Fig.* 3.] Let two equal Bodies, P and Q, fall from contrary Sides with equal Velocities, and as ſoon as they meet they will be at reſt.

134 Caſe 4.] *Two Bodies moved with different Velocities contrariwiſe, after having ſtruck one another, will both together continue their Motion in the ſame Direction, towards that Side where there is moſt Motion; and the Quantity of Motion, after their meeting, is equal to their Difference of Motion before the Stroke.*

The greateſt Motion overpowers; therefore the Bodies muſt be carried together the Way that the Motion is directed; * and a Body, which * 168 has a leſs Quantity of Motion, is carried in the 169 ſame Line (but in a contrary Direction) as before the Stroke; for this is required, that by the Action of one Body, the whole Motion of the other be deſtroyed, which cannot be done, unleſs that Body by the Re-action loſes an equal Quantity of Motion; there remains therefore only the Difference of the Motions.

Multiplying the Maſs of each Body by its Celerity, we have the Quantities of Motion; the leaſt of which muſt be ſubſtracted from the greater, to have the Difference of the Motions; which Difference, if it be divided by the Maſs of both Bodies, will give the Celerity after the meeting of the Bodies.

Experiment 4. *Plate* 10. *Fig.* 4.] Let the Body Q be moved with the Celerity 14, and an equal Body P in a contrary Direction with the Celerity 6; after meeting, the Body Q continues its Motion, and carries along with it the Body P with the Celerity 4.

Becauſe of the Equality of the Bodies, the Quantities of Motion will alſo be 14 and 6; ** 63 and their Difference is 8; which Number being divided by 2, the Maſs of both Bodies, the Quotient 4 will be the Celerity after the Stroke.

DEFINITION III.

We call Relative Celerity, *that with which one* 175 *Body is carried towards another, or with which two Bodies are ſeparated*; in Motions directed the ſame Way, it is the Difference of the Celerities of the Bodies; and, in contrary Motions, it is the Sum of the Celerities.

In

176 *In the Congreſs of Bodies, the Stroke is proportional to that Relative Celerity.* For the Force of Bodies, ſtriking againſt each other, is increaſed or diminiſhed, according to the Celerity with which two Bodies come towards one another.

CHAP. XXI.

Of the Congreſs of Elaſtic Bodies.

AN Elaſtic Body, whoſe Figure is changed by any Force, will, when the Action of that Force ceaſes, by its Elaſticity or Spring, return to its firſt Figure.* *44

DEFINITION.

177 *A Body has* perfect Elaſticity, *when it returns to its firſt Figure with the ſame Force with which it was preſs'd in.*

178 In that Caſe, *the Stroke, ariſing from the Reſtitution of the Spring, is equal to the Stroke by which the Figure of the Body was alter'd.*

In this Chapter we ſuppoſe this Sort of Elaſticity, tho' we know no Bodies perfectly elaſtic: In different Bodies, the Force by which the Parts return to their former Figure is very unequal, for which Reaſon we can give general Rules only, concerning perfect Elaſticity; the nearer Bodies approach this Elaſticity, the more exactly will their Motion agree with theſe Rules.

The Experiments, that we ſhall mention in this Chapter, are to be made with the ſame Machine* that the Experiments of the laſt Chapter were made with; and here we are to uſe Ivory Balls, ſuch as are mention'd in the Deſcription of the Machine; for the Want of perfect Elaſticity, and the Reſiſtance of the Air, do not make a ſenſible Error in the Experiments; which alſo, when *170 when Neceſſity requires, may be corrected by determining the Difference ariſing from it.

Which Way ſoever two Bodies ſtrike againſt each other, the mutual Actions of the one againſt the other are always equal*. By that Action* the Parts of Bodies are puſh'd inwards, and that with equal Force in both Bodies; by their Elaſticity alſo they return with equal Force to the firſt Figure. The Action of Bodies upon each other, from their Reſtitution by their Spring, is equal to the firſt Action from the Stroke*;* whence it follows, that *the Action of Bodies upon each other is double in elaſtic ones*; *that is, double in reſpect of each Body conſider'd ſingly*, becauſe of the Equality of the Action in each. *The Change* therefore, *which in that Caſe is produced in the Motion of each Body by the Stroke, is double that which the Stroke would by the ſame Motion produce in Bodies that have no Elaſticity*; and as, in reſpect of theſe Bodies, the Change (both in reſpect to the Quantity of Motion, and in reſpect to the Celerity) is determined in the foregoing Chapter, we may alſo determine what the Change will be in thoſe, that is, in elaſtic Bodies: In which the following *Rules* are to be obſerv'd.

* 126

* 178

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RULE I.

When Bodies that are not elaſtic ſtrike againſt each other, if one Body acquires a certain Quantity of Motion, it would require twice as much, if the Bodies were elaſtic; and this double Quantity is to be added to the firſt Motion, in order to determine the Motion after the Stroke. 180

RULE II.

When two Bodies that are not elaſtic ſtrike againſt each other, if one Body loſes a certain Quantity of Motion, as would loſe twice as much, if the Bodies were elaſtic; 181

elaſtic; and that double Quantity muſt be ſubſtracted from the firſt Motion, in order to determine the Motion after the Stroke.

What is ſaid of Motion muſt alſo be underſtood
of Velocity; becauſe in the ſame Body the Mo-
* 63 tion is proportionable to the Velocity *.

Experiment 1. *Plate* X. *Fig.* 5.] Let the Body
P, whoſe Maſs is 2, and Celerity 9, ſtrike againſt
Q, a Body at reſt, whoſe Maſs is 1; after the
Stroke, Q moves with the Velocity 12, and P
continues its Motion with the Velocity 3; which
agrees with the former Rules: For if the Bodies
were not elaſtic, the Celerities of both after meet-
* 171 ing would be 6 *; and ſo the Body Q would ac-
* 180 quire 6 Decrees of Velocity, and, by *Rule* I.* there-
fore it muſt acquire 12 Degrees; the Body P loſing
* 181 3 Degrees of Velocity, by *Rule* II.* muſt loſe 6,
which if you take from 9, the former Velocity,
there remain 3 Degrees of Velocity.

Experiment 2. *Plate* X. *Fig.* 6.] If the Body P, whoſe Maſs is 2, and Velocity 8, ſtrikes the Body Q, whoſe Maſs is 1, and which is carried the ſame Way with the Velocity 5; after the Stroke, the Body Q moves with the Velocity 9, and P with the Velocity 6; which again might have been determined by the foregoing Rules.

If the Bodies had not been elaſtic, both would
have moved after the Stroke with the Celerity
* 172 7 *: the Body Q would acquire 2 Degrees of Ce-
lerity, which, by *Rule* I. muſt be doubled, and
added to 5, the firſt Celerity, which gives us 9:
The Body P loſt one Degree of Velocity, and, by
Rule II. it muſt loſe 2; therefore it has 6 left.

RULE III.

When a Body loſes its whole Motion, and acquires a Motion the contrary Way, thoſe two Motions muſt be collected into one Sum, in order to have the Motion that is loſt. 182

When the Quantity, which is to be ſubſtracted by Rule II. *exceeds the Quantity of Motion before the Stroke, from which it muſt be ſubſtracted, that whole Quantity of Motion is deſtroyed, and what remains,* (that is, its Difference from that which it ſhould have been ſubſtracted from) *gives the Motion the contrary Way.*

Experiment 3. *Plate* X. *Fig.* 7.] Let the Body P ſtrike with the Velocity 12 againſt another Body Q, which is three times as heavy, and at reſt, and it will return with the Velocity 6. In this Caſe, Bodies not elaſtic would move with the Celerity 3; therefore the Body P would have loſt 9 Degrees of Velocity; but by *Rule* II. *it muſt loſe 18; which if you ſubſtract from the former Velocity 12, you have 6 Degrees the contrary Way, by *Rule* III.* In this Manner may be determined, by the following *Experiments*, what is laid down in the *Rules.* * 182

Experiment 4. *Plate* X. *Fig.* 8.] Let the Body P be carried with the Velocity 19, the ſame Way as Q, that is, three times as heavy, and move with the Velocity 3; after the Stroke the Body returns with the Velocity 5.

Experiment 5. *Plate* X. *Fig.* 9.] Let the two Bodies P and Q come towards one another with equal Quantities of Motion; after the Stroke both will return with the ſame Celerities with which they came upon each other.

Experiment 6. *Plate* X. *Fig.* 10.] Let P with the Velocity 5, and Q of triple the Weight, with the Velocity 11, move in contrary Directions; after the Stroke, Q continues its Motion with the Celerity 3, and P returns with the Celerity 19.

Experiment 7. *Plate* X. *Fig.* 11.] Let the ſame Bodies P and Q be carried in contrary Directions, P with the Celerity 16, and Q with the Celerity 8; both will be reflected after the Stroke, P with 20, and Q with 4 Degrees of Velocity.

All the Caſes of the Percuſſions of elaſtic Bodies may be determined by the Rules above-mention'd; the following remarkable Obſervations are alſo deduced from them.

183 *When the Bodies are equal, and move the ſame Way, they continue their Motions, interchanging their Velocities; if they move contrariwiſe, then they are reflected from each other, likewiſe interchanging their Velocities.*

Caſe 1. *Plate* XI. *Fig.* 1.] Let the Bodies move the ſame Way, and let AB be the Velocity of one Body, and BC the Velocity of the other; the Velocities here are as the Quantities of Motion.* Let the Line AC be divided into two equal Parts at D, and let D *b* be equal to DB; AD or DC gives the Celerity of each Body after the Stroke, if they are not elaſtic;* ſo the Celerity BC is increaſed by the Quantity DB; but as it muſt be doubly increaſed becauſe of the Elaſticity, * it will become *b* C: The Celerity AB, in Bodies not elaſtic, is diminiſhed by the Quantity DB; but it muſt be diminiſhed by double that Quantity, for the Reaſon above-mention'd; * and therefore it will now become A *b*.

*63 *172 *180 *181

There-

Plate 10. p. 60

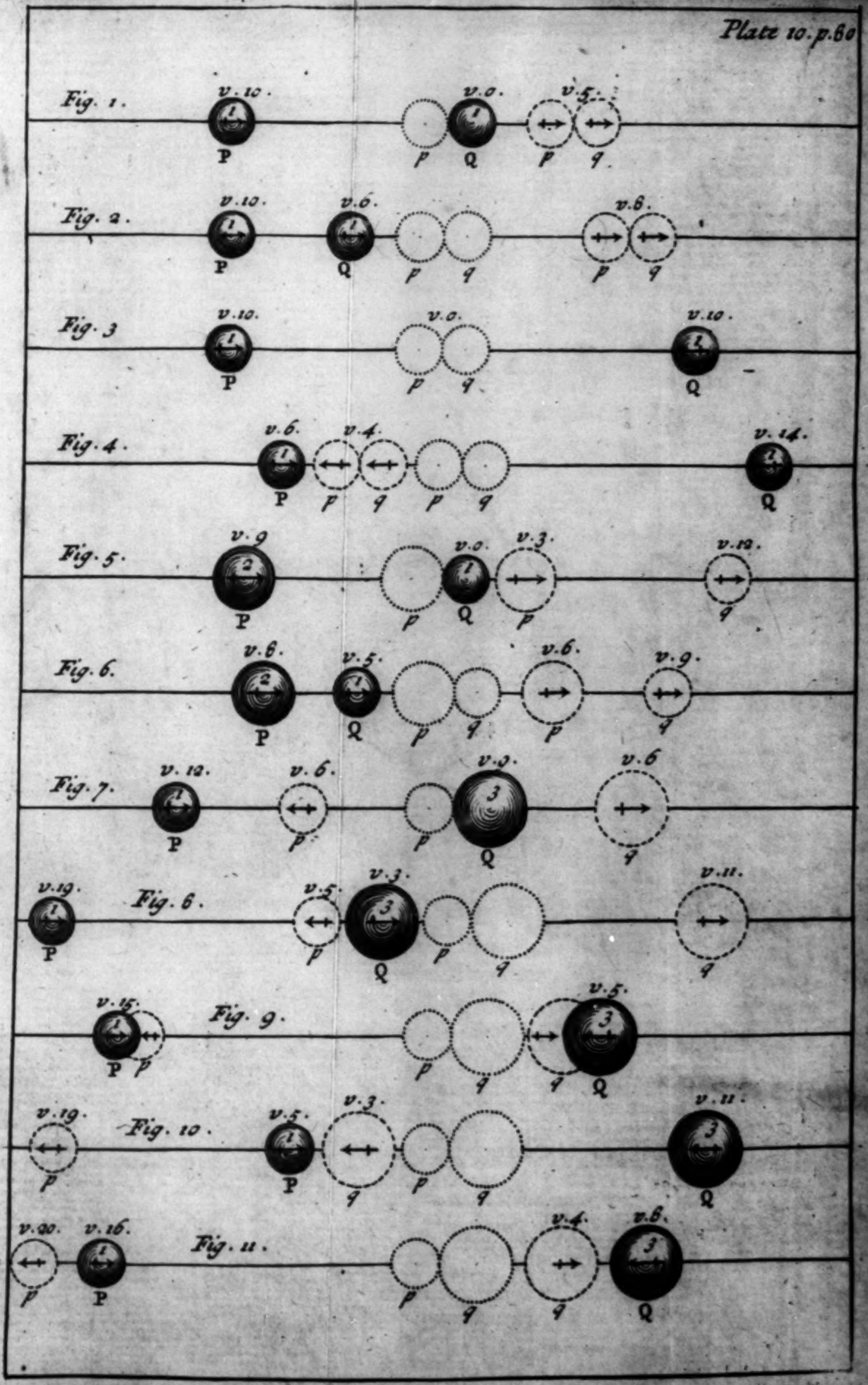
Fig. 1.
v. 10.
P
v. 0.
p
Q
v. 5.
p
q
Fig. 2.
v. 10.
P
v. 6.
Q
p
q
v. 6.
p
q
Fig. 3
v. 10.
P
v. 0.
p
q
v. 10.
Q
Fig. 4.
v. 6.
P
v. 4.
p
q
p
q
v. 14.
Q
Fig. 5.
v. 9
2
P
p
v. 0.
Q
v. 3.
p
v. 12.
q
Fig. 6.
v. 6.
2
P
v. 5.
Q
p
q
v. 6.
p
v. 9.
q
Fig. 7.
v. 12.
P
v. 6.
p
p
v. 0.
3
Q
v. 6
q
v. 19.
P
Fig. 8.
v. 5.
p
v. 3.
3
Q
q
v. 11.
q
v. 15.
P
p
Fig. 9.
p
q
q
v. 5.
3
Q
v. 19.
p
Fig. 10.
v. 5.
P
v. 3.
q
p
q
v. 11.
3
Q
v. 20.
p
v. 16.
P
Fig. 11.
p
q
v. 4.
q
v. 6.
3
Q

Therefore the Velocities AB and BC are changed into the Velocities A*b* and *b*C; but AB and *b*G, as alſo BC and A*b*, are equal to one another.

Experiment 8.] Let two equal Bodies P and Q, the firſt with the Velocity 10, and the other with the Velocity 5, be carried the ſame Way, they will continue their Motion after the Stroke, interchanging their Velocities: Which alſo agrees with the Computation by the foregoing Rules.

Caſe 2. *Plate* XI. *Fig.* 2.] Let AB be the Celerity of one Body, CB the Celerity of the other; let the Difference AC be divided into two equal Parts at D, and let A*b* be equal to CB. When the Bodies are not elaſtic, the Velocity of each of
them, after the Stroke towards the ſame Side, is
AD *; therefore the firſt Body has loſt the Velo- * 174
city DB, and the other has loſt the whole Velo-
city CB, and acquired DC the contrary Way;
therefore the whole Quantity loſt is alſo DB *; * 182
if this Quantity be doubled, it will be *b*B, the
Quantity of Celerity loſt by both Bodies *; the * 181
Difference of that Velocity with the Velocity of
each Body, does in each Body give a Velocity the
contrary Way *; that Difference for the Motion * 182
AB is *b*A, and for the Motion CB is C*b*; but C*b* and AB, as alſo *b*A and CB are equal to each other.

Experiment 9.] If the equal Bodies P, with the Celerity 10, and Q with the Celerity 5, are carried contrariwiſe, they will both be reflected after the Stroke, interchanging their Velocities.

184 *When an elaſtic Body ſtrikes another equal to it that is at reſt*, that the Velocities may be changed, *the percutient Body will be at reſt after the Stroke,*

and the other will go on with all the Velocity of the Percutient: Which is confirmed by

Experiment 10.] Let the Body P, with the Velocity 10, ſtrike the Body Q, which is at reſt; P will be at reſt after the Stroke, and Q will go forward with the Velocity 10. And this ſerves to explain the following

Experiment 11.] 1*ſt*, Let ſeveral equal Bodies P, Q, R, S, T, V, (*Plate* XI. *Fig.* 3.) be placed in the ſame Line, and touching one another; if the Body ſtrike againſt Q with any Velocity, after the Stroke, P, Q, R, S and T, will remain at reſt, and V only be moved.

2*dly*, Let P and Q move with equal Velocities, ſo that Q may ſtrike againſt R; after the Stroke P, Q, R, and S, will be at reſt; but T and V will move forward together.

3*dly*, If Three are let go together, they will alſo ſtrike off Three.

4*thly* and laſtly, If P, Q, R and S be moved at once, ſo that S ſtrikes T, after the Stroke P and Q will be at reſt, and R, S, T, V will move together. In general, let the Numbers of Balls be what it will, how many ſoever move on before the Stroke, ſo many alſo will move off in the ſame Direction after the Stroke.

In the firſt Caſe, the Body P (*Plate* XI. *Fig.* 3.) ſtrikes Q, and then is at reſt *, Q ſtrikes R, and is alſo at reſt after the Stroke; and ſo it happens to the others, till at laſt T ſtrikes V, which, having no Obſtacle to ſtop it, does alone continue in Motion. *184

In the ſecond Caſe, the Body Q (*Plate* XI. *Fig.* 4.) does in the ſame Manner drive forward the Body V; P immediately follows, and ſtrikes Q, which, on accouut of the firſt Stroke, was at reſt, but now communicates its new Motion forward to T, (in the Manner above-mentioned) which is not able to ſtrike V, that is already in Motion; and as the Motions of P and Q are equally ſwift, and thoſe Bodies follow one another very cloſe, there is no ſenſible Time between thoſe two Communications of Motion; which is the Reaſon that the Bodies V and T are moved equally ſwift, and not ſeparated from each other.

The relative or reſpective Velocity, with which 185
two elaſtic Bodies whatever recede from each other
after the Stroke, is the ſame as the reſpective Velo-
city with which the Bodies came againſt one ano-
ther.

If the Bodies were not elaſtic, they would
jointly continue their Motion*; and in that Caſe,* 168
by the Action of the Bodies upon each other, the
whole reſpective Celerity, by which they come
to one another, is deſtroyed; the Action from the
Reſtitution of the Spring is equal and contrary*,* 178
and therefore it muſt generate the ſame reſpe-
ctive Celerity with which they recede from each
other. Let the Inequality of the Bodies be what
it will, nothing is changed thereby, becauſe of
the Equality of the firſt and ſecond Action upon
each Body*. * 126

The Quantity of Motion towards the ſame Side, 186
or the ſame Way, is the ſame after as before the
Stroke.

For Bodies that are not elaſtic, this Propoſi-
tion is proved in all Caſes*; for when the Mo-* 171
tions do not conſpire, the contrary Motion muſt 172
be ſubſtracted from the Motion one Way, in or- 173
der to determine the Motion that Way. By the 174
Reſtitution on account of the Elaſticity, equal
Quantities are generated towards each Side*,* 120
by which the Quantity of Motion towards one
Side is not changed. Theſe two laſt Propoſitions

are fully confirmed by the above-mentioned Experiments.

187 *When a ſmall Body ſtrikes againſt another greater Body, which is at reſt, the great Body acquires a greater Quantity of Motion than the ſmall one had before the Stroke.*

The Quantity of Motion acquired by the great Body, is double the Quantity which the little one would loſe, if the Bodies had no Elaſticity; but in that Caſe, the little Body would loſe more than half its Motion *.

* 180 171

Experiment 12. *Plate* XI. *Fig.* 7.] Let the Body P, with the Velocity 15, (which Number alſo does here expreſs the Quantity of Motion) ſtrike the Body Q, which weighs four Times as much, and is at reſt; the Body Q acquires the Velocity 6 *; that is, 24 Degrees of Motion. But the Body P returns with 9 Degrees of Velocity; and ſo the Quantity of Motion, towards that Way where P was firſt directed, continues to be 15 *.

*171 150

*186

The Motion is more increaſed in the Body Q, by the Interpoſition of a Body of a mean Bigneſs between the Bodies P and Q.

188 *Experiment* 13. *Plate* XI. *Fig.* 8.] Let there be the two Bodies P and Q before-mentioned, and between them the Body R double the Body P; if the Body P, with 18 Degrees of Velocity, comes upon R which is at reſt, it will communicate to it 12 Degrees of Velocity *; with which if this Body ſtrikes upon Q that is at reſt, it will communicate to it the Celerity 8, that is, 32 Degrees of Motion; but in this Experiment, becauſe of the double Percuſſion, the Error ariſing from the Want of perfect Elaſticity, is more ſenſible than in the others, where there is but one Percuſſion, and

* 171 180

and ſo the Quantity of Motion acquired by the Body Q, is about 30.

189 *The greater the Number is of unequal Bodies, which are interpoſed between two Bodies, if the Maſſes always increaſe from the firſt to the laſt, ſo much the greater will the Quantity of Motion be in the greateſt; and it will be the greateſt of all (the Number of Bodies interpoſed remaining the ſame) when the Maſſes of all the Bodies increaſe in a Geometrical Progreſſion.*

Tho' the Quantity of Motion, directed the ſame Way in the Congreſs of Bodies, whether elaſtic or not elaſtic, remains the ſame, the Quantity of the Motion itſelf does not always remain the ſame, but is often diminiſh'd *, and alſo often increaſed *; ſo that there is no Reaſon to ſay, that there is always the ſame Quantity of Motion in the World. *173 174 *187

CHAP. XXII.

Of compound Motion, and oblique Percuſſion.

A Body in Motion may be acted upon by a new Force, and driven according to another Direction; in that Caſe the Change of Motion follows the Proportion and Direction of that Force *: And as the firſt Motion is not deſtroy'd by that Action, from theſe two Motions a third ariſes, according to a new Direction. *125

190 *Let the Body* P (*Plate* XI. *Fig.* 9.) *be driven by any Force, according to the Direction* P C, *and at the ſame time let it be driven by another Force, according to the Direction* P B; *and let the Celerities ariſing from thoſe Forces, be as thoſe Lines* P C, P B. In order to determine what will happen, let the Parallelogram P B A C be compleated, by drawing the Lines B A, C A, parallel to the Lines above-mention'd; let P A be the Diagonal of

that Parallelogram. Let the Body be ſuppoſed by the firſt Force, that is, with the Celerity P C, to deſcribe the Line P C, and let that whole Line be carried along in the Direction, and with the Celerity P B; when that Line is come to *b a*, the Body will be at *p*, ſo that P *b* will be to P B, as *b p* to *b a*, or A B, that is, it will be in the Diagonal P A, and ſo always. When the Line P C is at B A, the Body will be at A; therefore *from a Motion compounded of the two Motions abovementioned, there ariſes a Motion along the Diagonal* P A, *whoſe Celerity is proportional to the Length of* *53 *the Diagonal* *; for the Diagonal will be run thro' in the ſame Time by the Body P with a compound Motion, as the Line P B or P C would have been gone thro' by the ſame Body, by only one Motion; that is, acted upon by one of the Forces.

To confirm this Propoſition experimentally, we muſt make uſe of the following Machine.

191 It conſiſts of two Boards, or wooden Planes, C D E, C D E, (*Plate* XII. *Fig* 1.) of the Figure of a right-angled Triangle, whoſe Side C D is in Length about 3 Foot and a half, and the Side D E about 1 Foot and a half; theſe Boards are fixed ſo as to move in a vertical Situation about the Hinges A and B.

The Experiments upon this Machine are made with Ivory Balls of an Inch and a half Diameter. The Planes are ſo joined, that if you conceive two other Planes to run parallel to them, at the Diſtance of a little more than a Semidiameter of the Balls; the Lines, in which theſe imaginary Planes interſect, ſhall be the Axis of their Circumvolution: Which is brought to paſs by the Contrivance of the Hinges, (*Fig.* 2.) whoſe Pares *b*, *b*, are let into the Wood to be the firmer. In the Center of the upper Hinge A, there is a ſmall Cylinder *a*, (*Fig.* 2.) in whoſe Caſe there is a Hole, which meets another in the Side, thro' which the Thread *i h* is to run; at one End of this Thread a Ball, as P, hangs, and the other End is joined to the Key *l*. By Help of the Screws F, F, F, F, F, this Machine is ſet perpendicular, ſo as to have the Thread *h i* hang in the Axis of the Machine.

At *m*, *m*, there are two Pins fix'd to the two Planes, from which Pins the Balls Q, Q, hang, at ſuch a Diſtance from the Planes, that they may almoſt touch them; ſo that if you ſuppoſe a Line to paſs thro' the Centers of the Balls P and Q, it ſhall be parallel to the Plane on that Side: Beſides, it is required, that, when thoſe Balls hang at the ſame Height, they ſhall touch one another.

The Threads which are tied to the Balls go thro' the Holes in the ſaid Pins, and are fix'd to the Keys *ll*, ſo that the Balls may be raiſed or let down eaſily, and have all their Centers brought to the ſame Horizontal Plane. There is a Braſs Ruler, or graduated Limb R, bent up in the Arc of the Circle, ſo as to have the Ball P riſe along it in its Motion; and this Limb turns one of its Ends on a Center which is in the Axis of the Machine. This Piece of Braſs ſerves to ſhew to what a Height the Ball P aſcends.

Each Ball Q, when it ſwings, moves along the Plane to which it is applied; and the Height, from which it is made to fall, is ſhewn by an Index fixed to the Plane; to which End there are four Holes in each Plane containing equal Angles, in reſpect to the Motion of the Threads.

When the Ball Q is let fall from any Height, it ſtrikes upon the Ball P, and drives it to the ſame Height in the ſame Direction*. *184

Experiment 1.] The Horizontal Section of this Machine is here repreſented, (*Plate* XII. *Fig.* 3 and 4.) the Body P may be driven by either of the Bodies Q, with any Direction and Velocity. If the Bodies Q and Q are let fall at the ſame time, the Body P has two Motions impreſs'd upon
155 it at the ſame time, and therefore runs in the Diagonal P*p* of the Parallelogram made in the
190 Manner above mentioned, to expreſs thoſe two Motions, and runs up to an Height proportionable to the Length of that Diagonal.

The Experiment anſwers very exactly, whether the Balls Q and Q are let fall from the ſame Height, or from unequal Heights; and whatever the Angle be that is made by the two Planes, that is, by the Directions of the Motions, whether the Angle be right, acute, or obtuſe.

192 *A Body moved any how in the Right Line* P A, (*Plate* XI. *Fig.* 10.) *may always be conſider'd as acted upon by two Motions*; and that as many Ways as you pleaſe: For you may draw as many different Parallelograms as you pleaſe, as P B, A C, pbac, *pbac*, whoſe Diagonal is the Line abovementioned; and, in every one of them, if there be ſuppoſed two Forces acting in the Directions P B and P C, from which the Celerities which the Body would have are as the Sides P B and P C, a Motion will always be produced by the Action of them both at once, which will give a Celerity proportional to the Diagonal.

From this Reſolution of Motion into two other Motions, may be determined the Motion of Bodies that ſtrike one another obliquely.

Let Q (*Plate* XI. *Fig.* 11 and 12.) be at reſt, and P with the Direction and Celerity P A, ſtrike againſt it. When P is come to A, draw thro' the Centers of both Bodies the Line D B, and then PB perpendicular to it, and compleat the Parallelogram ABPC, the Motion along PA is reſolved into two others along PB and PC, or BA, CA: By the Motion in the Direction CA, the Body P does not act upon the Body Q; the Action therefore ariſes ſolely from the Motion in the Direction along BA, that is, the Body P, 193 *by the oblique Stroke along* PA *with the Celerity* PA, *acts upon the Body* Q, *in the ſame Manner, as if it ſhould ſtrike it directly along* BA *with the Celerity* BA. And ſo the Motion of the Body Q from that Action, whether the Bodies be elaſtic or not, is determined from what has been ſaid of direct Percuſſion.

The Motion of the Body P (*Plate* XI. *Fig.* 11 and 12.) after the Stroke, is deduced from the ſame Principle; the Motion along CA is not changed; therefore by that Motion, with an equal Celerity, the Body P is carried in the Direction AE. Now let AE be equal to CA; the Change in the Motion BA is determined, in reſpect of the Body P, in the ſame Manner as the Motion of Q, by the two foregoing Chapters; let the Celerity of that Motion be AD, in *Fig.* 11, when the Body goes forward, and in *Fig* 12, when it returns back; from that Motion, and the Motion along AE, ariſes a compound Motion in the Diagonal A*p*, which, by its Situation and Length, denotes the Direction and Celerity of the Body P after the Stroke *. *190

194 When *Bodies* are *equal and elaſtic*, the whole Motion along BA is deſtroyed by the Percuſſion*, and only the Motion along CA is left, and *148 the Body P is alſo carried in that Direction. In that Caſe, both the Bodies *do* always *fly from each other in Directions that are at Right Angles with one another, which Way ſoever the Body* P *comes upon the other Body.*

Experiment 2. *Plate* XII. *Fig.* 5.] In the Machine deſcribed *Numb.* 191, let the Ball Q and P hang; having ſet the Planes at Right Angles, let the Body Q with any Direction, and from any Height, come down upon P, and ſtrike againſt it; after the Stroke the Bodies will follow the Directions of the Planes, and riſe to Heights, which may be determined by what has been ſaid hitherto.

195 We may alſo the ſame Way determine the Motion of two Bodies after the Stroke, when both Bodies are moved, which Way ſoever they come upon one another. The chief Caſes are repreſented in *Plate* XIII, and all of them are explained exactly the ſame Way.

Plate XIII. *Fig.* 1, 2, 3, 4, 5, and 6.] Let the Body P be moved with the Direction and Celerity P A, and the Body Q, with the Direction and Celerity Q *a*; draw the Line B *b*, which goes through the Centers of both Bodies where they touch one another, and let C A and *c a* be drawn perpendicular to the Line above-mentioned, and let the Parallelograms P B A C and Q *b a c* be compleated. The Motion of P is reſolved into two others, of which the Celerities and Directions are expreſs'd by C A, B A. The Motions, into which the Motion of Q is reſolved, are expreſs'd by *c a*, *b a*; by the Motions along C A and *c a*, the Bodies do not act upon one another; therefore theſe Motions are not changed, and after the Stroke are expreſs'd by A E and *a e*, which are equal to A C and *a c*; the Percuſſion, from the Motions in the Lines B A *b a*, is direct, and determined in the foregoing Chapters: Let the Body P move towards D, and its Celerity be A D, and the Body Q move towards *d* with the Celerity *a d*. After the Stroke therefore, the Motion

Plate 11. p. 90

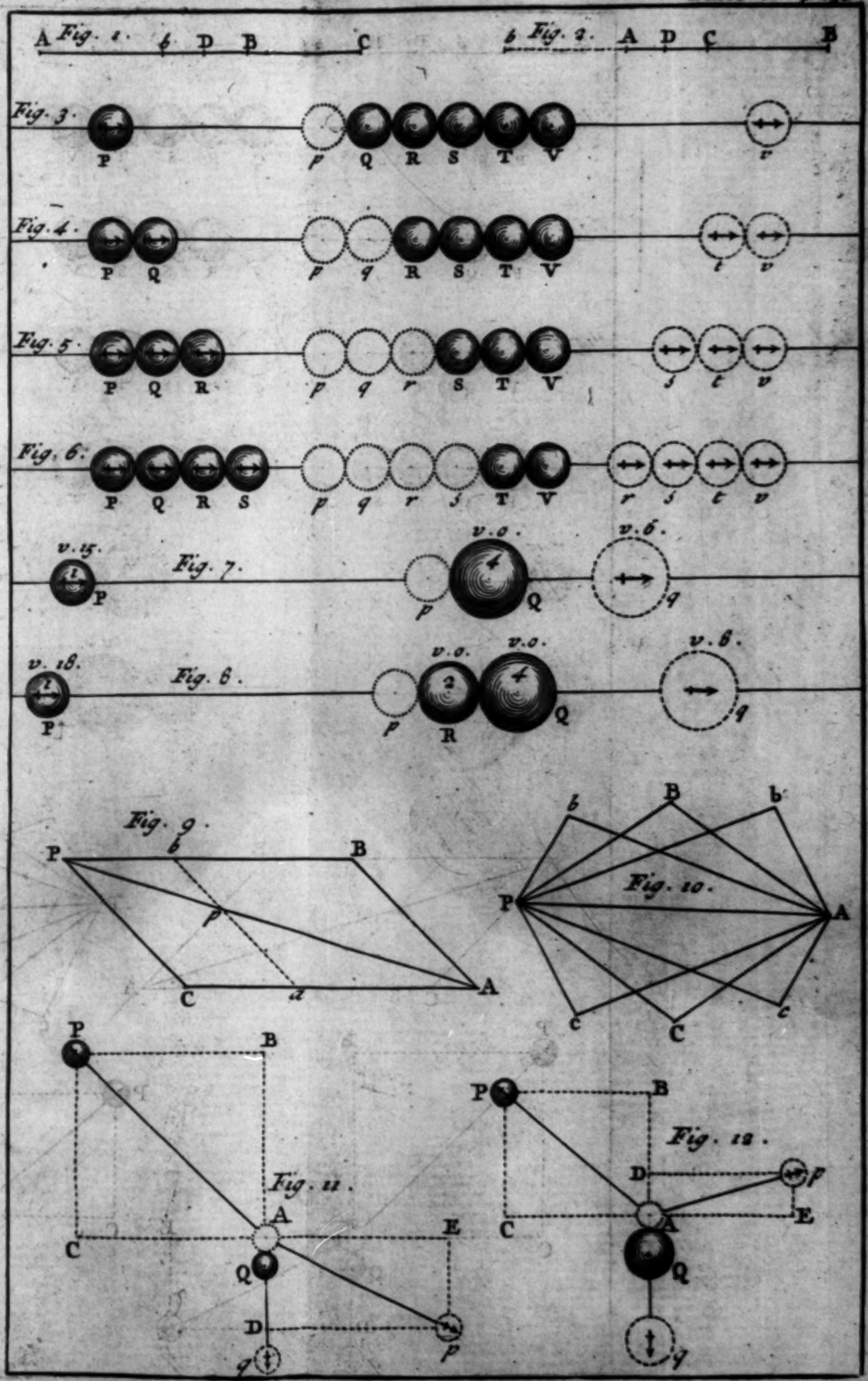
Fig. 1.
A b D B C
Fig. 2.
b A D C B
Fig. 3.
P p Q R S T V v
Fig. 4.
P Q p q R S T V t v
Fig. 5.
P Q R p q r S T V s t v
Fig. 6.
P Q R S p q r s T V r s t v
Fig. 7.
v. 15.
1
P
p
v. 0.
4
Q
v. 6.
q
Fig. 8.
v. 18.
1
P
p
v. 0.
2
R
v. 0.
4
Q
v. 6.
q
Fig. 9.
P b B p C a A
Fig. 10.
b B b P A c C c
Fig. 11.
P B A C E Q D p q
Fig. 12.
P B D p A C E Q q

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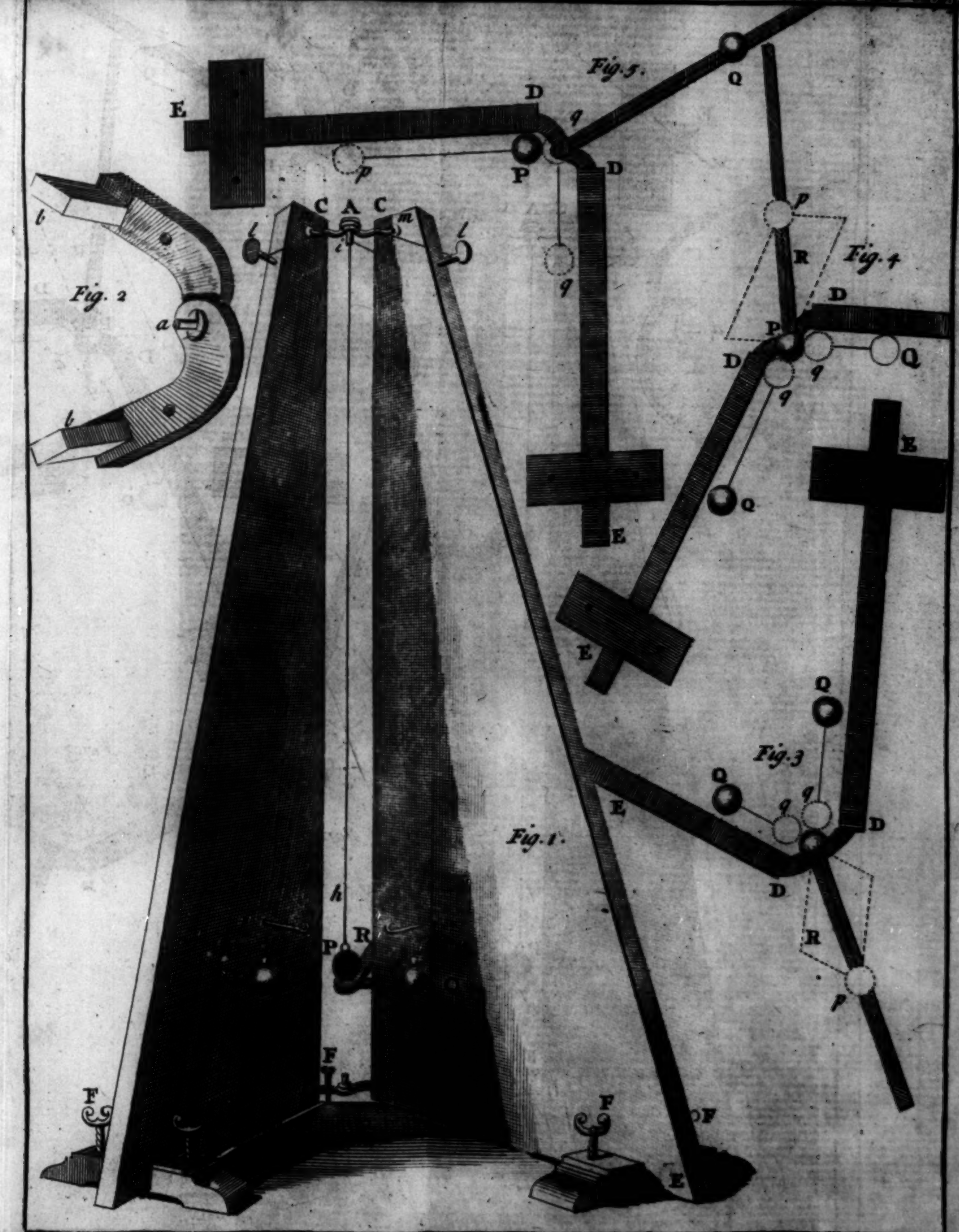
Fig. 5.
E
D
q
Q
P
p
D
q
C A C
l
m
l
i
b
Fig. 2
a
b
Fig. 4
p
R
D
P
D
q
Q
q
q
Q
E
E
E
Fig. 3
Q
Q
q
q
D
D
E
Fig. 1.
R
p
h
P
R
F
F
F
F
E

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Motion of the Body P is compounded of the Motions along AE and AD, and moved in the Diagonal A*p*. The Motion of the Body Q, after the Stroke, is compounded of the Motions along *ac* and *ad*, whence that Body is carried in the Diagonal *ae*; and the Lengths of thoſe Diagonals expreſs the Celerities of the Bodies after their Meeting. In the 1ſt, 2d, and 3d Figures the Bodies are ſuppoſed not elaſtic; and in the 4th, 5th, and 6th, the ſame Baſes are put, ſuppoſing the Bodies elaſtic. There are ſome Letters wanting in the firſt Figure, becauſe the Points, which are marked with thoſe Letters in the other Figures, do here coincide with other Points, and are not neceſſary for determining the Motions.

CHAP. XXIII.

Of Oblique Powers.

THE Body P (*Plate* XIII. *Fig.* 7.) being driven in the Directions PB and PC, with Celerities proportionable to thoſe Lines; from thence ariſes a Motion along P, the Diagonal of the Parallelogram PBAC, with a Celerity that is denoted by that Diagonal*: If there be a *190
third Force acting along the Line P*a*, ſo that the Celerity ariſing from it be PA; by that Action the Actions of both the ſaid Forces are deſtroyed, and the Body comes to reſt: If the aforeſaid Actions continue, the Body will continue at reſt; which happens when the Body is drawn towards C, B and *a*, with the ſaid Forces pulling by Threads. Whence it follows, That *a Body will* 196
be at reſt, which is drawn by three Powers, that are to one another, as the Sides of a Triangle made by Lines parallel to the Directions of the Powers.

This

197 This Propoſition is confirmed experimentally, by the Machine repreſented in *Plate* XIII. *Fig.* 8. It conſiſts of a round Board of about 8 Inches Diameter, which is in a horizontal Poſition, and ſuſtain'd by a Foot: Round the Edge of it, within the Thickneſs of the Wood, is a Groove whereby Pullies are applied at Pleaſure to any Part of the Circumference; for each Pulley has a Braſs Plate perpendicular to it, which fits into the Groove, when the Pulley is applied. See the Pulley with the Plate repreſented by F.

The Board above-mentioned is a little hollowed in, in the upper Part, ſo as to receive a leſs orbicular Board DFA, whoſe Thickneſs is about a Quarter of an Inch, and its Surface riſes a little above the firſt Board; ſo that a Thread that runs over any of the Pullies, being extended horizontally, may juſt touch the ſaid Surface.

You muſt have ſeveral of the leſſer round Boards for making different Experiments. They have Paper paſted upon them on both Sides, that the Lines (to be mentioned hereafter) may be the more eaſily drawn upon them.

Experiment 1.] Let C be the Center of the ſmall Board, and let there be drawn upon it the Triangle ABC, whoſe Sides are to one another as 2, 3, and 4: Let CE be parallel to the Side AB of the Triangle, and let the Side AC be continued towards D.

Now if there be three Threads joined together at C, and ſtretched over the Pullies faſtened to the greater Board, ſo as to be in the Lines CD, CE, and CB; if to the Thread CD you hang 4 Pounds, to CE 3; and laſtly, but 2 to the Thread CF, the Threads will not be mov'd, and the Knot remains over C; but if it be mov'd out of that Point, it will not be at reſt.

In this Propoſition any two Powers are balanced by a third, that is, act but as one; which acts contrariwiſe in the Direction of that third. Therefore *the Actions of two Powers may be reduced to the Action of one.* 198

So, when a Point is drawn by four Powers, there will be an Æquilibrium, if reducing two Powers to one, this new Power, with the other two remaining, be in the Poſition of *Numb.* 196; that is, if thoſe remaining Powers being alſo reduced to one, the Power ariſing thence be equal with, and acts contrary to the new Power mentioned.

Experiment 2. *Plate* XIII. *Fig.* 9.] The Point C is drawn by four Threads; towards B by the Weight of 1 Pound, towards F by 3 Pounds, towards E by 2 Pounds; and, laſtly, towards D by 4 Pounds; and this produces an Æquilibrium. Having drawn the Triangle C F *a*, or the Parallelogram C F *a* E, the aboveſaid Powers, drawing in C F and C E, are reduced to one that acts in the Direction C *a*, with the Force of 4 Pounds; and then the three Powers, drawing in the Lines C B, C D, C *a*, give us the Caſe of *Numb.* 196: And therefore if the Powers, drawing along C B and C D, be reduced to one drawing along C A, it will act in the ſame Direction, but pull againſt the Power pulling in C *a*, and be equal to it. 199

What is here ſaid of the four Powers, might be ſaid of five or more; for of five, if two be reduced to one, we come to the laſt mentioned Caſe.

Experiment 3. *Plate* XIV. *Fig.* 1.] The Point C is drawn by 5 Powers, pulling by the Threads C A, C B, C D, C E, and C F; the Powers are 200 are to one another as the Weight by which the Threads are drawn, and they have the ſame Proportion to one another as the Numbers that you ſee at the Pullies in the Figure, and you have an Æquilibrium.

The Powers, drawing in C B and C D, are reduced to one drawing in C G.

The Powers, drawing in C E and C F, may be reduced to one acting in C H; which brings us to the Caſe of *Numb.* 196. Laſtly, thoſe two new Powers, drawing in C H and C G, are reduced to one acting in C *a*, which are equal to the fifth drawing along C A, and pulls in the ſame Line, but contrariwiſe.

201 Beſides this, we deduce from the Propoſition
mentioned *Numb.* 169, that the ſame Thing may
be ſaid of *the Action of the Power* which has been
ſaid concerning Motion in the foregoing Chap-
192 ter; namely, that it *may be reſolved into the
Actions of two other Powers*, and that in num-
berleſs Manners, becauſe Triangles of numberleſs
Kinds may be made, tho' you keep one Side ſtill
the ſame. Thus in all Engines we can reduce a
Power, that acts obliquely, to a direct one; and
can determine the Proportion between a direct
and an oblique one: Which will appear by the
following Examples, that are confirmed by Ex-
periments.

202 *Experiment* 4. *Plate* XIV. *Fig.* 2 *and* 3.] To the Lever A B, whoſe Brachia are equal, apply at B the Weight P of two Pounds, and at A a Power acting obliquely in the Direction A D, and which is repreſented by the Weight M. If you imagine a Line, as D E, parallel to the Lever in a horizontal Poſition, and A E perpendicular to that Lever; and if A D be to A E as 3 to 2, and the

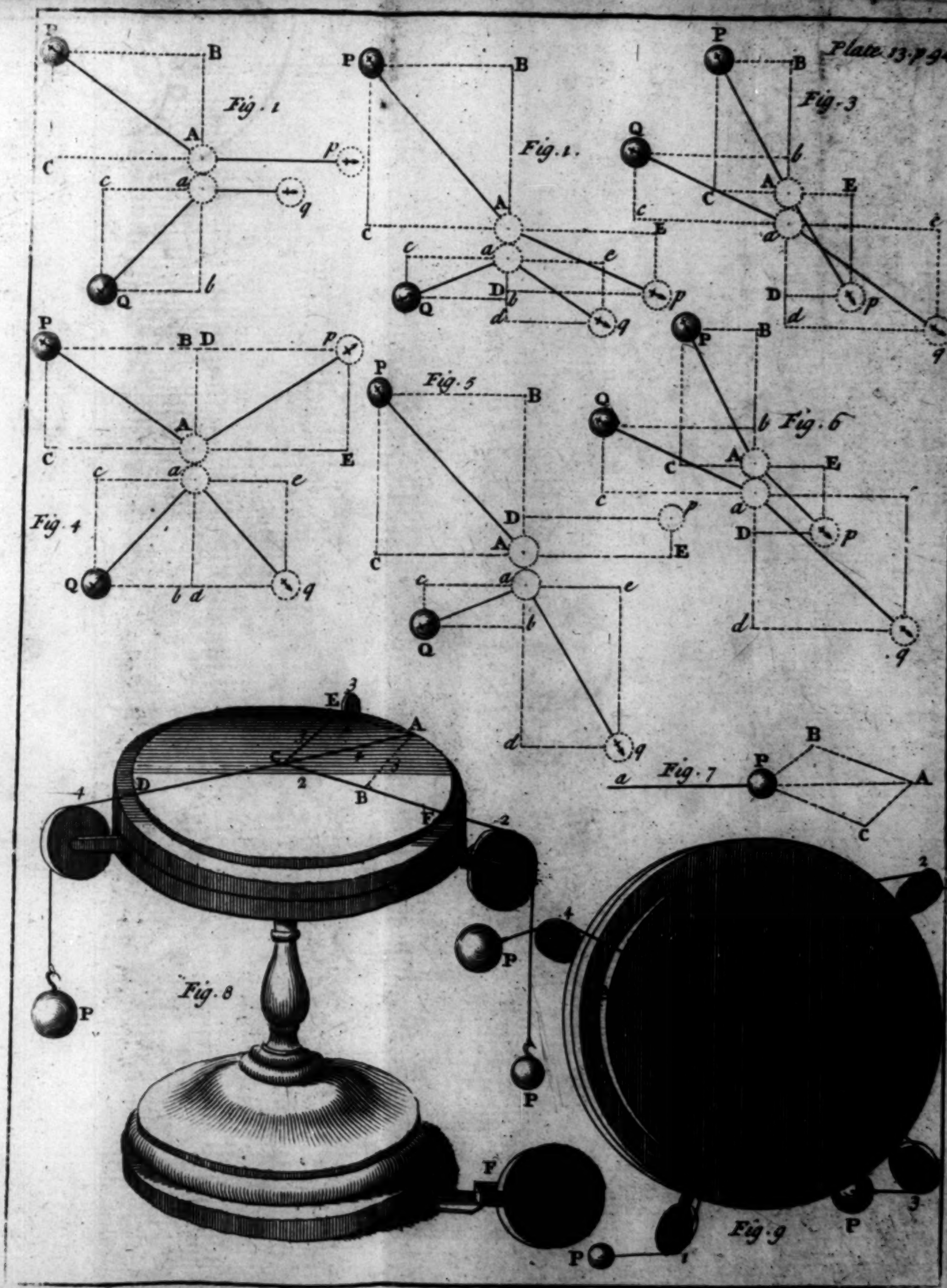
Fig. 1
P
B
A
C
p
c
a
q
Q
b
Fig. 2
P
B
A
C
c
a
e
E
D
b
d
Q
p
q
Fig. 3
P
B
Q
b
A
E
C
c
a
e
D
d
p
q
Fig. 4
P
B D
p
A
C
E
c
a
e
Q
b d
q
Fig. 5
P
B
D
A
C
E
p
c
a
e
b
Q
d
q
Fig. 6
P
B
Q
b
A
E
C
c
a
e
D
p
d
q
Fig. 7
a
P
B
A
C
Fig. 8
E
3
A
C
D
2
B
F
4
2
P
P
F
4
P
P
Fig. 9
2
3
P
P
1

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the Weight M be of three Pounds, there will be an Æquilibrium.

The Direction of the Motion of the Point A, by the Motion of the Lever, is perpendicular to the Lever, therefore it acts in the Line E A produced. As the Diſtance B A always remains the ſame, the ſecond Figure A is hindered from coming towards B, and as it were repelled in the Direction B A; in *Fig.* 3. the Point A is hindered from receding from B, and ſo A is as it were drawn towards B. Beſides, the Point A is by M drawn towards D; therefore that Point is drawn by three Powers, whoſe Directions are parallel to the Sides of the Triangle A E D; which therefore, to produce an Æquilibrium, muſt be to one another as thoſe Sides.

The Point A, by reaſon of the Equality of the Diſtances of the Points A and B from the Fulcrum, moving along E A, is drawn with the ſame Force as P deſcends, that is, with the Force of two Pounds; the Force therefore along A D muſt be of three Pounds, becauſe the Sides A D and A E are to one another as 3 to 2. The Side D E expreſſes what the Fulcrum ſuſtains by the Force with which the Point A in *Fig.* 2, is puſhed towards B, or is drawn from it in *Fig.* 3.

The ſame alſo may be ſaid of an oblique Power to the Axis in Peritrochio.

Experiment 5. *Plate* XIV. *Fig.* 4.] Let the 203
Weight P, fixed to a Pulley, be ſuſtained by Powers applied on both Sides to the running Rope, but drawing obliquely in the Directions C A and C B; theſe Powers are equal to one another, becauſe no Part of the Rope, that goes about the Pulley, can be at reſt, unleſs it be equally drawn on both Sides*. The Weight P is as it were a *83
third

third Power, and ſo the Point C is drawn by three Powers; ſuppoſe the Line CE perpendicular to the Horizon, and the Line AE parallel to CB: If CE be to AE or AC (for theſe two Lines are equal, becauſe of the above-mentioned Equality of the Powers drawing along CB, *196 CA*;) as 6 to 5, the Weight P of 6 Pounds will be ſuſtained by the Weight Q and Q of 5 Pounds each; the Reaſon of which is evident by *Prop.* 196.

If one End of the Running Rope is faſtened to a Pin, the Weight P will be ſuſtained by only one of the Weights Q.

204 *Experiment* 6. *Plate* XIV. *Fig.* 5.] If the Weight P be not joined to the Pulley, but ſuſtained by the Rope CA and CB faſtened to it, it may be ſuſtained by two unequal Powers. Draw the Triangle CAE, as was done in the foregoing Experiments, and let AE be 11, and CA $12\frac{1}{2}$, and CE 12; you ſhall have an Æquilibrium, if the Weights Q and Q are to P, as the firſt Numbers to the laſt; the Reaſon of which Experiment is alſo evident from *Numb.* 196.

205 Here we are to obſerve, that from the given Inclinations of the Threads CA and CB to the Horizon, the Proportion of the Weight Q and Q to the Weight P may be determined by Trigonometrical Tables. If in the Triangle ACE you conceive a Line A*e* drawn thro' the Point A parallel to the Horizon, and that Line be taken for a Radius of a Circle, CA will be the Secant, and *e*C the Tangent of the Angle which CA makes with the Horizon, and AE will be the Secant, and *e*E the Tangent of the Angle of Inclination of the Thread CB to the Horizon: Whence it appears that the Weights Q, Q are proportional to the ſaid Secants, and that the

Weight P follows the Proportion of the Sum of the aforeſaid Tangents.

On the Machine with which theſe laſt Experiments are made, (the Make of which the Figure alone ſufficiently expreſſes, eſpecially if it be compared with the 4th *Fig.* of *Plate* IV.) draw Lines along, that the Threads which go over the Pullies may be ſtretched; in the Middle of the Lines write down the Numbers, which expreſs the Secants of the Angles which thoſe Lines make with the Horizon; and, at the Ends of the Lines, write down the Numbers expreſſing the Tangents of theſe Angles.

Now in every Caſe where there is an Æquilibrium, the Weights Q and Q are as the Numbers in the Middle of the Lines along which the Threads are ſtretched; and the Weight P as the Sum of the Numbers at the Ends of thoſe two Lines.

Experiment 7. *Plate* XIV. *Fig.* 6.] For this Experiment we muſt make uſe of the Machine of *Numb.* 143. *Plate* VII. *Fig.* 7. the Body M, being laid upon an inclined Plane A B, is ſuſtained by a Power drawing along M S; let M R be a Line perpendicular to the Horizon, and A S R perpendicular to the Surface of the Plane; in every Caſe where the Weight P is to the Weight of the Body M, as M S to M R, the Body will be at reſt. 206

The Body M by its own Weight is drawn in the Direction R M; by the inclined Plane it is ſuſtained in a Direction perpendicular to the Plane, and ſo that Experiment is reduced to the Propoſition of *Numb.* 196.

Experiment 8. *Plate* XIV. *Fig.* 7.] The Brachia of the Lever A C B are equal, and form ſuch an 207

Angle, that if A C be continued towards D, and B D be drawn perpendicular to C D, D C ſhall be the Half of B C or C A. At A hang one Pound *p*, and at B the two Pound Weight P; then ſetting the Brachium C A in a horizontal Poſition, you will have an Æquilibrium*; because the Weight P hangs, as it would do upon a ſtreight Lever hanging at the Point D. *105

Change the Weights, and let the greater hang at A, and the leſſer be laid upon the Brachium B C at B; (*Plate* XIV. *Fig.* 8.) if by a vertical Plane you hinder this laſt Weight from falling off, you will again have an Æquilibrium.

The Brachia of the Lever are equal, and by the Motion of the Lever more equally; therefore, by the Force of the Weight P, the Weight *p* is as it were drawn towards E, in the Direction perpendicular to the Brachium B C; by the Action of the vertical Plane, that Weight is puſh'd horizontally; and at laſt is puſh'd vertically by the Force of Gravity. Therefore the Weight *p* is drawn by three Powers, which are to one another as the Sides of the Triangle B E D*. *196 Therefore the Force tending towards the Earth, (that is, the Weight *p* to the Force drawing towards E, namely, the Weight P) is as B D to B E, or D C to C B or C A; that is, as 1 to 2. Which is alſo the Ratio between the Weights *p* and P. And here therefore the Reaſon of the Experiment is deduced from the often mentioned Propoſition of *Numb.* 196; to which all other Caſes imaginable, relating to oblique Powers, alſo belong.

CHAP.

CHAP. XXIV.

Of the Projection of heavy Bodies.

A Body, moved by two Impreſſions, has a Motion compounded of both*; *if a Body be projected*, or thrown in the Line AB, (*Plate* XV. *Fig.* 1.) in the Time in which it could run the Length AB, it is by the Force of Gravity carried towards the Center of the Earth the Length BF, and ſo, by a Motion compounded of both, it is moved in AF; and by that Motion the ſecond Moment it would run through FC, equal to AF, if that ſecond Moment it was not by the Force of Gravity carried in CG, ſo that the Motion in the ſecond Moment is in FG: After the ſame Manner, the Motion in the third Moment is in GH, and the fourth Moment in HI; but as Gravity acts continually, thoſe Moments of Time are to be look'd upon as infinitely ſmall, and ſo you will every where have a Motion compounded in different Directions; that is, an Inflection of Direction in the Body's Motion; in that Caſe therefore *it will move in a Curve Line*. 208 *190

This Motion of a projected Body, or Projectile, may be conſider'd more ſimply in all Projections which we make; becauſe all Lines which tend towards the Center of the Earth, may be look'd upon as parallel, and the Direction from that Motion is always the ſame: When the projectile Motion *is made up of two Motions, the firſt equable in the Line of the Projection, and the ſecond accelerated towards the Earth**. 209 *129

Let a Body be projected in the Line AD, parallel to the Horizon; in equal Times, by that

Motion, it will run thro' the equal Spaces A B, B C, C D: By Gravity it will, in a Motion perpendicular to the Horizon, be carried in the Direction B F, C G, or D H, which here are ſuppoſed parallel; this Motion is accelerated, and therefore if after the firſt Moment the Body be at the Point F, after the ſecond it will be at G, after the third at H; ſo that if you call B F one, C G will be four, and D H nine*. The Body will run in a Curve, which goes through all the Points that may be determined in the ſame Manner as F, G, H, and that Line is called a *Parabola*. *131

The Machine made uſe of, for proving this Propoſition experimentally, is made of three Parts, as may be ſeen in *Plate* XV. *Fig.* 3. A *b* is 6 Inches high, D E is exactly of the ſame Height: The Length *b* H is of 12 Inches, ſuppoſing the Point H to be diſtant 1 Inch from the End of the Cavity in which it is taken.

Let E A be hollowed circularly; or in any other Curve; and let this hollow Channel be overlaid with a Plate of very ſmooth Tin or Braſs, that a Braſs Ball may freely roll down it; but Care muſt be taken that the lower Part of the Curve, at A, ſhall have a horizontal Direction, that the Ball may quit it in that Direction.

A *b* muſt be divided into 9 equal Parts, of which A *f* is 1, and A *g* contains 4.

When, to this firſt Part of the Machine, you add the ſecond B, (*Plate* XV. *Fig.* 4.) it reaches to *g*, and *g* G is 8 Inches long: If upon this you lay the third Part C, this laſt reaches to *f*, and *f* F is of 4 Inches.

The Diameter of the Ball P, which in making the Experiments is to be let fall along the Curve E A, is of about half an Inch; neither muſt a leſs

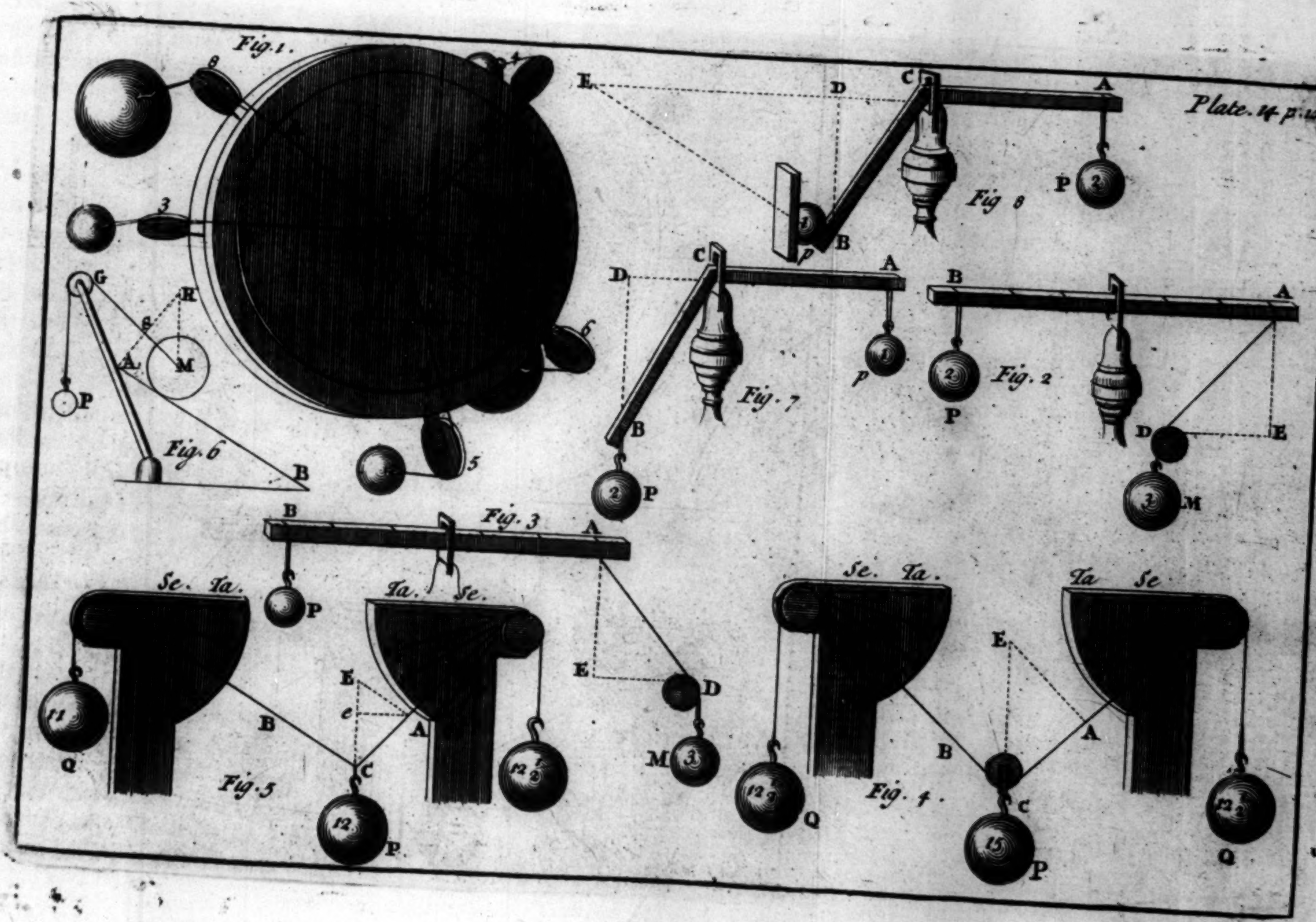


Plate. 14 p. 100

Ball, or a bigger Machine than what is here mentioned, be made uſe of; for the leſs the Bodies are, and the ſwifter their Motion, the more in Proportion is the Motion retarded by the Air's Reſiſtance; as ſhall be ſhewn in its Place.

When the Ball P is let fall from E, running down the Curve E A, it acquires ſuch a Degree of Velocity, as appears to be always the ſame in ſeveral Trials; and with that Velocity and horizontal Direction, it continues its Motion.

Experiment.] Having joined together the three Parts of the Machine, as in *Fig.* 4. let go the Ball P from E, and it will ſtrike the Point F. Take away the leaſt Part C, and let the Ball come down as before, and it will ſtrike G. Laſtly, take away the Part B, and the Ball, deſcending as before, will ſtrike againſt H.

If you ſtick on a Piece of ſoft Clay upon H and G, the Point of the Stroke will be exactly mark'd; this will not do ſo well in the Point F, becauſe of the great Obliquity of the Motion there; but, by repeating the Experiment, the Point F will be well enough determined by Sight only.

The Propoſition of *Numb.* 134 may be experimentally confirmed by this Machine; for, as we have already ſaid, the Ball running down E A will ſtrike the Point H. 211

Coming down E A, it acquires a Celerity which it could have acquired in falling in E D*; with that Celerity it is horizontally projected from the Point A, and it moves that Length *b* H equably according to that Direction, whilſt by its Fall it goes thro' A *b* equal to E D; but *b* H is double the Length A *b* or E D. *150

212 What has been ſaid of the Curve, deſcribed by a Body projected horizontally, may alſo be applied to any Projection.

Let the Body be projected in the Direction AE, (*Plate* XV. *Fig.* 5.) and let AB, BC, CD, DE, be equal; the Body will go through the Curve AFGHI, ſo that BF, CG, DH, EI, will be to one another, as 1, 4, 9, and 16; in which Caſe the Curve is alſo called a *Parabola*.

DEFINITION.

213 Let AI be drawn horizontal, and the Curve above-mentioned will cut it at I; AI is called the Amplitude of the Projection.

The Motions of Bodies projected with the ſame Celerity, but different Directions, may be compared together.

Let AL be the Height to which a Body, thrown up with a determinate Degree of Celerity, may riſe: Let the Body with the ſame Celerity be thrown along AB, cutting in B the Semicircle deſcribed in the Diameter AL; let AB expreſs that Celerity, and MB be parallel to the Horizon. The Motion in AB may be reſolved *192 into two others*; the firſt along MB, a horizontal, and the ſecond along AM, a vertical Line; and it is only by that ſecond Impreſſion that the Body aſcends: The Height therefore, to which the Body aſcends in that Caſe, is to the Height to which it would aſcend with the Celerity AB, as *138 the Square of AM to the Square of AB*; that is, as AM to AL; but this is the Height to which the Body aſcends with the Celerity of the Projection; therefore alſo AM is the greateſt Height to which the Body comes in that Projection. In the Time of the Aſcent in AM, the Body might by an uniform Motion, with the ſame Celerity with which it moves in A M, in a horizontal Motion go thro' twice the Length of the Line MB; and as the Time of the Fall is equal to the Time of the Aſcent*, the Amplitude AB is four Times the Length of the ſame Line M B. Now this Demonſtration will ſerve, whatever the Inclination of the Direction of AB is. Whence we deduce, *136

1. *That the Amplitude is the greateſt, with the ſame Celerity, when the Angle of the Projection is a Half Right Angle.* For then the Line *m b*, being a Radius of the Semicircle, is the greateſt of all. 214

2. *Except this Caſe, there are always two Inclinations that give the ſame Amplitude*; for if thro' B, B *b* be drawn parallel to A L, cutting the Semicircle at *b*, and *m b* parallel to the Horizon, this Line will be equal to M B; therefore the Amplitude of the Projection, in the Direction A *b*, will alſo be A I. In the ſecond Part of the following Book, all this will be confirm'd by Experiments.

If the Celerity be changed, and the Body projected in the ſame Direction, the Amplitude is changed, in the ſame Ratio as the Diameter A L; that is, *the Amplitudes, the Direction remaining the ſame, are as* the Heights to which Bodies, with the ſame Celerities, being thrown up, may aſcend; and therefore they are as *the Squares of the Celerities.* 215

CHAP. XXV.

Of Central Forces.

A Body in Motion continues its Motion in a Right Line*, and does not recede from *124
it, unleſs a new Impulſe acts upon it; after ſuch an Impulſe the Motion is compound, and ſo from the two there ariſes a third Motion in a
190 Right Line alſo. If therefore a Body is moved in a Curve, it receives a new Impulſe every Moment; for a Curve cannot be reduced to Right Lines, unleſs you conceive it divided into Parts infinitely ſmall. We have an Example of that
208 Motion in the Projection of heavy Bodies; and another in all Motions round a Point as a Center.

216 *If a Body, that is continually driven towards a Center, be projected in a Line that does not go thro' that Center, it will deſcribe a Curve; and, in all the Points of it, it endeavours to recede from that Curve, according to the Direction* of a Curvature; that is, *of a Tangent to the Curve*; ſo that if the Force driving towards the Center ſhould immediately ceaſe to act, the Body would continue its Motion in a Right Line along the Tangent.

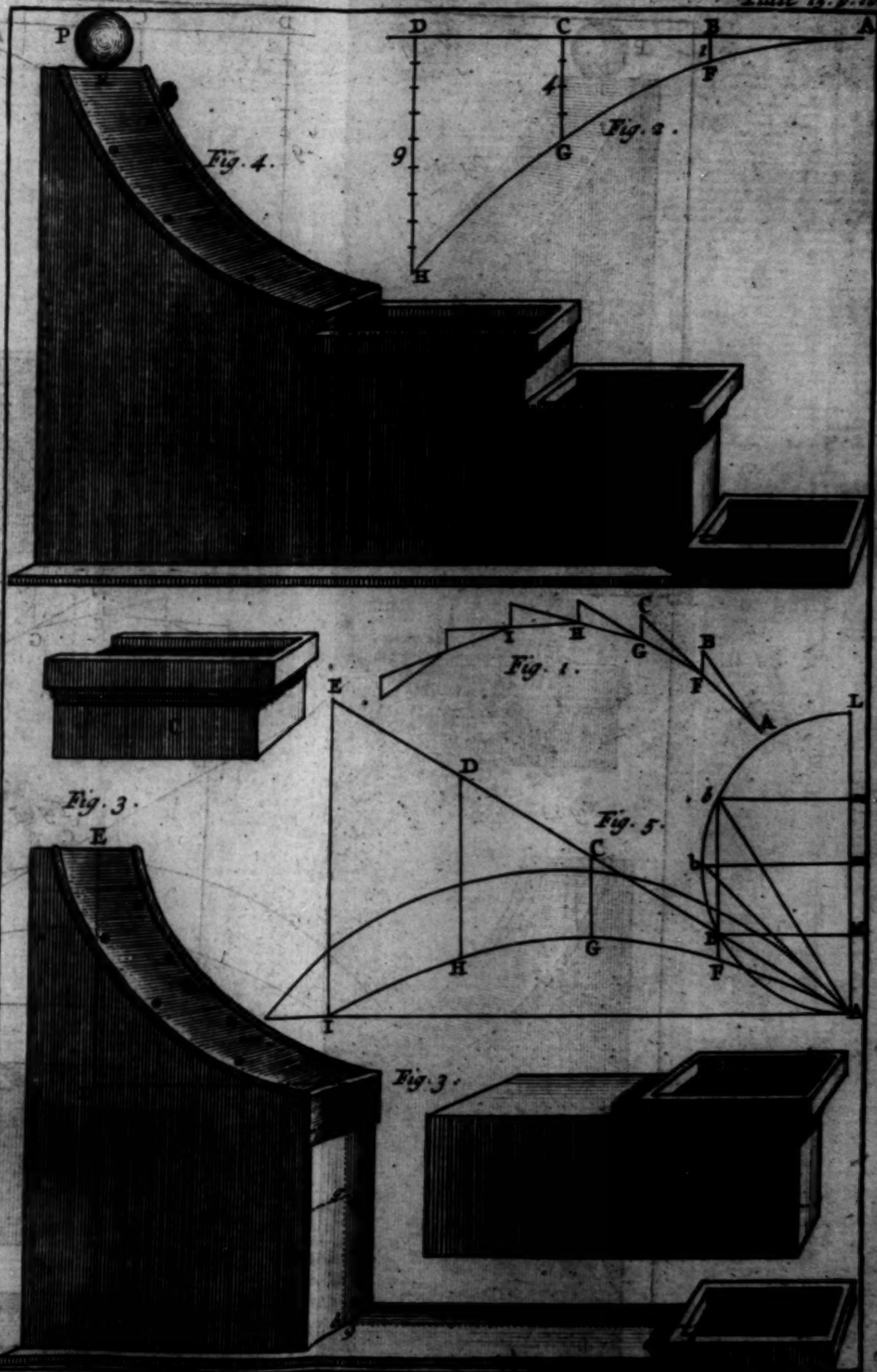
A Stone whirl'd round in a Sling deſcribes a Curve, becauſe the Sling does every Moment, as it were, draw it back towards the Hand; but, if you let the Stone go, it will fly out in the Tangent of the Curve.

DEFINITION I.

218 *The Force with which a Body in the Caſe abovementioned endeavours to fly from the Center*, ſuch as the Force by which the Sling in Motion is ſtretch'd, *is call'd* a Centrifugal Force.

DEFI-

Plate 15. p. 104



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DEFINITION II.

But *the Force, by which a Body is drawn or impell'd towards that Center, is call'd* a centripetal Force. 219

DEFINITION III.

Theſe Forces are by a common Name call'd *Central Forces.* 220

In all Caſes, the centrifugal and centripetal Forces are equal to one another; for they act in contrary Directions, and deſtroy one another. 221

The whirl'd Sling is equally ſtretch'd both Ways*, and the Stone endeavours to recede from the Hand with as much Force as it is drawn towards it. *126

Central Forces are of great Uſe in Natural Philoſophy; for all the Planets move in Orbits, and moſt of them, if not all, turn upon their Axes.

I ſhall choose out the chief Propoſitions relating to theſe Forces, and confirm them by Experiments; but firſt we muſt deſcribe the Machines with which theſe Experiments are performed.

Plate XVI. *Fig.* 1 and 2.] A is a round Board or Table of 2 Feet and a half Diameter; whoſe vertical Section is ſeen in *Fig.* 2. in which *aa* repreſents the Section of the Table itſelf, and *gh* the Section of its turn'd Foot, which is joined perpendicularly to its Center: This Foot or Supporter of it conſiſts of two Pieces ſeparated at D, which are fix'd together by four ſmall Irons, whoſe Ends are rivetted to Rings of the ſame Metal. 222

The upper Part of the Foot has a Groove round it at *cc*; and has a cylindric Hole thro' it at *fg* of three Quarters of an Inch Bore.

The Frame of the whole Machine, repreſented at C, is very ſolid; and one Side receives the Foot of the Table A, which paſſes freely thro' an Hole in the upper Part of the Frame C; to which is firmly joined the wooden Collar F of *Fig.* 3. which fits in the ſaid upper Part of the Frame.

The Table with its Foot bears upon the croſs Piece ED, which has a Plate of Iron to receive the Braſs Center *b*. This tranſverſe Piece is fix'd but juſt above the Feet of the Frame, that, when the Table-Foot is let down upon it, the Groove *cc* may be but juſt above the wooden Collar; to the Top of which are ſcrewed down two Iron Plates RR (*Fig.* 3.) by four Screws, ſuch as *ss*. In this Poſition the Table will very freely move horizontally about its Center; and, that it may move the more eaſily, there is ſlipped on upon the Foot cloſe to the Table (where it is not round, but ſix or eight ſquare) a ſmall Wheel or Pulley, whoſe Section you ſee in *bb*, and which is joined to the Table by means of the Screws *el*, *el*. There muſt be three other ſuch Wheels whoſe Circumferences, taking them at the Bottom of their Grooves, are to one another as one, two, three; the leaſt of all the Wheels is of about 5 Inches Diameter.

Another Table B, made juſt like the firſt, is to be whirl'd about round its Center in the oppoſite Part of the Frame C. Tho' there is a ſmall Difference between them; for in this the lower Part of the Foot has an Hole thro' it as well as the upper, (ſee *ih Fig.* 4.) yet it is turn'd freely, having fix'd to its Bottom a Braſs Plate with a Hole in it to receive the little Pipe of another Plate M, whoſe vertical Section is ſeen at L, and which is fixed to ED, the other croſs Piece of the Frame, at the Place *m*. This croſs Piece muſt be bored thro' to anſwer to the Hole of the Plate, in ſuch Manner that a Thread may go from the Top of the Table quite thro' the whole Foot and the Piece ED. Such a Wheel as *bb* is made faſt to the Table immediately under it, and is in Bigneſs juſt equal to the leaſt of thoſe which are made to take off and on, belonging to the Table A.

The two Tables A and B may be whirl'd, very ſwiftly, either ſeparately, or both together, by help of the great vertical Wheel Q: For performing of which, you muſt make uſe of the Machine of *Fig.* 5. which is a wooden Plane, or ſtrong flat Board, to which is perpendicularly fixed a Parallelopiped, in whoſe upper Surface are vertically fixed the two Pullies *vv*, at the Ends, and ſideways at one End there is another Pulley, as *t*, which is horizontal. The Surface, when the Machine is applied to the Frame, is in the ſame Plane with the ſmall Wheels of the Tables.

If B alone is to be whirl'd, the Piece of *Fig.* 5. is to be fixed to the Frame C, by help of two Screws going thro' ſuch Holes as *x* in the lower Part of the Piece, which muſt be ſo fixed, in reſpect to the little Wheel of the Table, as is repreſented in *Fig.* 6. where *b* repreſents that little Wheel: The Rope goes round the great Wheel Q, and from its lower Part goes down *d* towards *v*, goes round the Wheel *b*, and againſt the Pulley *t*, towards *c*, and ſo comes back to the upper Part of the Wheel Q.

The ſeventh *Figure* repreſents the Poſition of the Machine or Piece of *Fig.* 5. when both Tables are to be whirl'd round at once. A Sight of the Figure ſhews the Way of the Rope, which from *v* goes down to *d*, and ſo to the lower Part of the great Wheel.

Beſides this, in ſeveral Experiments you muſt make uſe of long Boxes or Troughs IF, IF, which are laid upon the Tables, and fix'd to them with Screws; the Center of every one of theſe Boxes lies juſt over the Center of the Table where there is a Hole equal to the Hole *g*, *f*, (*Fig.* 2.) and exactly anſwering to it, in which Hole of the Box is thruſt a wooden Cylinder N, (*Fig.* 2.) as may be ſeen at G: Thro' the Middle of this Cylinder goes a little Glaſs Tube of about a Quarter of an Inch Bore, whoſe Ends are thicken'd at the Flame of a Lamp, ſo as to make the Hole ſomething leſs and ſmooth; that a Thread or ſmall String may run up and down thro' it, without any ſenſible Friction.

One of the Troughs holds a Ball, tied to a Thread which goes thro' the above-mentioned Tube, and is alſo faſtened by a Screw to the Weight O, (*Fig.* 7.) which lies in D the Separation of the Foot. This Weight is laid upon the lower Half of the Foot, from whence it is raiſed up, as the Ball recedes from the Center of the Trough.

This Weight is a round Plate of Lead, and of about 2 Inches Diameter; it has a Braſs Cylinder fix'd to its Center, whoſe upper Part, in order to receive the String, is cleft into two Parts, which are drawn together by means of a Screw: This Plate of Lead with its Cylinder weighs half a Pound; and there muſt be two ſuch Weights.

There are ſeveral other Weights, ſome of half, ſome of a quarter of a Pound, repreſented by P, (*Fig.* 7.) which are to be laid upon the ſaid Weight O; that one may at Pleaſure vary the Weight to be raiſed by the Ball.

223 *When a Body, laid upon a Plane, does in the ſame Time, and about the ſame Center, revolve with that Plane,*

Plane, and ſo deſcribe a Circle; if the centripetal Force, by which the Body is every Moment drawn or impelled towards the Center, ſhould ceaſe to act, and the Plane ſhould continue to move with the ſame Celerity, the Body will begin to recede from the Center (in reſpect of the Plane) in a Line which paſſes thro' the Center.

Experiment 1.] Take a Ball which is tied to a Thread, the other End of the Thread being faſtened to the Center of one of the Tubes A or B, and lay it on the Table, which muſt be whirl'd ſingly ſo long, till the Ball is carried round with it; here the Ball is at reſt, in reſpect of the Table, and in that Situation it is retained only by the String faſtened to the Center; therefore it ſuffers no Impreſſion in that Plane, except that by which the String is ſtretched, that is, whoſe Direction paſſes thro' the Center of the round Table; and ſo, if it be left to itſelf, it cannot the firſt Moment move in any other Direction in that Plane.

When a Body moves about a Center, if in its Motion it comes nearer to the Center, its Motion is accelerated; but on the contrary retarded, if it recedes from the Center. 224

Experiment 2.] Let the Trough F I, through whoſe Center the Cylinder G, with its Glaſs Tube, is fixed into the Center of the Table B, be faſtened to the ſaid Table.

Let the Ball L, tied to a Thread, be laid in the Trough, and the Thread put thro' the Tube abovementioned, as alſo thro' the whole Foot of the Table, and the croſs Piece at Bottom that ſuſtains the Foot, and then with your Hand hold the End of the Thread.

Let the Table be turned round, and you will obſerve, that, during that Motion, the Ball will apply itſelf to one Side of the Trough, and is carried round ſo as to move with the ſame Velocity as the Trough. Let the Thread be pull'd, ſo as to bring the Ball nearer the Center, and it will immediately ſtrike the oppoſite Side of the Trough, becauſe it moves faſter than the Trough. If you bring your Hand nearer to the Foot of the Table, ſo as to give more String, the Ball recedes from the Center, and ſtrikes the firſt Side of the Trough, as moving more ſlowly than the Trough.

This Acceleration when a Body approaches nearer to the Center, and Retardation when it recedes from it, is determined by Geometricians: If a Body, for Example, which is driven towards the Center C (*Plate* XVI. *Fig.* 11.) be moved in the Curve A E, it will move faſter at E and ſlower at A: Draw the Lines A C, B C, and E C, D C, ſo that the Areas A B C and D E C may be equal to one another, the Parts A B and D E of the Curve are deſcribed in equal Times by 225 the Body; and therefore, *a Body that is retained in a Curve, by a Force tending towards a Center*, is ſaid *to deſcribe round that Center Areas proportionable to the Times.*

226 The inverſe Propoſition is alſo demonſtrated, namely, *That a Body which is moved in any Curve in a Plane, and deſcribes about any Point Areas proportionable to the Times, is turned out of the right Line, and urged by a Force tending to that Point.*

227 *The greater the Quantity of Matter in any Body is, the greater is its centrifugal Force*; which ariſes from a greater Quantity of Motion.

228 If Liquors of different Denſities be included in a determinate Space, ſo that the heavier cannot recede from the Center, unleſs the lighter

come towards it, and they be ſo diſpoſed, that by their Weight the heavy Fluid comes to the Center; upon moving the Whole about that Center, the light Fluid will come towards the Center, and the heavy one fly from the Center.

If a Solid be included with a Liquid in a determinate Space, the ſame may be ſaid as was ſaid of the two Liquids: If it be lighter than the Liquid, it will come towards the Center; if heavier, it will recede from it. All which ariſes from the great Centrifugal Force in the heavier Body.

Experiment 3. *Plate* XVI. *Fig*. 8.] Take four Glaſs Tubes of about one Inch Diameter each, and a Foot long, and having hermetically ſealed them, let them be firmly tied to an inclined Plane. In the firſt, you muſt have Quickſilver and Water; in the ſecond, Oil of Tartar *per deliquium*, and Spirit of Wine; and in the third Water with a Leaden Bullet; and laſtly, in the fourth, Water with a Piece of Cork; and all of them muſt be about half empty.

This inclined Plane muſt be faſtened to the whirling Table A or B, (*Plate* XVI. *Fig*. 1) ſo that the lower Part of the Plane may come almoſt to the Center of the Table, by means of two Screws, one of which goes through *x* (*Fig*. 8.) Let the Table be whirled round, and immediately the lower Part of the Tubes will remain empty, and the heavier Bodies will go to that Part of the Tube which is fartheſt from the Center; the Cork deſcends and ſtrikes to the lower Part of the Water, whilſt the Leaden Bullet goes to the Top of the Tube.

Central Forces not only differ on account of 229
the Quantity of Matter, but the Diſtance does alſo cauſe a Change, and likewiſe the Celerity with which the Body is moved round. There is nothing

nothing elſe that can make any Difference in theſe Forces: and theſe are all the Things to be conſidered, when we compare them together.

Definition IV.

230 The Periodical Time *is the Time, in which a Body going round a Center performs one whole Revolution*; that is, if it deſcribes a Curve that returns into itſelf, the Time elapſed between its Departure from, and Return to a Point: If the Curve does not return into itſelf, inſtead of a Point we muſt take a Line paſſing thro' its Center.

231 The Periodical Time depends upon the Celerity of the Body; and therefore, in comparing central Forces, it muſt be taken for the Velocity.

When the Periodical Times are equal, and the Diſtances from the Center are alſo equal; the central Forces are as the Quantities of Matter in the revolving Bodies.

Experiment 4. *Plate* XVI. *Fig.* 1.] Of the three Wheels or Pullies *b b*, mentioned in the Deſcription of the Machines, apply the leaſt to the Table A; ſo that if the two Tables A and B be whirled at the ſame Time by the Motion of the Wheel Q, they may run round in equal Times; to each of them fix the long Troughs I F, I F, and the Cylinders G G, that contain Glaſs Tubes, muſt be thruſt thro' the Center Holes of the Troughs quite into the Feet of the Tables.

Put a Ball L of half a Pound into the Trough of the Table B, and a Ball L of one Pound in the Trough of the Table A: Threads tied to the Balls go thro' the little Tubes G G, and are faſtened to Weights placed in the Separations or Hollows of the Feet of the Tables, in ſuch Manner, that the Diſtances of the Balls from the Center, when the Threads are ſtretched and the Weights not raiſed, may be equal; now if the Weight in the Separation or Hollow of the Foot of A be one Pound, that in the Separation of B muſt be half a Pound; or if this laſt ſhould be one Pound, the other muſt be two Pounds.

Let the Wheel Q be turn'd round faſter and faſter, till by the centrifugal Force of the Balls the Weights above-mentioned be raiſed, and both Weights will be lifted up preciſely at the ſame Time; therefore Weights that are as the Bodies, will, *cæteris paribus*, be overcome by the centrifugal Force.

When the Quantities of Matter in the revolving 232
Bodies are equal, and the periodical Times alſo equal, the Forces are at the Diſtances from the Center.

Experiment 5. *Plate* XVI. *Fig.* 1.] This Experiment is made in the ſame manner as the foregoing; inſtead of a Ball of half a Pound, put in the Trough of the Table B a Ball equal to the other that is of one Pound. Let the Diſtances from the Center be taken in any Proportion; if the Weights joined to the Balls in the ſame Proportion, and the Wheel Q be moved faſter and faſter, you will ſee the two Weights riſe exactly at the ſame Time. As, for Example, if the Diſtance of the Ball upon A be of 12 Inches, and the Weight joined to it of 1 Pound and a half; and the Diſtance of the other Ball of 8 Inches, and the Weight joined to it of 1 Pound, the Experiment will ſucceed.

When the periodical Times are equal, but the Di- 233
ſtances and the Quantities of Matter in the revolving Bodies differ, the central Forces are in a Ratio compounded of the Quantities of Matter and the Diſtances; which follows from the two laſt Propoſitions. To determine that compound Ratio, the Quantity of Matter in each Body muſt be multi-

plied by its Diſtance from the Center, and the Products will be to each other in the Ratio aforeſaid.

Experiment 6.] If in the laſt Experiment the Ball upon B be changed, and you place a Ball of half a Pound at 8 Inches from the Center, and you alſo change the Weight joined to it, and half a Pound be uſed inſtead of a Pound, the Experiment will alſo then ſucceed, and the Weights will begin to riſe at the ſame time. If you multiply the half Pound Ball by its Diſtance of 8 Inches, the Product is 4, and multiplying the 1 Pound Ball by 12 Inches, its Diſtance from the Center, the Product is 12; which Products are to one another as 1 to 3; that is, as the Weight of half a Pound to that of 1 Pound and a half, which are in this Experiment both lifted up at the ſame Moment.

The Differences of central Forces ariſing from the different Diſtances from the Center, and the different Quantities of Matter may compenſate
234 one another; for *ſuppoſing the Quantities of Matter in the revolving Bodies to be in an inverſe Ratio of the Diſtances from the Center, the central Forces will be equal*; as much as one Force is greater in reſpect of the Quantity of Matter, ſo much does the other exceed it by reaſon of its greater Diſtance.

Experiment 7.] Let a Ball of half a Pound be placed at the Diſtance of 14 Inches, and a Ball of one Pound at the Diſtance of 7 Inches; every thing elſe being as in the foregoing Experiment; if the Weights in the Foot or Spindle of each Table be alike, they will riſe at the ſame Moment.

235 There is a Caſe of this Propoſition, *when two Bodies joined by a Thread revolve about their common*

Center of Gravity. For the Diſtances from that Center are in an inverſe Ratio of the Weights of the Bodies*, and therefore the central Forces are *95.90 equal. By the Force by which one Body endeavours to recede from the Center, the other is drawn towards it; and by reaſon of the Equality of the Forces, *they retain one another*, *and continue their Motion:* If they revolve about any other Point, they do not continue their Motion; and the Body whoſe centrifugal Force overpowers, recedes from the Center, and carries the other Body along with it.

Experiment 6. *Plate* XVI. *Fig.* 10.] Let two unequal Bodies P and Q be joined by a Thread, in which you muſt mark the Point C, which is the common Center of Gravity of thoſe Bodies, when the Thread is ſtretch'd.

In this Experiment you muſt uſe but one Table, and fix upon it a long Trough that reaches beyond the Diameter of the Table both Ways, and whoſe middle Point is over the Center of the Table. In this Trough you muſt place the Bodies above-mentioned, and the Thread that joins them being ſtretched, the Point C muſt be put in the Middle of the Trough. When the Table is whirl'd round, the Bodies are carried round with it, and remain at reſt in it. If the Point C be removed from the Middle of the Box, upon whirling the Table, both Bodies will be carried to that End of the Box which the Point C was placed neareſt to.

The Difference of the central Forces is alſo determined from the Difference of the periodical Time.

When the Quantities of Matter in the Bodies 236 *whirled round, and the Diſtances from the Center are equal, the central Forces are in an inverſe Ratio of* *the Squares of the periodical Times*: That is, directly as the Squares of the Revolution made in the ſame Time.

Experiment 9. *Plate* XVI. *Fig.* 1.] To the Table A apply a Wheel or Pulley, ſuch as *bb*, (*Fig.* 2.) whoſe Circumference is double the Circumference of the Wheel which is fix'd to the Table B; ſo that when the two Tables are whirl'd both together, B ſhall go round twice for A once; that is, the periodical Time of that ſhall be double the periodical Time of this.

In each Trough IL, IL, lay a Ball of one Pound at equal Diſtances from the Center. The Ball laid on the Table B muſt be tied to two Pounds in the Foot, and the other Ball upon A is joined by its Thread to half a Pound in the Foot. Upon whirling the Tables, both Weights will riſe at the ſame Time: Which Weights are here as 1 to 4, the periodical Times being as 2 to 1, whoſe Squares are reciprocally as 1 to 4.

237 *However the central Forces differ from one another*, they may, according to what has been ſaid, be compared to one another; for *they are always in a Ratio compounded of the Ratio of the Quantities of Matter in the revolving Bodies**, *and the Ratio of the Diſtances from the Center**, *and alſo an inverſe Ratio of the Squares of the periodical Times**. If you multiply the Quantity of Matter in each Body by its Diſtance from the Center, and divide the Product by the Square of the periodical Time, the Quotients of the Diviſion will be to one another in the ſaid compound Ratio, that is, as the central Forces.

*231 *232 *236

Experiment 10.] Every thing being prepared as in the former Experiment, ſet a Ball of half a Pound, at the Diſtance of 8 Inches from the

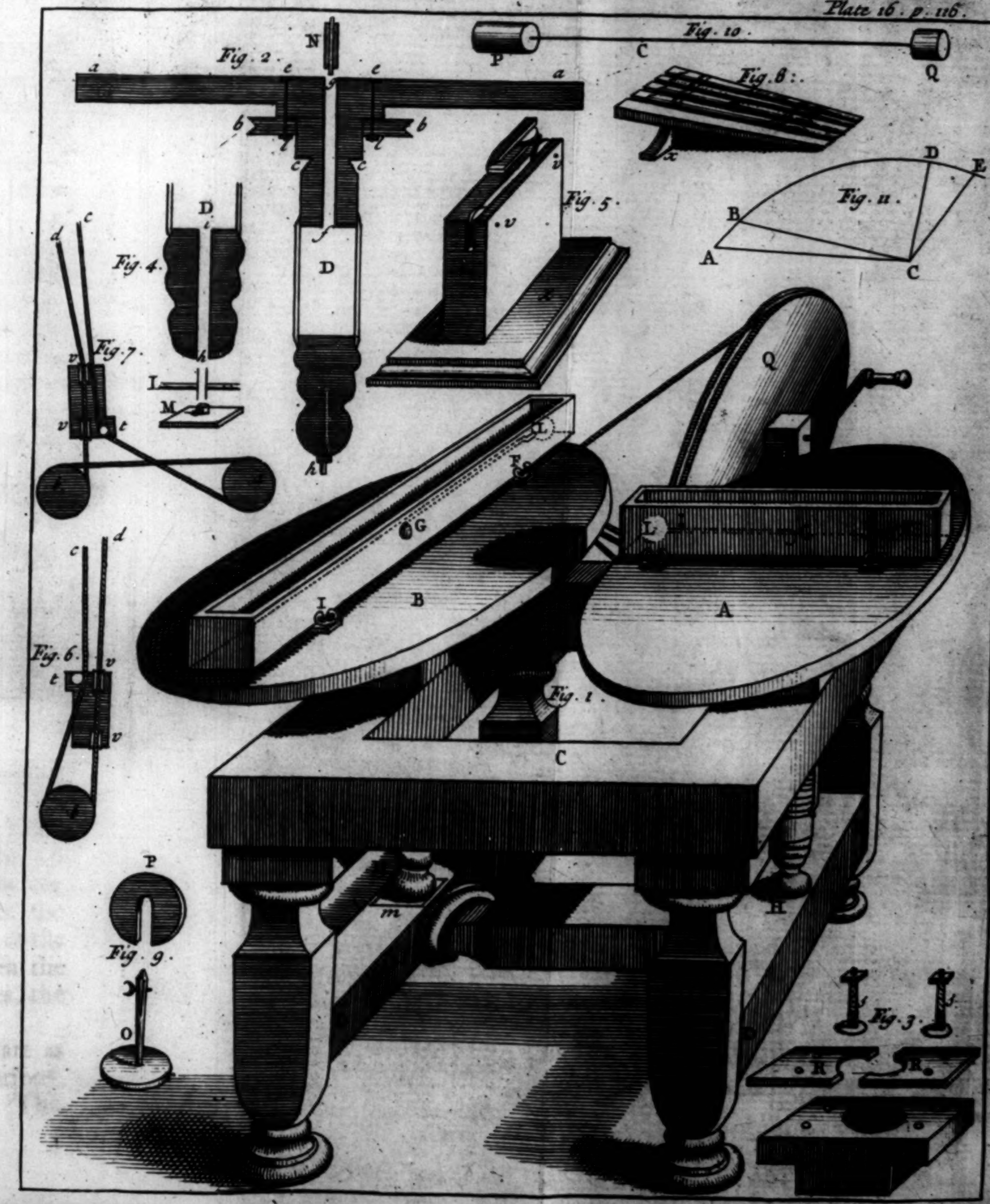
Fig. 10
P
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Q
Fig. 2
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Fig. 8
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Center of the Table B, and let it by the Thread be joined to one Pound in the Foot; let another Ball of two Pound be placed at the Diſtance of 12 Inches from the Center of the Table A, and joined with a Weight of 3 Quarters of a Pound; whirl the Tables, and the Weights will be raiſed juſt at the ſame Time.

Here the Bodies are as $\frac{1}{2}$ to 1; the Diſtances as 8 to 12; the Squares of the periodical Times as 1 to 4; multiplying one Half by 8, and dividing the Product by 1, the Quotient of the Diviſion is 4; multiplying 1 by 12, and dividing the Product by 4, the Quotient is 3. Therefore the central Forces are to one another as 4 to 3, which Ratio alſo the Weights in the Feet have to one another.

When the Quantities of Matter are equal, the Diſtances themſelves muſt be divided by the Squares of the periodical Times, to determine the Proportion of the central Forces. 238

In that Caſe, *if the Squares of the periodical Times be to one another as the Cubes of the Diſtances*, the Quotient of the Diviſions, as well as *the central Forces, will be an inverſe Ratio of the Squares of the Diſtances.* 239

Experiment 11.] Let the periodical Times of A and B be as 1 to 2, in the ſame manner as in the two laſt Experiments. Take two equal Balls, and let the Diſtance from the Center on B be 10 Inches, and the other Ball's Diſtance from the Center be of 16 Inches: To the Thread of the firſt, faſten one Pound and a Quarter, and to the Thread of the other faſten half a Pound in the Hollow of the Foot A; whirling the Tables, the Weights will riſe the ſame Moment.

In that Experiment the central Forces are as 5 to 2, which you alſo find by Calculation*. *238

This Ratio differs very little from the inverſe Ratio of the Squares of the Diſtances, which are to one another as 200 to 512; the Cubes of the Diſtances are alſo almoſt as the Squares of the periodical Times: Theſe Squares are as 1 to 4, and thoſe Cubes as 125 to 512, which Ratios do not much differ. If you take other Numbers, theſe Ratios will be exactly the ſame, and the Experiment will ſucceed in the ſame Manner; but it is not eaſy in the Experiment to vary the periodical Times or the Weights in what Ratio you pleaſe.

240 When the Force, by which a Body is carried towards a Point, is not every where the ſame, but is either increaſed or diminiſhed in Proportion to the Diſtance from the Center, ſeveral Curves will thence ariſe in a certain Proportion.

241 If the Force decreaſes in an inverſe Ratio of the Squares of the Diſtances from that Point, the Body will deſcribe an *Ellipſis*, which is an oval Curve, in which there are two Points called the *Foci*, and the Point towards which the Force is directed falls into one of them: So that in every Revolution the Body once approaches to, and once recedes from it. The Circle alſo belongs to that ſort of Curves, and ſo in that Caſe the Body may alſo deſcribe a Circle: The Body may alſo (by ſuppoſing a greater Celerity in it) deſcribe the two remaining Conic Sections, *viz.* the *Parabola*, or *Hyperbola*, Curves, which do not return into themſelves.

242 On the contrary, if the Force increaſes with the Diſtance, and that in the Ratio of the Diſtance itſelf, the Body will again deſcribe an *Ellipſe*; but the Point, to which the Force is directed, is the Center of the *Ellipſe*, and the Body in each Revolution will twice approach to, and again, twice recede from that Point. In this

Caſe alſo a Body may move in a Circle, for the Reaſon above-mentioned.

Experiment 12.] Hang up a leaden Ball with a long Thread; if the Ball be drawn back from its Point of Reſt, it is always carried towards it by its Gravity, and from equal Sides with equal Force, if the Diſtance be equal. The Ball in its Motion deſcribes an Arc of a Circle, which Way ſoever it falls when you let it go: If thoſe Arcs are not very great, they coincide with a *Cycloid*, and the Force, with which the Ball in any Point is carried towards the loweſt Point, is as its Diſtance from that Point*; therefore here the Force increaſes in the Ratio of the Diſtance. *156

Let the Ball be pull'd back from the loweſt Point, and projected obliquely; then it will deſcribe an oval Curve about that Point, which (when the Ball does not run out to a great Diſtance) will hardly differ at all from an *Ellipſe*, becauſe of the Proportion of the Forces, and becauſe in that Caſe the Ball does ſenſibly move in the ſame Place.

The Center of the *Ellipſe* is the Point in which the Ball is at reſt when it is not projected; and in every Revolution the Ball does twice approach to, and twice recede from it. If the Ball hangs over a Table ſo as almoſt to touch it when it is at reſt, and the Point, over which it is, be mark'd upon the Table, the Experiment will become more ſenſible if you draw an Oval upon the Table with Chalk, by following the Body with your Hand.

243 If the Proportion (mentioned *Numb.* 241, and 242.) of the Forces by which a Point is driven towards a Center, be a little changed, the Body will no longer deſcribe an *Ellipſe*; but ſuch a Curve as may be reduced to an *Ellipſe*, by ſup-

poſing the Plane in which the Body moves agitated by ſome Motion, which therefore will make the *Ellipſis* moveable.

Experiment 13.] Every Thing being as in the former Experiment, let the Ball be ſo thrown that it may run out to a greater Diſtance; and then it will deſcribe a Curve which may be referr'd to the moveable Oval: It will indeed twice in every Revolution come towards the Center, and twice recede from it; but the Place of the Points in which it is leaſt, or moſt diſtant, is changed every Revolution, and theſe Points are always carried the ſame Way, their Motion conſpiring with the Motion of the Ball.

CHAP. XXVI.

Of the Laws of Elaſticity.

*44 WE have already ſhewn what *Elaſticity* is, and whence it ariſes*; and what is its Effect in the Congreſs of Bodies, whether they ſtrike one another directly or obliquely; what remains is to examine the *Laws* of *Elaſticity* itſelf, which we ſhall do from Phænomena.

All Bodies, in which we obſerve *Elaſticity*, conſiſt of ſmall Threads or Filaments, or at leaſt may be conceived as conſiſting of ſuch Threads; and it may be ſuppoſed that thoſe Threads laid together make up the Body; therefore that we may examine *Elaſticity* in the Caſe which is the leaſt complex, we muſt conſider Strings of muſical Inſtruments, and ſuch as are of Metal; for Catgut-Strings have a ſpiral Twiſt, and cannot be conſider'd in the ſame manner as thoſe Fibres of which Bodies are form'd.

244 *The Elaſticity of Fibres conſiſts in this, that they can be extended, and taking away the Force by which they*

they are lengthen'd, they will return to the Length which they had at firſt.

Fibres have no Elaſticity, *unleſs they are extended with a certain Force*; as it appears in Strings that have their Ends fix'd without being ſtretch'd; for if you remove them a little from their Poſition, they do not return to it: But what the Degree of Tenſion is, which gives Beginning to *Elaſticity*, is not yet determined by Experiments. 245

When a Fibre is extended with too much Force, it loſes its Elaſticity; and this Degree of Tenſion is alſo unknown; this we know, that the Degree of Tenſion in Fibres, which conſtitutes Elaſticity, is confined to certain Limits. 246

Hence appears the Difference of Bodies that are elaſtic, and ſuch as are not ſo; why a Body loſes its Elaſticity, and how a Body deſtitute of Elaſticity acquires that Property. A Plate of Metal, by repeated Blows of a Hammer, becomes elaſtic, and being heated, does again loſe that Virtue. 244

Between the Limits of Tenſion, that terminate Elaſticity, there is a different Force required for different Degrees of Tenſion in, or to ſtretch Chords to certain Lengths: What this Proportion is, muſt be determined by Experiments, which muſt be made with Chords of Metal, as was ſaid before. But as theſe Wires are ſcarce ſenſibly lengthen'd, the Proportions of the Lengthening cannot be directly meaſured; therefore they muſt be meaſured by another Method.

Let AB (*Plate* XVII. *Fig* 1.) be a ſmall Wire ſtretch'd horizontally with a certain Force, whoſe Ends are fix'd at A and B: Let it be bent by a Weight hanging in the Middle of it, ſo that it may come to the Poſition ACB.

DEFI-

DEFINITION.

248 *The Line* C*c drawn from the middle Point of a String or Chord after its Inflection, to the middle Point of the ſame when it was in its natural State, is call'd* the Sagitta (*Arrow*) of the Chord.

Let *ce* be an Arc of a Circle deſcribed about the Center B, with the Radius B*c*. Half the Chord or Wire by the Inflection was ſtretch'd the Length C*e*, which Quantity has a certain Relation to the *Sagitta* C*c*.

The Weight alſo, by which the String is ſtretch'd, has a certain Relation to the Force with which the Fibre is lengthen'd, that is drawn along BC: And ſo in ſeveral Experiments, by comparing the *Sagitta* C*c*, and the Weights with which the Chords are inflected, the Proportions of the Lengthenings are determined; as will be ſhewn in the following Experiments.

249 The Machine for performing them is a vertical Board, about 3 Foot long, and 1 Foot high. See *Plate* XVII. *Fig*. 2.

The Rulers of Wood *mn*, *mn*, are fix'd to the Board like a Moulding, and carry two Priſms H, H, made like a Wedge, which ſlide along upon the Rulers, and are fix'd any where upon them by means of Screws, which hold them behind the Board, their Shanks being moved backwards and forwards, by means of a Slit in the Board.

Between A and B there are equal Diviſions reckon'd from the Middle on either Side, in order to determine the Places where to fix the Priſms.

At O there is a Groove, to hold the Pulley T in the Side of the Board; which Pulley is repreſented in *Plate* XIII. *Fig*. 8.

The Wire, with which the Experiments are made, is fix'd at one End of the Ruler *mn*, and at the other End goes over the Pulley T, the Weight P ſtretching it, and the Priſms, H, H, ſuſtaining it in Points, which are equally diſtant from the Middle of the Machine.

There is a Braſs Plate *de* let into the Middle of the Board, and mark'd with very ſmall Diviſions, along which moves another Braſs Plate or Index *fg*, which hangs upon the Wire, having a Hole thro' which it runs: This Index has a Scale hanging from it, which, together with the Index *fg*, weighs juſt an Ounce. The Length of the Wire is determined in each Experiment by the Diſtance of the Edge of the Priſms H, H; for in the ſmall Inflections made by hanging on Weights in C, concerning which alone Experiments are made, the String is not moved upon the Priſms, nor is the Weight P raiſed up, but only the Part A B is extended by theſe Inflections.

In the Inflections of the String, the *Sagittæ* are meaſured by the Diviſions on the Plate *ed*; for the End *g* of the Index *gc* does always deſcend equally with the Point C in every Inflection.

Experiment 1.] Let P be a two Pound Weight, 250 and let the Wire be inflected at C with the Weight of an Ounce, that is, with the Weight of the Scale and Index *fg*; and obſerve the Diviſion of the Plate *ed*, to which the End *g* of the Index *fg* deſcends. Change the Weight P to 4 Pounds, and alſo double the Weight by which the String is inflected, that *g* may deſcend to the ſame Diviſion, and this Weight will be two Ounces: Three Ounces will give the ſame Inflections, when the Weight P is of 6 Pounds.

From this Experiment it follows, that *the* 251 *Weight, by which a Fibre is increaſed a certain Length by its ſtretching, is in the different degree of Tenſion, as the Tenſion itſelf*; if, for Example, there be three Fibres of the ſame Kind, Length and Thickneſs, whoſe Tenſions are as 1, 2 and 3; any Weights in the ſame Proportion will equally ſtretch thoſe Fibres.

252 *The leaſt Lengthenings of the ſame Fibres are to one another nearly as the Forces by which the Fibres are lengthened.* As for Example, let a Fibre be ſtretched with the Weight of 100 Ounces, if it be ſeparately lengthen'd with the Weights of 1 Ounce, 2 Ounces, and 3 Ounces, the Lengthenings will be nearly as 1, 2, and 3; that is, each Ounce ſuperadded does equally lengthen the Fibre: For the Tenſions by the Weights of 100, 101, and 102 Ounces, by which the Fibre is ſtretch'd in each Caſe, when an Ounce is ſuperadded, do not ſenſibly differ from each other.

The Property of Fibres may be applied to their Inflection, and is of great Uſe. Let the Wire AB (*Plate* XVII, *Fig.* 3.) be ſo inflected, as to acquire the Poſitions A*c*B, A*c*B, and ACB, yet ſo that in the greateſt Inflection the *Sagitta* may not be $\frac{1}{4}$ Inch long, ſuppoſing the Wire 2 Feet and a Half: In thoſe Caſes the Lengthenings of the String are very ſmall, therefore they are in the Ratio of the Forces that
252 produce them, and they ſerve to expreſs them; let cD expreſs the Force by which a String is ſtretch'd when it is not inflected, and with the Center B deſcribe the Circle D*d*; the Lines *dc*, *dc*, *d*C, which are longer than *c*D by the Quantity by which the Fibre was lengthened in every Caſe, expreſs the whole Forces by which the Fibre is ſtretched in every Caſe. But here the Arc D*d* is hardly of one Degree, and D is always far enough diſtant from the Point c, wherefore D*d* may be looked upon as a Right Line parallel to cC, and the Lines *cd*, *cd*, C*d* have the ſame Ratio to the Lines *c*B, *c*B, CB. Therefore

fore the Point C is always drawn towards B*c* and A, by Forces proportionable to the Line C B or C A, and the Force by which the Wire is inflected, whoſe Direction is along c C, is as the double *Sagitta**, or as the *Sagitta* itſelf. *Therefore in all the leaſt Inflections of a Chord, Muſical String or Wire, the* Sagitta *is increaſed and diminiſhed in the ſame Ratio as the Force with which the Chord is inflected.* *203 253

Experiment 2. *Plate* XVII. *Fig.* 2.] Let the Wire A B, ſtretched by any Weight, be inflected by the Weight of 1, 2, and 3 Ounces; the Deſcents of the Point *g*, that is, the *Sagittæ* themſelves are to one another as 1, 2, and 3.

254 *In Chords of the ſame Kind, Thickneſs, and which are equally ſtretched, but of different Lengths; the Lengthenings, which are produced by ſuperadding equal Weights, are to one another as the Lengths of the Chords.* This is plain, becauſe the Chord is equally ſtretched in all its Parts; therefore the Lengthening of a whole Chord is double the Lengthening of half of it, or of a Chord of half the Length.

As to the Inflection of thoſe Chords, let A B, *a b*, (*Plate* XVII. *Fig.* 4.) be Chords of the ſame Kind and Thickneſs, but of different Length, equally ſtretched, and ſo inflected, that A C B ſhall be the Poſition of the firſt, and *a d b* that of the laſt; and let the Triangles B C *c*, and *b* D *d* be ſimilar: *c* B is to D *b*, that is, the Lengths of the Chords are as C B to *db*; therefore the Chords are lengthened in Proportion to their firſt Length, and conſequently they are drawn by equal Force in the Directions *bd*, *ad*, B C, A C*: But by the Likeneſs of the Triangles above-mentioned, the Forces alſo acting along *c* C and D *d* are equal to one another*, and *the* Sagittæ *c* C, D *d*, *are as the* *254 255 *203 *the Lengths of the Chords*; which does therefore, *cæteris paribus*, obtain *in unequal and inflected Chords.*

Experiment 3. *Plate* XVII. *Fig.* 2.] Let the Chord AB be ſtretched by any Weight, having fixed the Priſms H, H, at the ſixth Diviſion on each Side: Now let it be inflected with any Weight, ſo that the *Sagitta* may be equal to ſix Diviſions of the Plate *e d.* Let the Priſms be brought to and fixed at the fourth Diviſion on each Side, and the *Sagitta* will be equal to four Diviſions of the Plate; and ſo on for any Poſition of the Priſms.

One may compare together Fibres of the ſame Kind, but different Thickneſs; they may be looked upon as made up of ſeveral very fine Fibres of the ſame Thickneſs, whoſe Number in the abovementioned Fibres muſt be taken in a Ratio of the Solidity of thoſe Fibres, that is, as the Squares of the Diameters, or as the Weight of the Fibres when their Lengths are equal. Therefore theſe Fibres will be equally ſtretched by Forces that are in the ſame Ratio of the Squares of the Diameters; which Ratio alſo is required between the Forces by which the Chords are inflected, that the *Sagittæ* may be equal in the given Fibres. But by diminiſhing the Force by which the Fibre is ſtretched in the ſame Ratio as the Force by which
*250 it is inflected, the *Sagitta* is not changed*. There-
256 fore, *if the Forces by which the Fibres are ſtretched be equal, and they are inflected by equal Forces, even in that Caſe alſo the* Sagittæ *will be equal, however different the Thickneſs be.*

Experiment 4. *Plate* XVII. *Fig.* 2.] Take any Chords of the ſame Kind, and unequal Thickneſs;

neſs; and let them be ſeparately applied to the Machine, leaving the Priſms H H in the ſame Place; if they be ſtretched by the ſame Weight P, and alſo be inflected by the ſame Weight L, the *Sagittæ* will be equal.

Let the Chord A B (*Plate* XVII. *Fig.* 1.) *ſtretched any how, be* ſo *inflected* as to acquire the Figure A C B, *then left to itſelf*, and by its Elaſticity it will return to its firſt Figure, and in that Caſe the Motion of the Point C is accelerated; for when the Chord is let go from the Poſition A C B, the Point C is moved with the Force that is able to retain it in that Poſition. This Motion is not deſtroyed, but there is ſuperadded to it, in all the Points of the *Sagitta*, the Force by which the Point C might be retained in them. The Celerity is the greateſt of all at *c*, and by that Celerity the Point C is carried farther, and then returning, *it will perform ſeveral Vibrations*, in which the Point C runs out but ſhort Spaces; for which Cauſe the Force, by which the Point C is acted upon in all Diſtances from *c*, is as the Diſtance * in each Point. Therefore the Motion agrees with the Motion of a Body vibrating in a Cycloid, *and how unequal ſoever the Vibrations are, they are performed in the ſame Time**. 257 *253 *155

If there be two equal and ſimilar Chords, but unequally ſtretched, unequal Forces are required to inflect them equally; therefore the Vibrations are performed in unequal Times. One may compare their Motions with the Motions of the Pendulums which vibrate in Cycloids*, and deſcribing ſimilar Cycloids by different Forces; which Forces are inverſly as the Squares of the Times of the Vibrations*. In Chords therefore likewiſe *the Squares of the Times of the Vibrations are to one another inverſly*, as the Forces by which they are equally 258 *257 *165

equally inflected; which are *as the Weights by*
*250 *which the Chords are ſtretched**.

259 *When the Chords are ſimilar, equally ſtretched, but of different Lengths*, their Motion muſt be compared with that of Pendulums by another Method; for as the Times of the Vibrations are to be conſider'd, the Celerities alſo, with which the Chords are moved, muſt alſo be conſidered: And in the Chords A C B, *a d b* (*Plate* XVII. *Fig.* 4.) whoſe *Sagittæ* are equal, and in which the Points C and *d* may be conſidered as deſcribing ſimilar Cycloids, the Celerities, with which thoſe Points are moved in correſpondent Points, are to each other in an inverſe Ratio of the Squares of the Times
165 of the Vibrations. In Pendulums and equal Chords, the Forces are taken for the Celerities;
77,63 becauſe in thoſe Caſes they are in the ſame Ratio.

Let the Chords A C, *a b*, be divided into very ſmall Parts, but each into an equal Number of Parts; the Ways to be run through by correſpondent Parts, ſuppoſing the *Sagittæ* equal, will be equal, and theſe ſmall Parts will perform ſimilar Vibrations; but the Particles of Matter in the correſpondent Particles are as the whole Chords: That therefore their Celerities may be determined in correſpondent Points, the Forces with which the Chords are inflected, when the *Sagittæ* are equal, muſt be divided by the Quantity of Matter in the Chords, as it follows from *Numb.* 64. It is therefore plain, that thoſe Celerities are to one another directly as the Weights by which the Chords are inflected, and inverſly as the Quantities of Matter in thoſe Chords, that is, inverſly as their Lengths: But thoſe Weights are alſo in an inverſe
255 Ratio of the Lengths of the Chords; therefore
253 the Celerities are in an inverſe duplicate Ratio of thoſe Lengths, that is, inverſly as the Squares of the Lengths;

Lengths; and then, as was ſaid before, the Squares of the Times of the Vibrations will alſo be in that inverſe *Ratio. The Lengths* therefore *of the Chords will be as the Times of the Vibrations.*

One may, in the ſame manner, compare *the Times of the Vibrations of Chords of different Thickneſs, ſuppoſing the Chords equal, and ſtretched with equal Weights*; the Quantities of Matter *are as* the Squares of the Diameters therefore to determine the Celerities of the correſpondent Points, the Weights, by which the Chords are inflected, are to be divided by thoſe Squares, when the *Sagittæ* are equal*; the Celerities therefore are inverſly as the Squares of the Diameters, and therefore *the Diameters* are as the Times of the Vibrations. 260 *256

Any Chords of the ſame kind being given, the Durations of the Vibrations may be compared together; for they *are in a Ratio compounded of the inverſe Ratio of the ſquare Roots of the Weights, by which the Chords are ſtretched*, and the Ratio of the Lengths of the Chords*, and the Ratio of the Diameters**. If you multiply the Diameters by the Lengths, and divide the Product by the ſquare Root of the Weight that ſtretches the Chord, and go through the ſame Operation for ſeveral Chords; the Quotients of the Diviſion will be to one another as the Times of the Vibrations. 261 *258 *259 *260

Elaſtic Plates may be conſidered as a Congeries, or Bundle of Chords; when the Plate is inflected, ſome Fibres are lengthened, and there are unequal Lengthenings in ſeveral Points of the Plate; now the Curve, which is formed by the inflected Plate, may be diſcovered from what has been ſaid concerning Chords. 262

By comparing together *the Inflexions of the ſame Plate*, they *are proportional to the Forces by* 263

K *which*

which the Plate is bent. Let A B (*Plate* XVII. *Fig.* 5.) be an elaſtic Plate or Spring, whoſe End A is fixed, and let it be inflected by two Forces,
264 ſo as to be brought into the Poſition *a b* and *a b*; if the one be doubled, the other, *b b* and *b* B, will be equal; and therefore in the Vibrations the Motion of the Spring is accelerated in the ſame
253 manner as the Motion of a Chord, and the Mo-
156 tion of a Pendulum in a Cycloid; *and the Vibrations of this Plate are performed in the ſame Time.*

Experiment 5. *Plate* XVII. *Fig.* 6.] The Spring A is made up of ſeveral elaſtic Plates, and put into the Box B, and there moves on each Side; between the Rulers *c d*, *c d*, two Strings are fixed to the upper Part of the Spring, and run through the Hole *e*, *e*, in the Bottom of the Box. If you hang half a Pound upon the Threads, it will deſcend half an Inch; add another half Pound, and it will deſcend half an Inch more; and ſo on, 'till the Spring can be compreſſed no farther.

Each ſmall Plate is bent in Proportion to the Weight; and the Motion of the Weight, on account of all the Inflections together, follows the ſame Proportion. The Experiment is made with ſeveral Plates joined together; becauſe in various Inflections the Direction of the Action of the Weight on the Plates is not ſenſibly changed.

265 What has been ſaid of the Inflection of Plates, may be applied to the curve Plate or Spring A C B (*Plate* XVII. *Fig.* 7.) If it be preſſed by two Weights, ſo as to acquire the Poſition *a c b*, *a e b*, and the Weights are to each other as 1 to
*263 2, the Diſtances *c c* and *c* C will be equal*: Therefore the bending in of the Spring, or Spaces

gone through by the Point C, are as the Weights with which the Plate is preſſed. Which may alſo be applied to the bending in of ſeveral Plates joined together.

The Ball ACB (*Plate* XVII. *Fig*. 8.) being 266 made of an elaſtic Subſtance, may be conſidered as conſiſting of ſeveral Plates; and the Introceſſions (or Yieldings inward) of the Point C will be proportionable to the Forces with which the Body is compreſſed.

Let the Point C of the Ball ACBE (*Plate* XVII. *Fig*. 9.) ſtrike ſeveral times againſt any Plane, and let that Point go inwards to *d*, *d*, and D; the Strokes will be to each other as the Lines C*d*, Cd, and CD. At the firſt Stroke the Part *abc* becomes flat, the ſecond Stroke *acb* is flattened, and the third ACB: As here we always conſider the leaſt Arcs, the Arcs (that is, the Diameters of the plane Surfaces made by the Strokes) are to one another ſenſibly as the Chords C*a*, Ca, and CA; therefore the Surfaces are as the Squares of thoſe Chords; in which Ratio alſo, from the Nature of the Circle, are the Lines C*d*, Cd, and CD, which are to each other as the Strokes. Therefore *in elaſtic Spheres, the Planes made by the Strokes follow the Proportion of the Strokes.*

Experiment 6.] Take a flat Piece of blue Marble made faſt in a horizontal Poſition, and a little wet, ſo as to make the Colour the more intenſe; if you let an Ivory Ball fall upon this Plane, that Part of the Ball, which by being made flat applies itſelf to the Stone, leaves a very round Spot in the Surface of it: Let the Ball fall from the Height of 9 Inches, and the Spot be E: Then let it fall from the Height of 3 Feet, which is the Quadruple of the other, and the 268

Spot will be F; laſtly, let it fall from the Height of 6 Feet and 9 Inches, which is nine times the firſt, and the Spot will be G. In that Experiment, the Strokes of the Body againſt the Stone
131 are to each other as 1, 2, and 3: In which
130 Ratio alſo are the Spots E, F and G; for if you draw the right-angled Triangles DAB, DBC. in which the Sides DA, AB, BC, are equal to one another, and to the Diameter of the Spot E, the Line BD will be exactly equal to the Diameter of the Spot F, and the Line CD to the Diameter of the Spot G.

END *of the* FIRST BOOK.

4 AP 64

MATHE-

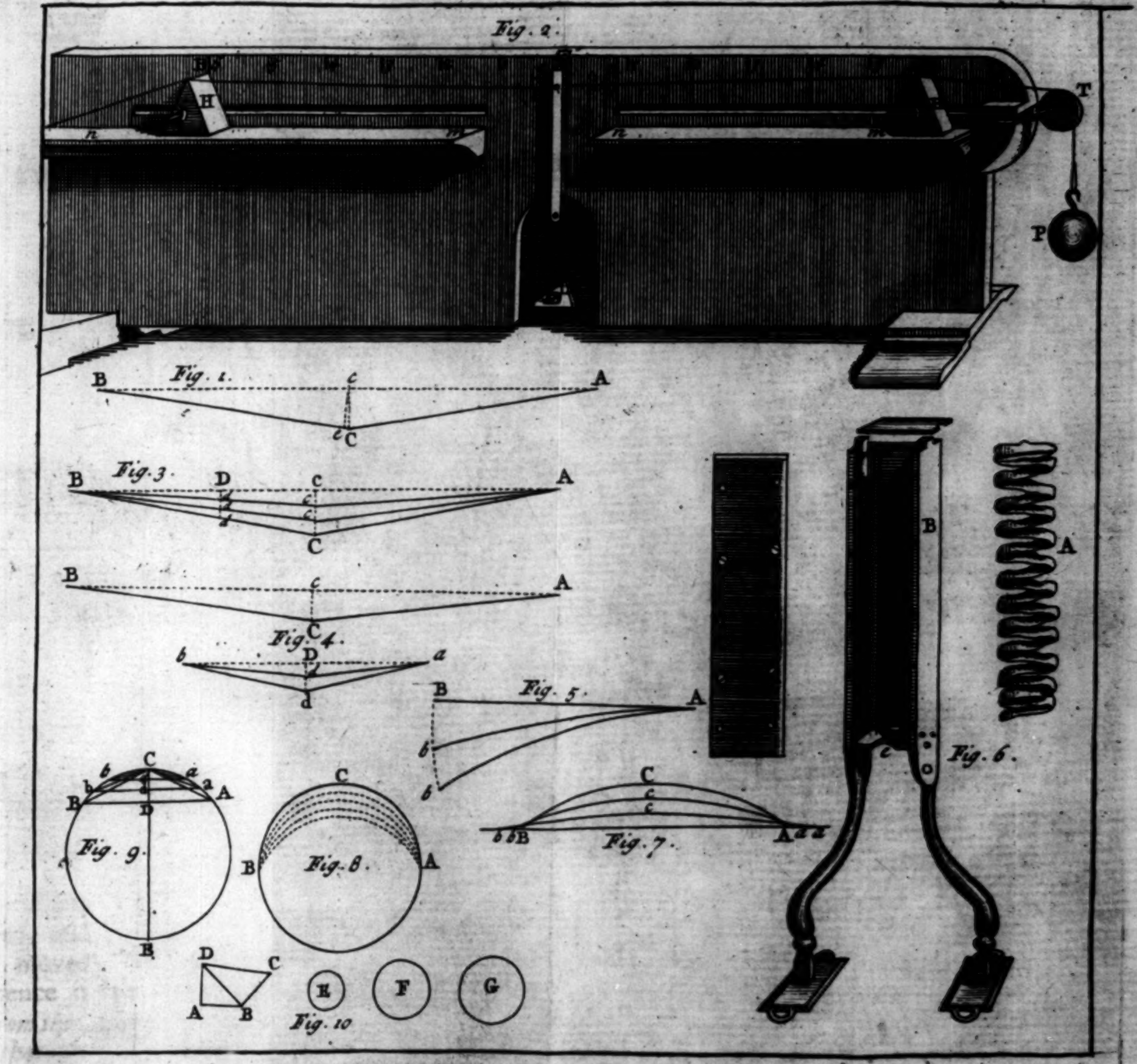
Fig . 2 .
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Fig . 10

MATHEMATICAL
ELEMENTS
OF
Natural Philoſophy,
Confirmed by
EXPERIMENTS.

BOOK II.

PART I. *Of the Gravity*, *Preſſure*, *and Reſiſtance of* FLUIDS.

CHAP. I.

Of the Gravity of the Parts of Fluids, and its Effect in the Fluids themſelves.

A FLUID is a Body whoſe Parts
yield to any Force impreſſed, and
by yielding are very eaſily moved
one amongſt another*. Whence it *30
follows, that *Fluidity ariſes from this*, 269
*That the Parts do not ſtrongly cohere, and that the
Motion is not hinder'd by any Inequality in the Surface
of the Parts*, as it happens in Powders.

But the Particles, of which Fluids conſiſt, are of the ſame Nature with the Particles of other Bodies, and have the ſame Properties; for Liquids are often converted into Solids, when there is a more ſtrong Coheſion of them, as in Ice. On the contrary, melted Metals give us an Inſtance of a Solid changed into a Fluid.

270 *Fluids agree in this with Solid Bodies,* viz. *That they conſiſt of heavy Particles, and have their Gravity proportionable to their Quantity of Matter, in any Poſition of the Parts.* If in the Liquid itſelf that Gravity be not ſenſible, it is owing to this, that the lower Parts ſuſtain the upper, and hinder them from deſcending: But it does not follow from thence, that the Gravity is taken away; becauſe a Liquid contained in a Veſſel will preſs down the End of a Ballance, which carries the Veſſel, in Proportion to its Quantity. The following Experiment will alſo ſhew, that the Gravity is preſerved in any Part of the Liquid.

271 In this, as well as in other hydroſtatical Experiments relating to the Gravity of Fluids, we uſe a very exact Pair of Scales, differing from common Scales only in this, that each Scale has a Hook V V under it, (*Plate* XVIII. *Fig.* 1.) for ſuſpending ſuch Bodies as are to be immerſed in Liquids.

272 The Ballance itſelf hangs by a Line which goes round two Pullies T T, and is faſtened to a Weight P, (*Plate* XVIII. *Fig.* 1.) that, by moving the Weight, the Ballance may be conveniently raiſed and depreſſed, and ſuſpended at any Height.

Experiment 1.] Immerſe in Water the Phial D cloſe ſhut, and hanging by a Horſe-hair, and ballance it with the Weight in the oppoſite Scale; then, without taking the Phial out of the Water, open it, and let it be filled with Water, and you will find, that the Water in the Phial will bring down the End of the Ballance, although it has no Communication with the external Water: If you reſtore the *Æquilibrium*, by putting more Weight into the oppoſite Scale, the Phial will remain ſuſpended in any Part of the Water.

From this Gravity it follows, that *the Surface of a Fluid contained in a Veſſel, to keep it from flowing out, if it be not preſſed from above, or if it be equally preſſed* (for that makes no Alteration) *will become plain*, or flat, *and parallel to the Horizon.* For, as the Particles yield to any Force impreſſed, they will be moved by Gravity, 'till at laſt none of them can deſcend any lower. 272

The lower Parts ſuſtain the upper, *and are preſſed* by them; *and this Preſſure is in Proportion to the incumbent Matter, that is, to the Height of the Liquid above the Particle that is preſſed*; but, as the upper Surface of the Liquid is parallel to the Horizon*, all the Points of any Surface, which *272 you may conceive within the Liquid parallel to the Horizon, are equally preſs'd. 273

If therefore *in a Part of ſuch a Surface there is a leſſer Preſſure than in the other Parts, the Liquid*, which yields to any Impreſſion *there*, will be mov'd, that is, *will aſcend, 'till the Preſſure becomes equal.* 274

Experiment 2. *Plate* XVIII. *Fig.* 5.] Take a Glaſs Tube C open at both Ends, and ſtopping one End with your Finger, immerſe the other in Water; when the Tube is full of Air, the Water will riſe in it but to a very ſmall Height: If you take away your Finger, that the Air that is compreſſed may go out, the *imaginary Surface* (as Mr. *Boyle* uſed to call it) that you conceive in the Water, juſt at the Bottom of the Tube, and parallel to the Horizon, is leſs preſſed juſt againſt the Hole of the Tube, ſo that the Water will riſe up into the Tube till it comes up to the ſame Height with the external Water.

275 *The Preſſure upon the lower Parts, which ariſes from the Gravity of the ſuperincumbent Liquid, exerts itſelf every Way, and every Way equally.*

Which follows from the Nature of a Liquid, for its Parts yield to any Impreſſion, and are eaſily moved; therefore no Drop will remain in its Place, if, whilſt it is preſſed by a ſuperincumbent Liquid, it is not equally preſſed on every Side: But it cannot be moved on account of the neighbouring Drops, which are paſſed in the ſame manner, and with the ſame Force, by the ſuperincumbent Liquid; and therefore the firſt or loweſt Drop is at reſt, and equally preſſed on all Sides, that is, in all Directions.

Experiment 3. *Plate* XVIII. *Fig.* 2.] Let the Glaſs Tubes A, B, D, be immerſed in Water, in the ſame manner as in the laſt Experiment; and, upon taking away the Finger, the Water will riſe in all the Tubes to the ſame Height as in the Tube C: In C the Preſſure is directed upwards, in B downwards, in A ſidewiſe, and in D obliquely; yet the Preſſure is equal in each. If you pour in a greater Quantity of Liquid into the Veſſel, it will alſo riſe equally in each Tube.

276 Hence it follows, that *all the Particles of Liquids* are preſſed equally on all Sides, and therefore *are at reſt*; and that they do not continually move among themſelves, as ſeveral have ſuppoſed.

277 *In Tubes that have a Communication, whether equal or unequal, whether ſtrait or oblique, a Fluid riſes to the ſame Height*; that is, all the upper Surfaces are in the ſame horizontal Plane; which is eaſily deduced from what has been ſaid.

Plate XVIII. *Fig.* 3.] Let A be a Veſſel, and B a vertical Tube, and D an inclined Tube; they muſt communicate by Means of the Tube C, E: Let there be a Liquid poured into them, and let *fgh* be a Surface parallel to the Horizon. If the Heights *fi* and *gl* be unequal, the Water will aſcend where that Difference is leaſt*. For the *274 ſame Reaſon, unleſs the Preſſures at *g* and *h* be equal, the Water will not be at reſt; but they are equal when *l* and *n* are in the ſame horizontal Plane: For ſince the Preſſure ariſes from the Gravity of the Parts, which tends towards the Center of the Earth, the Height of the preſſing Liquid muſt be meaſured according to that Direction, that is, it will be *hm*; but the Obliquity of the Column *hn* cauſes no Change, becauſe at the ſame Depth the Preſſure every Way is equal*. *275

Experiment 4. *Plate* XVIII. *Fig.* 4.] Pour Water into the Machine repreſented by *Fig.* 3. and after any Agitation it will not reſt, unleſs all the Surfaces be in the ſame horizontal Plane. The Glaſs Veſſel A is joined to the Glaſs Tubes B and D, by help of the Braſs Tube CE. 278

All Liquids are not equally heavy, that is, have not the ſame Quantity of Matter in the ſame Space; but what has been ſaid will agree to every Liquid by itſelf.

When Liquids of different Gravities are contained in the ſame Veſſel, the heavieſt lies at the loweſt Place, and is preſſed by the lighter, and that in Proportion to the Height of the lighter. 279

Experiment 5. *Plate* XVIII. *Fig.* 5.] Take Water tinged with ſome Colour, and pour it into the Glaſs Veſſel A to the Height of *bc*; immerge into it the Glaſs Tube *d*E; the Water

*274 will riſe in it to the Height *b c**. Now pour in Oil of Turpentine, which is a Liquid lighter than Water, and immediately the Water will riſe in the Tube; and ſo much the higher as the Oil is poured in, to a greater Height: Yet the Water in the Tube does not riſe to the ſame Height as the Oil in the Veſſel; becauſe, ſince Water is heavier, there is not required the ſame Height of Water as there would be required of Oil to produce the ſame Preſſure.

If you have a Mind to try this Experiment with Mercury and Water, you will find a greater Difference in their Heights, by reaſon of their greater Difference of Gravity.

Experiment 6. *Plate* XVIII. *Fig.* 6.] Let the End of a Tube be immerſed in Water, and pour Oil into it. The Water in the Tube is depreſs'd as far as *d*; yet the Height of the Oil *d e* is greater than the Height of the Water in the Veſſel. If the Tube be immerſed deeper, the Water will run into it in greater Quantity; if you raiſe it up, the Water will again go out at *a*, and the Water in the Tube *d e* follow it, if it be raiſed to ſuch a Height that the Preſſure of the Oil may overcome the Preſſure of the Water in the lower Part of the Tube.

CHAP. II.

Of the Actions of Liquids againſt the Bottoms and Sides of the Veſſels that contain them.

280 THE *Bottom and Sides of a Veſſel, which contain a Liquid, are preſſed by the Parts of the Liquid which immediately touch them*; and be-
126 cauſe Re-action is equal to Action, thoſe Parts all ſuſtain an equal Preſſure. But, as the Preſſion of Liquids

Liquids is equal every Way, the Bottom and Sides are preſs'd as much as the neighbouring Parts of the Liquids; therefore *this Action increaſes, in Proportion to the Height of the Liquid**, *275
and is every Way equal at the ſame Depth, de- 273
pending altogether upon the Height, and not at all upon the Quantity of the Liquid. Therefore, when the Height of the Liquid, and Bigneſs of the Bottom remain the ſame, the Action upon the Bottom is always equal, however the Shape of the Body be changed. In every Caſe the Preſſure, ſuſtained by the Bottom, is equal to the Weight of a Column of Water, whoſe Baſe is from the Bottom itſelf, and the Height of the vertical Diſtance of the upper Surface of the Water from the Bottom itſelf.

Plate XVIII. *Fig* 7 and 8.] Take the hollow 281
Cylinder A, open at both Ends, and finely poliſhed within, whoſe Diameter and Height alſo are about three Inches and an half; the Ring E is faſtened to it by a Screw, ſo as it may be ſuſtained by a Trevet.

Let the Cylinder have a moveable braſs Bottom F, with which the Braſs Ring G, having a Screw in the Inſide, is joined: This Ring retains and fixes a Leather Ring, broader than the Bottom, all round by half an Inch; this Leather covers the external Surface of the Braſs Ring when the Bottom is thruſt into the Cylinder, and it hinders the Water from going out when it is moved up and down. This Leather muſt be ſoaked in Oil, and after a few Days it muſt be taken out and ſoaked as long in Water; after which Preparation the Leather muſt be well anointed with Oil and Water, and moved ſeveral times up and down the Cylinder, and left in it in that Condition two or three Days. When you are going to uſe the Machine, you muſt anoint the Leather again with Oil and Water, then the Bottom will move eaſily, and hold Water. The Leather muſt be neither too thick nor too thin, which muſt be left to the Judgment of the Workman.

The Bottom has in its Middle a ſmall Braſs Cylinder *bi* faſtened to it, by which the Motion of the Bottom is directed; for this Cylinder goes thro' the Hole *m* in the Plate *b*, which is laid upon the larger Cylinder *A*, and let into it by a Cut in the Edge. In the upper Surface of the Cylinder *bi* there is a Cavity which contains a Screw, by which the Bottom is joined to the Braſs Wire *n p*, which is carried through the Tube D, that the Bottom may be faſtened to the Brachium of a Ballance by the help of this Wire.

Let the Cylinder A have the Cover C laid upon it; and, to hinder the Water from running out, the Mouth of the Cylinder muſt be cover'd with a Leather Ring, which is ſtrongly preſſed by the Screw which joins the Leather Cover to the Cylinder. To the Cover and Cylinder itſelf may be added a Handle, that the Cylinder may the more eaſily be ſhut and open'd. The Cover has a Hole in the Middle, and the hollow Cylinder *l*, which has a Screw on the Outſide, is faſten'd to it, that the Tube *d* may be joined to the Machine with a Leather upon the Screw, to hinder the Water from coming out.

Experiment 1. *Plate* XIX. *Fig.* 1.] Having joined together all the Parts of the Machine in the manner juſt mentioned, hang upon one End of the Beam of a Ballance the Braſs Wire which is fixed to the moveable Bottom, ſo that the Beam may be exactly horizontal when the Bottom is two Inches diſtant from the Cover; then put

Plate 18. p. 14

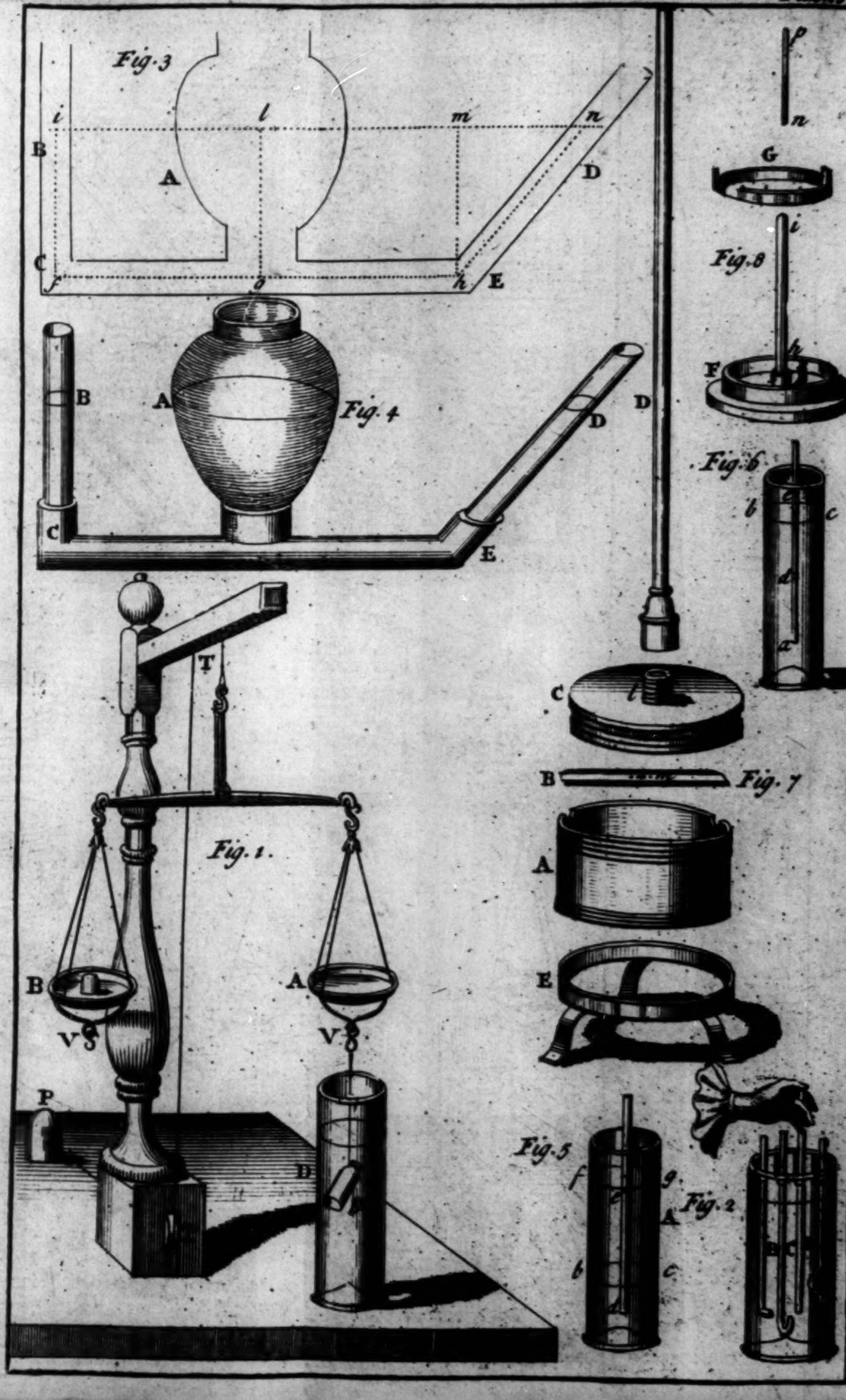
Fig. 3
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Fig. 5
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Fig. 2

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put ſuch a Weight in the oppoſite Scale as will make an *Æquilibrium* with the Weight of the Bottom only. Let the Tube be one Foot long; and, the Beam of the Ballance being placed horizontal, pour Water into the Tube D, ſo that it may riſe up to its upper End; another Weight of 4½ Pounds, being put into the upper Scale, will make an *Æquilibrium* with the Water; and, if you diminiſh or increaſe this Weight, the Bottom will move upwards or downwards. But you muſt obſerve, that in altering the Weight, you muſt put in, or take out a pretty conſiderable Weight; for Example, half a Pound, becauſe of the Friction of the Bottom.

The Diameter of the Bottom is almoſt 3½ Inches, and the Height of the Top of the Water, in this Experiment, is 14 Inches; the Weight of a Pillar of Water of that Height, whoſe Baſe is equal to the Bottom, is 4½ Pounds; and juſt ſo much does the Water preſs againſt the Bottom; tho' there be but a ſmall Quantity of Water in the Machine.

Since only the Motion of the Bottom is to be obſerved, the Machine is to be fix'd down, leſt it ſhould be wholly raiſed; which is done by laying ſuch Weights upon it as are repreſented by P P, *Plate* XX. *Fig.* 1.

Experiment 2. *Plate* XIX. *Fig.* 2.] Having taken away the Cover and the Tube, join the Cylinder A to the Veſſel DE, which has at the Bottom a Ring with a Screw. Into this Machine pour Water upon the Bottom as high as in the foregoing Experiment; the reſt of the Experiment is made in the ſame manner as the former, and the Succeſs is the ſame; for the Preſſure is not changed, tho' you alter the Figure of the

Veſſel and the Quantity of the Water, provided you keep the Water to the ſame Height.

Experiment 3. *Plate* XIX. *Fig*. 3.] Hang the Cylindrick Veſſel A to the End of a Ballance, which Veſſel muſt be filled in part by a wooden Cylinder D *e*, which Cylinder is fixed any how to the Piece of Wood BC, and neither touches the Sides nor the Bottom of the aforeſaid Veſſel; if you pour Water into this Veſſel to any Height, and make an Æquilibrium by putting Weights in the oppoſite Scale, that Weight will be the Weight of the whole Water which would be contained in the Veſſel, the Cylinder being taken away, ſuppoſing it filled to the ſame Height as in the Experiment. And ſo a ſmall Quantity of Water, whoſe upper Surface is raiſed, ſo as the Preſſure againſt the Bottom be increaſed, will ſuſtain a great Weight.

It will viſibly appear, that the lateral is equal to the vertical Preſſure, making uſe of the following Machine.

282 *Plate* XIX. *Fig*. 4.] The Veſſel DB is a Parallelopiped of Wood about a Foot and a half high; in the Side, towards the Bottom, there is a Hole in which there is a Braſs Ring containing a Screw, that the Cylinder A, mentioned in the firſt and ſecond Experiments, may be ſcrewed to it. Here you muſt take away the Trevet which ſuſtained the Cylinder in thoſe Experiments, and was fixed to the lower Ring by Screws. Now the Motion of the Bottom, in the Cylinder is horizontal. Two croſs Pieces of Wood are joined to the Sides of this Machine, one of which is ſeen in GH; along them the Ruler CC is moved horizontally, which is wider in the Middle towards F, that by its Motion the Bottom of the Cylinder may be thruſt inwards, which

the Ruler preſſes a little below the Center. At CC Ropes, as CE, are faſten'd to this Ruler, which are ſtretch'd along the Pieces, as GH, and going over Pullies at the Extremities of the ſaid Pieces, as T, have Weights joined to them, as P.

Experiment 4.] Pour Water into the Veſſel BD, ſo that the Surface of the Water may be higher by 14 Inches than the Ruler CC; let the Weights, as P, be of 2 Pounds and a Quarter each; ſo that both taken together ſhall amount to 4 Pounds and a Half, the Preſſure of the Water will ſuſtain that Weight, and the Bottom in that Caſe will be moved with the ſame Eaſe towards either Part.

The following Experiment proves that the Force, with which Water preſſes upwards, is equal to that with which it preſſes downwards and ſideways.

Experiment 5. *Plate* XX. *Fig.* 1.] In the Middle of the upper Surface of the Block or Foot B there is a Cylinder of about 2 Inches Diameter, on which you muſt put the moveable Bottom of the Cylinder A, ſo often mentioned; ſo that, the Bottom remaining fixed, the Cylinder may be moved. The Cylinder muſt have its Cover on, and to it the Tube D, 3 Feet and a Half long, muſt be faſtened; pour in Water, by which, the Bottom remaining fixed, the Machine will be raiſed; put the Weights PPP, which all together weigh 9 Pounds, upon the Cover, and they, with the Weight of the whole Machine, will be ſuſtained by the Water in the Tube; but the Weight of the Machine is more than 3 Pounds and a Half.

The Force, which acts againſt the Cover, is equal to the Weight of a Pillar of Water, whoſe Baſe is the Cover, excepting the Hole to which the Tube is fix'd, and whoſe Height is the Height of the Water-Tube above the inward Surface of the Cover; which agrees with this Experiment.

If you apply the ſame Tube to a greater Machine, the Action againſt the Cover will increaſe in the ſame *Ratio* as the Cover; ſo that a prodigious Weight may be ſuſtained, and even raiſed by a ſmall Quantity of Water.

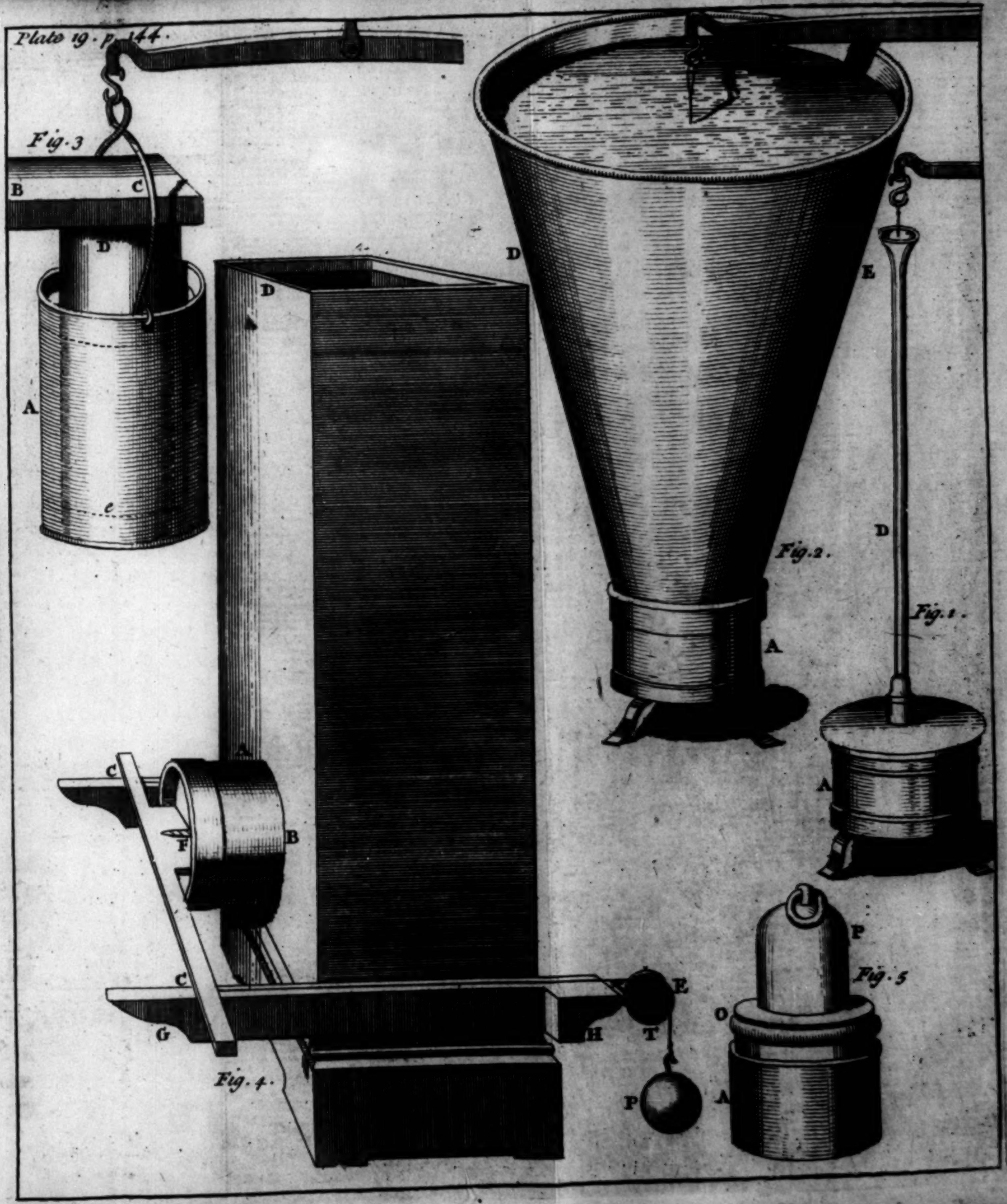
283 *Plate* XX. *Fig.* 2.] Take two round Boards AB, AB, of 15 Inches Diameter, and join them together with a Piece of Leather, ſo that they may make a Cylindrick Veſſel ſomething like a Pair of Bellows, ſo that it may contain Water.

There is a Hole *l* in the upper Board, to which is fix'd a braſs Cylinder that has a Screw, whereby the Tube D is fix'd to it, which is as long as the Tube uſed in the former Experiment.

Experiment 6.] Pour Water into theſe Bellows thro' the Tube, and the Water in the Tube will ſuſtain the Weights P, P, P, P, P, P, all which together weigh more than 250 Pounds. Theſe Weights will even be raiſed by continuing to pour Water into the Tube.

Though theſe are Paradoxes, they follow from the Nature of Liquidity; every Drop which is at reſt, endeavours to recede every Way with *75 equal Force*; if therefore it be preſſed on one Side, it endeavours to recede that Way with the ſame Force, becauſe Action and Re-action are equal, and with that very Force itſelf will preſs every Way. In the firſt Experiment, the Water which touches the Bottom, and correſponds with the

Plate 19. p. 144.



the Tube, ſuſtains the Weight of the Column of *Water* contain'd in the Tube, and reaching quite to the Bottom, and preſſes the Bottom with ſuch a Force, that it acts with the ſame Force upon the *Water* next to it, and ſince that *Water* cannot flow out againſt the Bottom, the *Water* next to that is alſo preſs'd with the ſame Force. The ſame may be ſaid of the *Water* next to that; and ſo in all Parts of the Bottom there is a Preſſure equal to that which lies under the *Water* in the Tube; and therefore the Bottom in this Caſe is as much preſs'd as if a Pillar of *Water*, of the ſame Height as the *Water* in the Tube, and of a Baſe equal to the Bottom, ſhould lie upon it.

The fifth and ſixth Experiments are illuſtrated by the ſame Reaſoning.

In the ſecond Experiment, ſuppoſe that the Cylinder A ſhould be continued, ſo as to reach up to the Surface of the *Water*; by this Means the external *Water* would be ſeparated from the *Water* contain'd in this Cylinder, and then no *Water* but this interior *Water* would preſs upon the Bottom, and the Bottom would ſuſtain it all. The *Water* in the Cylinder preſſes againſt the Sides of the Cylinder, and the external *Water* preſſes upon the external Surface of the Cylinder, and the outward Surface is preſs'd exactly in the ſame manner as the inward, and the Preſſures againſt oppoſite Points are preciſely equal; ſo that if the Surface was taken away, theſe Preſſures would deſtroy one another; therefore it is no matter whether there be ſuch a Surface or not, ſo that taking it away (that is, taking away the Continuation of the Cylinder) the Action againſt the Bottom is no way alter'd.

The third Experiment is alſo illuſtrated by what has been ſaid; for the Weight placed in the Ballance is not only ſuſtain'd by the *Water* in the Veſſel, but alſo by the Action of the inferior Surface in the Cylinder D *e* againſt the *Water*.

Tho' all that has been ſaid depends upon the Gravity of Liquids, their Actions muſt be diſtinguiſh'd from their Gravity, which laſt is always proportionable to the Quantity of Matter*. *270

CHAP. III.

Of Solids immerſed in Liquids.

WE have often ſaid, that the different Gravity of Bodies, whether Solids or Liquids, ariſes from this, that they contain a greater or leſs Quantity of Matter in an equal Space.

DEFINITION I.

284 *The Quantity of Matter in a Body being conſider'd in relation to its Bulk*, that is, in relation to the Space poſſeſs'd by it, *is call'd* the Denſity of the Body.

A Body is ſaid to have double, or triple, *&c.* the Denſity of another Body, when, ſuppoſing their Bulks equal, it contains a double, or triple, *&c.* quantity of Matter.

DEFINITION II.

285 *A Body is* ſaid to be *Homogeneous, when it is every where of the ſame Denſity.*

DEFINITION III.

286 Heterogeneous, *when the Denſity is unequal in different Parts of the Body.*

DEFINITION IV.

287 *The Gravity of a Body, conſider'd with relation to its Bulk, is called* the ſpecifick Gravity of a Body.

The ſpecifick Gravity is ſaid to be double, when under the ſame Bulk the Weight is double.

Therefore *the ſpecifick Gravities and Denſities of Bodies, in homogeneous Bodies, are in the ſame Ratio; and they are to one another as the Weights of equal Bodies, in reſpect to their Bulk.* 288

If homogeneous Bodies are of the ſame Weight, their Bulks will be ſo much leſs as their Denſities are greater, and under the ſame Weight the Bulk is diminiſhed in the ſame Ratio in which the Denſity is increaſed; therefore in that Caſe the Bulks are *inverſly as the Denſities.* 289

When a Solid is immerſed in a Liquid, it is preſs'd by the Liquid on all Sides, and that Preſſure increaſes in Proportion to the Height of the Liquid above the Solid; as it follows from what has been ſaid in the foregoing Chapter; and which may alſo be proved by a direct Experiment.

Experiment 1. *Plate* XX. *Fig.* 3.] Tie a Leather Bag S to the End of a Glaſs Tube B *m*, and fill it with Mercury; you may alſo make uſe of a Bladder; let this Bag be immerſed in *Water*, but ſo that the End B of the Tube may be above the *Water*; by the Preſſure of the *Water* againſt the Surface of the Bag, the Mercury in the Tube will riſe to *m*; and the Aſcent of the Mercury follows the Proportion of the Height of the *Water* above the Bag.

When a Body is immerſed in a Liquid to a great Depth, the Preſſure againſt the upper Part differs very little from the Preſſure againſt the under Part; whence *Bodies very deeply immerſed, are, as it were, equally preſs'd on all Sides*; which Preſſure may be ſuſtained by ſoft Bodies, without any Change of Figure, and by very brittle Bodies, without their breaking.

Experiment 2. *Plate* XIX. *Fig.* 5.] Take a Piece of ſoft Wax of an irregular Figure, with an Egg, and incloſe it in a Bladder full of Water, and the Bladder being exactly ſhut, muſt be put into a braſs Box A; let it be cover'd with a wooden Cover O, ſo that it may be ſuſtained by the Bladder, lay on a Weight P of 70 or 80 Pounds, and the Egg will not be broken, nor the Figure of the Wax any way changed.

292 *A Body ſpecifically heavier than a Liquid, being immerſed in a Liquid in any Depth, will deſcend.* The inferior Part of the Body preſſes the Surface of a Liquid which it touches, and this Preſſure is equal to the Weight of a Column made up of the Body itſelf and the ſuperincumbent Liquid, and with this Force the Body is carried downwards. The Weight of a like Column, but which conſiſts wholly of a Liquid, is the Force by which the Body is preſs'd upwards by the Liquid*. But when the Solid is ſuppoſed ſpecifically lighter than the Liquid, this Force is leſs than that, and therefore is overcome by it. *290 275

293 It is proved by the ſame Reaſoning, *That a Solid ſpecifically lighter than a Liquid, and immerſed into it, muſt aſcend to the higheſt Surface of the Liquid.*

294 But *ſuppoſe a Solid of the ſame ſpecifick Gravity with the Liquid, it will* neither aſcend nor deſcend, but *remain ſuſpended in the Liquid at any Height, and the Liquid will ſuſtain the whole Body*; in which Caſe, by reaſon of the Equality of the ſpecifick Gravities, the Liquid ſuſtains a Weight equal to the Weight of the Quantity of the Liquid, which would fill the Space taken up by the Body. But a Liquid acts in the ſame manner upon all equal Solids immerſed to the ſame Depth, and will

ſuſtain them equally; therefore they all loſe Part of their Gravity.

DEFINITION V.

A Weight, which keeps a Body immerſed in a Liquid, is called its reſpective Gravity. 295

And *this reſpective Gravity is the Exceſs of the ſpecifick Gravity of a Solid above the ſpecifick Gravity of a Liquid*; for ſince *a Solid immerſed in a Liquid* loſes that Part of its Weight which is ſuſtained by the Liquid, it *loſes the Weight of the Quantity of the Liquid, which could fill the Space taken up by the Body.* 296 297

Experiment 3. *Plate* XXI. *Fig.* 1.] Hang the hollow Braſs Cylinder E to the Ballance abovementioned*; hang the ſolid Cylinder C of the ſame Metal by a Horſe-hair to a Hook fix'd to the Bottom, which, if it be put into the other Cylinder E, will exactly fill it; ſo that E, when it is full of Water, will contain ſuch a Quantity of Water as will fill the Place taken up by C; put a Weight in the oppoſite Scale to make an Æquilibrium; let the Ballance deſcend, that the Cylinder C may be immerſed into the Water contained in the Veſſel D; by that Means the Æquilibrium is deſtroyed, becauſe C is partly ſuſtained by the Water; but is reſtored, if E be fill'd with Water.

*271

Hence it follows, that *all equal Solids, but of different ſpecifick Gravity, when they are immerſed into the ſame Liquid, they loſe equal Parts of their Weight.* The laſt mentioned Experiment will ſucceed in the ſame manner with a Cylinder of any other Metal, and by pouring in the ſame Quantity of Water, that is, ſo much as will fill the Veſſel E, the Æquilibrium will always be reſtored. 298

299 Moreover, from what has been ſaid it follows, that *however the Denſities of equal Bodies differ among themſelves, if they be immerſed in the ſame Liquids, the Weight which they loſe is in the Ratio of their Bulks,* for the Spaces which they take up in a Liquid are in the ſame Ratio.

Therefore Bodies of the ſame Weight, but of different Denſities, loſe an equal Part of their Weight, when they are immerſed in the ſame Liquid, becauſe of the Inequality of their Bulks.

Experiment 4. *Plate* XX. *Fig.* 4.] Let two Pieces of Metal of the ſame Weight, the one of Gold, and the other of Lead, *g*, *g*, be ſuſpended with the Hook VV of the Ballance above-mentioned with Horſe-hair*, and you will have an Æquilibrium; let the Ballance deſcend, and the Bodies *g*, *g*, be immerged in the *Water* contained in the Veſſel FF, and the Æquilibrium will be deſtroyed. When a Solid, ſpecifically heavier than a Liquid, is ſuſpended in a Liquid, the Liquid acts on every Side againſt that Solid, in Proportion to its Weight*, and the Solid re-acts equally againſt it; therefore thoſe Actions are the ſame as if the Space taken up by the Solid, were fill'd with the Liquid; therefore *it is no Matter, in reſpect of the Gravity of the Liquid, whether a Solid, ſpecifically heavier than the Liquid, be ſuſpended in it, or a Quantity of the Liquid be poured in, which takes up a Space equal to the Solid.*

*271

*290

300 *Experiment* 5. *Plate* XXI. *Fig.* 2.] Take the Veſſel A containing *Water*, hang it to one End of the Ballance, immerge into it the Braſs Cylinder C, which is ſuſtained by a Horſe-hair, leſt it ſhould touch the Bottom of the Veſſel, putting a Weight into the oppoſite Scale, and you will have an Æquilibrium; take the Cylinder Court

of the Water, the Æquilibrium will be deſtroyed; and it will be reſtored again by pouring in Water as much as can be contained in the hollow Cylinder E, which will be exactly filled by the following Cylinder C.

301 By comparing together the *Numb.* 297 and 300, as alſo the third and fifth *Experiments*, which confirm them, it appears, that *a Liquid acquires the Weight which the immerſed Solid loſes.* The Force of Gravity is always proportionable to the Quantity of Matter, and is not changed by the Immerſion of a Solid into a Liquid; wherefore the Sum of the Weight of the Solid, and of the Liquid, do not differ before and after the Immerſion.

Experiment 6. *Plate* XX. *Fig.* 5.] Hang the Solid C to the Ballance, and make an Æquilibrium, by putting into the oppoſite Scale B the Weights P and *p*, of which *p* equal to the Weight which the Body C loſes in Water. Take the Veſſel E, which contains Water, and is ſuſpended to the Ballance F F, and, putting a Weight into the oppoſite Scale, make an Æquilibrium here alſo; let the Ballance deſcend with the Body C, that it may be immerſed in the Water contain'd in D, by this Means you will deſtroy the Equilibrium in both Ballances, which will be reſtored by taking out of the Scale B the Weight *p*, and putting it into the Scale of the Brachium F.

302 A Body ſpecifically heavier than a Liquid, and which deſcends in it, is carried downwards with a greater Force than it is preſſed upwards, as has been explained before*; the Difference of which Forces is the reſpective Gravity of the Body. *292

The firſt Force in part conſiſts of the Weight of the Liquid incumbent over the Body, and the Body may be immerſed to ſuch a Depth, that

that Weight ſhall be equal to the above-mentioned ſpecifick Gravity: If in that Caſe you take away 303 the ſuperincumbent Liquid, the Body will be ſuſtained by the Preſſure of the Liquid under it. If the Body immerſed to a greater Depth, and the Liquid be alſo hindered from paſſing upon the upper Surface of the Body (becauſe the Preſſure by which a Body is puſhed up, increaſes as the *290 Depth to which it is immerſed)* the Body then will be carried upwards with greater Force than downwards by Gravity; wherefore, if it could move freely, it would aſcend.

Experiment 7. *Plate* XXI. *Fig.* 3.] To the Cylinder C, which is open at both Ends, apply at Bottom the Plate of Lead F, a Quarter of an Inch thick; if it fits ſo exactly to the Cylinder as to let no Water ſlip by, and the Plate be held up by a Thread faſtened to the Hook V in the Center of the Plate, until it be immerſed with the Cylinders to the Depth of about 3 Inches, the Lead will be ſuſtained by the Water, as appears by letting go the Thread. If you immerge it to a greater Depth, it will ſtick cloſer to the Cylinder; but if to a leſs, it will fall off.

If this Experiment was made with a Plate of Gold, it ought to be immerſed to a greater Depth; for the ſpecifick Gravity of Gold is to the ſpecifick Gravity of Water as 19 to 1; and therefore its reſpective Gravity is to that of Water as 18 to *296 1*. Therefore to have a Pillar of Water equal in reſpective Gravity to the Plate of Gold, that Pillar muſt be above 18 times its Height; and therefore the Height of the Water, above the upper Surface of the Plate of Gold, muſt be at leaſt equal to as many times its Thickneſs.

Experiment 8. *Plate* XXI. *Fig.* 3.] Take a Cylinder A with a moveable Bottom, that has alſo a Cover with the Tube D joined to it, as was before deſcribed*; immerge it in Water, and when *381 it comes to be a Foot under Water, the Bottom will riſe, although it weighs a Pound and a Quarter, and has P a Pound Weight ſcrewed to it at Bottom.

If the ſame Solid be immerſed into Liquids 304 of different Denſity, it will loſe different Parts of its Weight* : And therefore, when two Bo- *297 dies of the ſame Denſity and Weight are immerſed in Liquors of different Denſity, they will loſe their Æquilibrium.

Experiment 9. *Plate* XXI. *Fig.* 4.] Take two flat Pieces *gg*, of the ſame Metal and equal, and hang them upon the Hooks V V of the Scales A and B; then by the Deſcent of the Ballance immerge them in the Liquids contained in the Veſſels F F, the one in Water, the other in Oil of Turpentine, and the Æquilibrium will be deſtroyed, the Piece which was immerſed in Oil becoming lighter.

A Solid lighter than a Liquid, and immerſed in it, 305 aſcends and remains at the upper Part of the Liquid*, ſo as to be immerſed only in Part; but the *293 greater is its ſpecifick Gravity, the more it deſcends, and the Body *will not be at reſt till the immerſed Part takes up ſuch a Space in the Liquid, that the Bulk of the Liquid, which would fill that Space, ſhall weigh as much as the whole Body.* For in another Caſe the Solid does not act with the ſame Force againſt the neighbouring Parts of the Liquid as the Liquid would act, if it ſhould take up the Place of the Body; therefore in this Caſe alone the Liquid and the Body can be at reſt*. *276

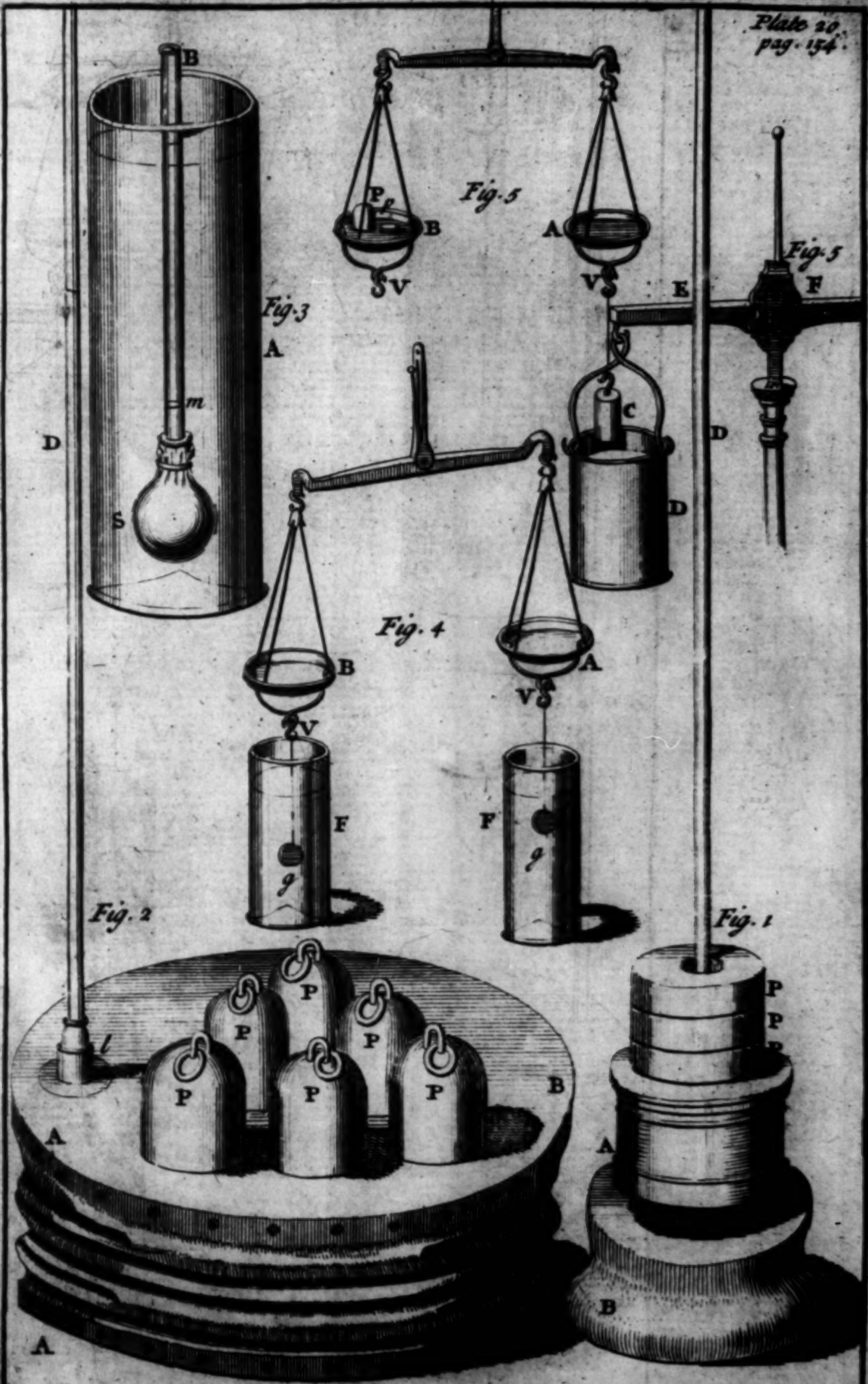
It follows from this Propoſition, that *the immerſed Parts of the Bodies, ſwimming on the Surface of the ſame Liquor, are to one another as the Weights of the Bodies.* Therefore if, by ſuperadding a Weight, the Gravity of the Body is changed, the immerſed Part is increaſed in the ſame Proportion, and *the Parts, which deſcend into the Liquid by laying on of different Weights, are to one another as thoſe Weights.*

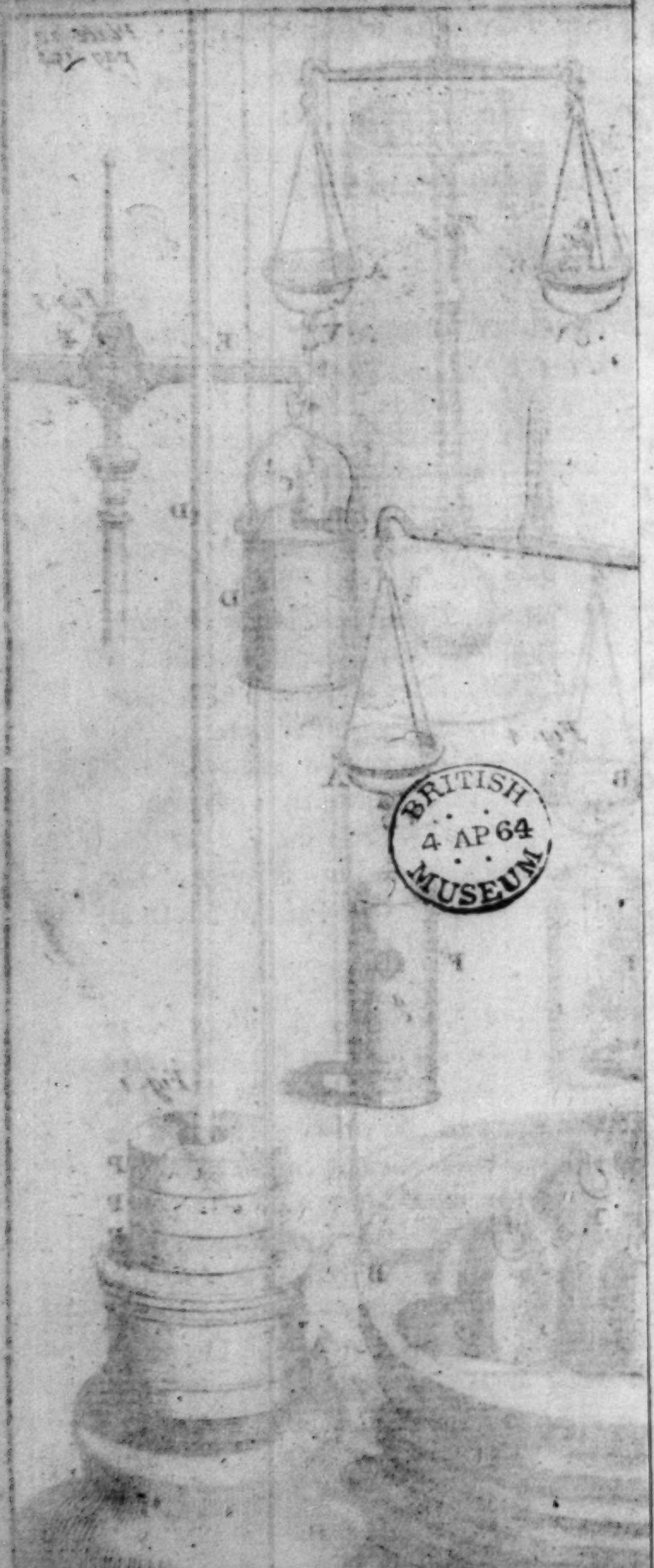
Experiment 10. *Plate* XXII. *Fig.* 1.] Take a Veſſel A containing Water in it; let C be a hollow Cylinder of any Metal; lay upon it the Weight *p*, that it may deſcend into the Water with its Part *bd*; adding the Weight of one Pound, meaſure how far it will deſcend; then adding another equal Weight, you will find that it will deſcend equally every time.

308 In *Numb.* 302 and 303, confirmed by the *Experiments* 7 and 8, it appeared how a Body, ſpecifically heavier than a Liquid, may be made to ſwim; by the ſame Method a Body, ſpecifically lighter than a Liquid, may be retained at the Bottom: In that Caſe the Preſſure of the ſuper-incumbent Water is taken off; but here you muſt take off the Preſſure of the inferior Water whereby the Body is puſhed upwards.

Experiment 11. *Plate* XXII. *Fig.* 2.] Upon the Foot D, which is fixed at the Bottom of the Veſſel A, there is a Braſs Plate *bc* exactly flat and poliſhed; there is another Braſs Plate *bc*, like the former, faſtened to a large Piece of Cork E, ſo that together with the Cork it ſhall make up a Body ſpecifically lighter than Water: Lay this Plate upon the other, ſo that they may fit, and keep the Cork down with a Stick while you pour in Water; leaving the Cork, it will not aſcend until, by

Plate 20.
pag. 154.





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by moving it out of its Place, the Plates are ſeparated, ſo that the Water may exert its Preſſure againſt the Plate joined with the Cork, and puſh it upwards together with the Cork.

CHAP. IV.

Of the Manner of comparing the Denſities of Liquids.

SInce the Denſity of Bodies is in Proportion to their Gravity, by comparing the Weights of equal Bodies we diſcover their Denſities*. *If* therefore *any Veſſel be exactly filled with a Liquid, and that Liquid be weighed; and if you make the ſame Experiment with other Liquids, their Weights will be as their Denſities.* But, as this Method is liable to ſeveral Difficulties in Practice, I ſhall not ſpend any Time in explaining it here. 309 *288

When the Preſſures of two Liquids are equal, the Quantities of Matter, in Columns that have equal Baſes, do not differ*; wherefore the Bulks, that is, *the Heights* of the Columns, *are inverſly as the Denſities**; whence may be deduced the Method of comparing them together. 310 *273 *289

Experiment 1. *Plate* XXII. *Fig.* 3.] Pour Mercury into a curve Tube A, ſo as to fill the lower Part of the Tube from *b* to *c*; pour in Water in one Leg from *b* to *e*; in the other Leg pour in Oil of Turpentine, till both the Surfaces of the Mercury *bc* be in the ſame horizontal Line, and the Height of the Oil be *cd*: Theſe Heights will be as 87 to 100, which is the inverſe Ratio that the Denſity of the Water has to the Denſity of Oil of Turpentine, and therefore theſe Denſities are to each other as 100 to 87.

The Mercury is poured in, leſt the Liquids ſhould be mix'd in the Bottom of the Tube.

The Denſities of Liquids are alſo compared together, by immerging a Solid into them; for
311 if *a Solid, lighter than the Liquids to be compared together, be immerſed ſucceſſively into different Liquids, the immerſed Parts will be inverſely as the Denſities of the Liquids*; for, becauſe the ſame Solid is made uſe of, the Portions of the different Liquors, which in every Caſe would fill the Space taken up by the immerſed Part, are of the
305 ſame Weight; therefore the Bulks of thoſe Portions, that is, the immerſed Parts themſelves,
289 are inverſely as the Denſities.

312 *Plate* XXII. *Fig.* 4.] Take the Glaſs A, which is a hollow Ball that has a Tube divided into equal Parts; at the Bottom of the great Ball there is a ſmall one, Part of which is fill'd with Mercury, or very ſmall Shot, whoſe Weight ſerves to make the Tube deſcend vertically in Liquids, and ſtand in that Poſition: Care muſt be taken not to have too much Weight in the little Ball, for the whole Glaſs muſt be lighter than the Liquids to be compared together. The *Hydrometer* (for ſo it is called) deſcends to different Depths in different Liquids; and thoſe Denſities, as we have already ſaid, are inverſely as the Parts immerſed, which therefore are to be compared together. Tie a Thread to the Hydrometer, and weigh it together with the Thread; the Weight (if it be like mine) will be 573 Grains; if put into Water, it will deſcend to *b*; therefore a Bulk of Water, equal to the immerſed Part of the Hydro-
305 meter, weighs 573 Grains, and may be expreſſed by that Number. Faſten the Thread above-mentioned to the Hook V of the Scale A of the Ballance repreſented in *Plate* XVIII. *Fig.* 1. the Hydrometer

ter remains immerſed; put 20 Grains in the Scale B, and let the Weight P be moved gently to raiſe the Ballance, (by which the Tube will be drawn a little way out of the *Water*) till there be an Æquilibrium, and the Surface of the *Water* then will be given with the Point *d*; the *Water* ſuſtains the Weight of the whole Machine, except 20 Grains; that is, it ſuſtains 553 Grains; and the Weight of the ſame Bulk of *Water*, which is now immerſed, weighs juſt ſo many Grains, and is expreſſed by that Number; wherefore one may call the Bulk of the Parts *d b* of the Tube 20; if the Space *d b* be divided into 10 equal Parts, and you continue the Diviſions upwards beyond *b*, and downwards below *d*, each Diviſion may be called 2; and by obſerving the Diviſion to which the Inſtrument deſcends in a Liquid, you will have the Bulk of the immerſed Part; ſo, if the whole Tube ſtands out above the *Water*, the immerſed Bulk will be 549; if it riſes to the upper Diviſions, the immerſed Bulk will be 579; and the Denſities of the Liquids, in which this happens, will be inverſly as thoſe Numbers, that is, as 579 to 549, and only the intermediate Denſities may be compar'd by this Inſtrument: If the Ball was leſs in Proportion to the Tube, it would ſerve for comparing together Liquids whoſe Denſities differ more than this. When ſeveral Liquids are compared together, the Numbers which expreſs the Bulk of the immerſed Parts are the Denominators of Fractions, which have 1 for their Numerator, and theſe Fractions expreſs the Ratio of the Denſities; for they are to one another inverſly as the Denominators.

Experiment 2.] Let the Denſities of Waters, containing different Quantities of Salts, be to be compared, the Hydrometer deſcends in the one to the Diviſion *a*; if it be immerſed in another, it only deſcends to the Diviſion *c*, their Denſities will be to one another as $\frac{1}{369}$ to $\frac{1}{363}$ as may be eaſily deduced from what has been ſaid.

This Method is alſo liable to ſeveral Difficulties beſides this, that it is difficult to compare together Liquors very different in Denſity by the ſame Hydrometer.

313 The beſt Method of all is, to make uſe of a Solid heavier than the Liquids. *When the ſame*
314 *Body is immerſed in different Liquids, the Weights, which it loſes in the Liquids, are to each other as*
*297 *the Denſities of thoſe Liquids**. Here you muſt
288 uſe a hydroſtatical Ballance, and beſides a ſolid
*271 Piece of Glaſs, as C, which may hang to one of the Scales by a Horſe-hair, *Plate* XXI. *Fig.* 5, and 6, you muſt have a Weight, as P, which æquiponderates with the Glaſs C, when it is immerſed in Water, as is repreſented in *Fig.* 4. The Difference between the Weight P, and the Weight of the Glaſs C, when it is taken out of the *Water*, is the Weight which the Body has loſt, when weigh'd in *Water*: This muſt be obſerved, that it may ſerve in all the Experiments; in our Ballance it weighs 722 Grains. Suſpend the Body in any other Liquid, unleſs it be of the ſame Denſity as *Water*, the Æquilibrium will not be preſerved: Let it be reſtored by putting Grain Weights in either of the Scales; if they be put into the Ballance A, add them to the above-mentioned Difference of 722 Grains; if the Weights be put into B, ſubſtract them from that Number; and by that Means, in each of thoſe Caſes, as it appears, the Weight loſt by a Body is determined, that is, the Weight which expreſſes the Denſity of the Liquid.

Experiment 3.] Let the Weight C, which hangs from the Scale A, be immerſed in Oil of Turpentine, whilſt the Weight P hangs from the Scale B; put 94 Grains in the Scale B, and you will have an *Æquilibrium*. Then immerge the ſame Weight in Milk, that the Ballance may return to an *Æquilibrium*, a Weight of 22 Grains muſt be put in the Scale A. Subſtracting the firſt Number from 722, and adding the ſecond to it, you will have 628 and 744, expreſſing the Denſities of the above-mentioned Liquids, whilſt 722 ſhews the Denſity of the Water itſelf.

CHAP. V.

Of the Hydroſtatical Compariſon of Solids.

IN all homogeneous and equal Bodies, the Denſities are as the Weights*; in unequal Bodies *288 of the ſame Weight, the Denſities are inverſly as the Bulks*; if therefore both the Bulks and *289 Weights differ, the Ratio of the Denſities is compounded of the direct Ratio of the Weights, and the inverſe Ratio of the Bulks; and therefore, *dividing the Weights by the Bulks, you have the* 315 *Denſities*; that is, you will have Numbers that are to each other as thoſe Denſities.

The Weight of all Bodies may be compared 316 by Means of the Ballance; the Bulks are found by *immerging Bodies in the ſame Liquid*; for *the Weights which they loſe, are as the Bulks**. *29

Plate XXI. *Fig.* 7.] Here alſo the hydroſtatical Ballance is to be uſed, as likewiſe the Glaſs Veſſel D, in which the Bodies to be compared are to be placed; you muſt alſo have ſuch a Weight as is repreſented by P in *Fig.* 6. that is, equal to the 317

Weight of D; and laſtly, the Weight *p* (*Fig.* 8.) equal to the Weight which the Glaſs D loſes, when it hangs in the *Water*.

With a Horſe-hair you muſt faſten the Glaſs D in the Place of the Body C (*Fig.* 4.) in the Scale A, and hanging the Weight P in the Scale B, you will have an Æquilibrium. The Body, whoſe Denſity is required, is placed in the Glaſs D (as we have ſaid before) and weighed, the Veſſel and Body being immerſed in *Water*; putting the Weight *p* into the Scale B, the Æquilibrium is reſtored in reſpect to the Glaſs D; you muſt add as much Weight beſides as is required to make an Æquilibrium, and that will be the Weight loſt by the Body weighed; by this Weight therefore you muſt divide the Weight of the Body itſelf, to have
315 the Denſity.

Experiment 1.] A Piece of Gold, weighing 137 Grains, loſt in *Water* $7\frac{1}{4}$ Grains. A Piece of Silver, weighing 248 Grains, loſt in *Water* 24 Grains. Therefore their Denſities are as $18\frac{26}{29}$ to $10\frac{1}{2}$, that is, nearly as 11 to 6. By faſtening a Body, whoſe Denſity is required, and is heavier than a Liquid, to a Body lighter than the Liquid, the Denſity is alſo diſcovered.

318 *Plate* XXII. *Fig.* 5.] Take the Machine A, like the Machine deſcribed in the preceding Chap-
312 ter, and let it have fixed to its Bottom the Ring DE, and to its Top the Ring FG; then the Ball will, by its own Weight, be in part immerg'd in *Water*. This Machine cannot be apply'd to Uſe, unleſs you know, by ſome other Method, how much of its Weight any Body loſes in *Water*; therefore we lay down as known, that $109\frac{2}{10}$ Grains of Lead weigh in *Water* but 100 Grains; therefore lay juſt as many Grains upon the Ring

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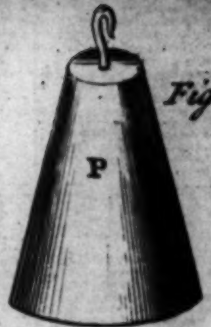
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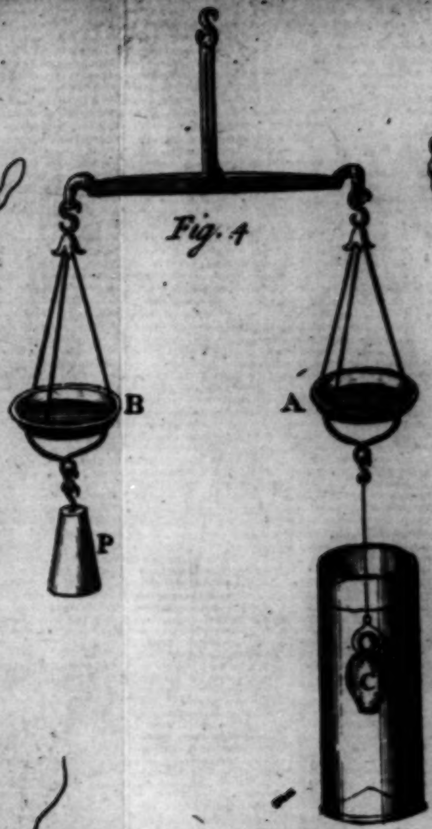
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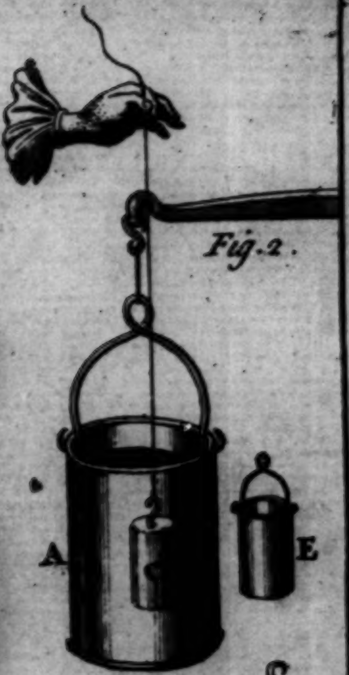
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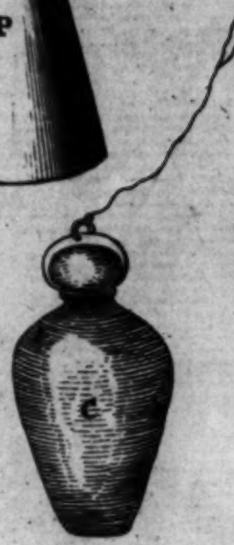
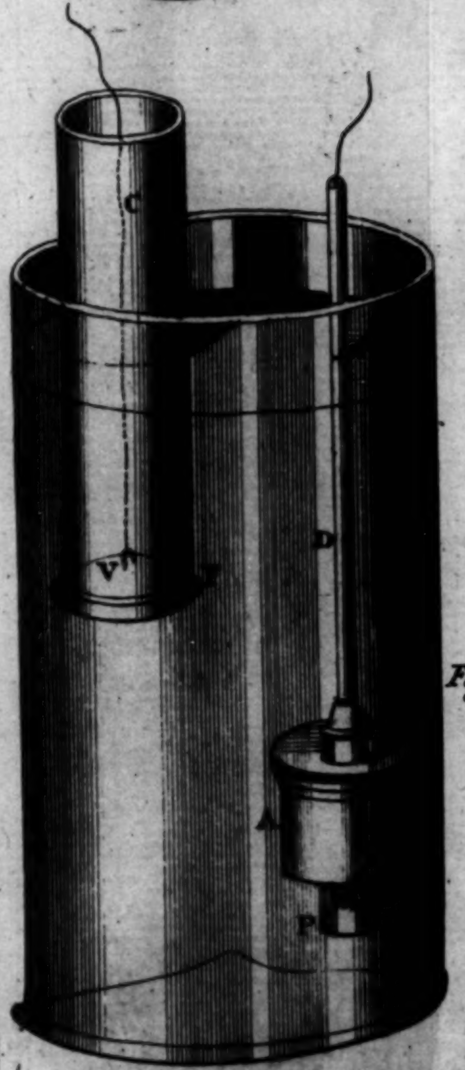


Fig. 5.

Fig. 8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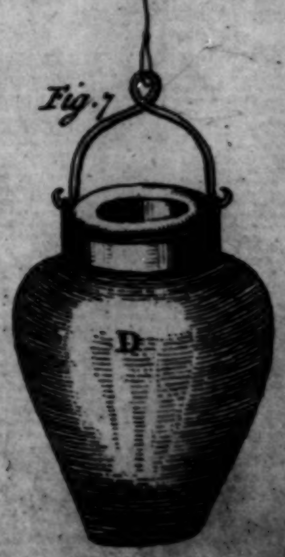
Fig. 7

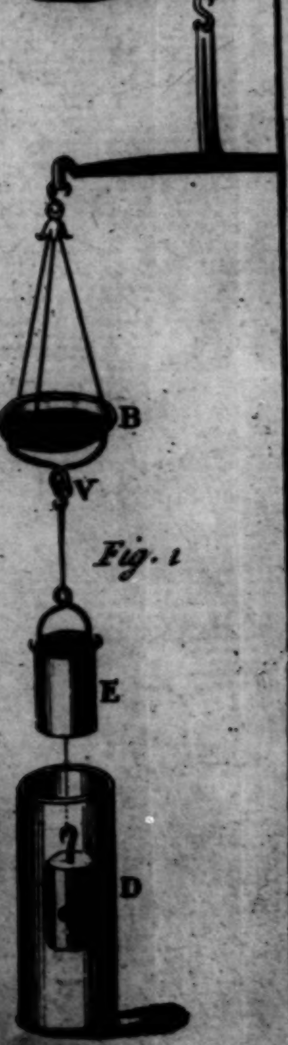
Fig. 1

D E as will make the Machine, when immerg'd in Water, deſcend to *a*; lay what Number of Grains you pleaſe upon the Ring F G, for Example, Eight, and the Machine deſcends to *c*; the Space *a c* muſt be divided into Eight Parts, and the Diviſions muſt be continued upwards and downwards; if you make *a* the hundredth Diviſion, *c* will be the hundred and eighth Diviſion, and the loweſt of all in this Figure will be the 97th. 'Tis plain, that if the Propoſition of *Numb.* 307 be compar'd with the aforeſaid Preparation, the Diviſion, to which the Machine deſcends in Water, ſhews the Weight of the Grains which preſs down the Machine; therefore laying a Body upon the Ring D E, its Weight in Water will be determined; by ſubſtracting this Weight from its Weight out of the Water, you will have the Weight loſt in the Water; by which if the Weight out of the Water be divided, the Denſity is diſcover'd, as has been ſaid in the Beginning of this Chapter.

Experiment 2.] Lay a Piece of Braſs, weighing, for Example, 100 Grains, on the Ring D E, by which the Ball of the Machine is not immerg'd; lay any Weight, for Example, of 17 Grains, on the Ring F G, and the Machine deſcends to *b*, that is, to the 105th Diviſion; which proves, that the Machine is preſs'd down by ſo many Grains; from this Number of Grains ſubſtract the 17 laſt-mentioned, the remaining 88 are the Weight of the Piece of Braſs in Water, which therefore loſes 12 Grains. If again the Weight 100 Grains be divided by the 12 Grains, you have 8$\frac{1}{3}$, expreſſing the Denſity of the Braſs. The Denſities of any other Bodies may be found after the ſame Manner.

This Method has ſeveral Difficulties. The foregoing is the beſt of all.

CHAP. VI.

Of the Reſiſtance of Fluids.

319 *ALL Bodies moved in Fluids ſuffer a Reſiſtance, which ariſes from two Cauſes. The firſt is the Coheſion of the Parts of the Liquid.* A Body in its Motion, ſeparating the Parts of a Liquid, muſt overcome the Force with which thoſe Parts cohere, and thereby its Motion is retarded.
12 *The ſecond is the Inertia,* or Inactivity *of Matter,* that belongs to all Bodies, which is the Reaſon, that a certain Force is required to remove the Particles from their Places, in order to let the Body paſs. The Body acts upon the Parts to remove them, and they diminiſh its Motion by Re-action.

320 The *Retardation from* the firſt Cauſe, that is, *the Coheſion of Parts,* is always the ſame in the ſame Space, the ſame Body remaining, be the Velocity of the Body what it will. The ſame Coheſion is to be overcome in every Caſe; therefore this Reſiſtance increaſes as the Space run through, in which Ratio the Velocity alſo in-
53 creaſes; therefore it *is as the Velocity* itſelf.

321 *The Reſiſtance ariſing from the* Inertia, *or Inactivity of Matter, when the ſame Body moves through different Liquids with the ſame Velocities,* follows the Proportion of the Matter to be removed in the ſame Time, which *is as the Denſity of the Liquid.*

322 *When the ſame Body moves thro' the ſame Liquid with different Velocities, this Reſiſtance increaſes* in Proportion to the Number of Particles ſtruck in an equal Time, which Number is as the Space run through in that Time, that is, as the Velocity. But this Reſiſtance does farther increaſe in Proportion to the Force with which the Body runs againſt every Part; which Force is alſo as

the Velocity of the Body. And therefore, if the Velocity is triple, the Reſiſtance is triple from a triple Number of Parts to be removed out of their Places. It is alſo triple from a Blow three times ſtronger againſt every Particle; therefore the whole Reſiſtance is ninefold, that is, *as the Square of the Velocity.*

A Body therefore moved in a Liquid is reſiſted 323 partly in a Ratio of the Velocity, and partly in a duplicate Ratio of it. *The Reſiſtance from the Coheſion of Parts in Liquids, except glutinous ones, is not very ſenſible* in reſpect of the other Reſiſtance; which as it increaſes in a Ratio of the Square of the Velocities*, but the firſt in a Ratio of the *222 Velocity itſelf*: By how much the Velocity in- *220 creaſes, by ſo much more do theſe Reſiſtances differ amongſt themſelves; wherefore, *in ſwifter* 324 *Motions the Reſiſtance alone is to be conſidered, which is as the Square of the Velocity.*

I ſhall not now treat of tenacious or glutinous Liquids, nor of ſlow Motions, in which the Reſiſtance, ariſing from the Coheſion of the Parts, muſt be conſidered.

If a Liquid be included in a Veſſel of a priſma- 325 tical Figure, *and there be moved along in it with equal Velocity,* and a Direction parallel to the Sides of the Priſm, *two Bodies, the one ſpherical, and the other cylindric, ſo that the Diameter of the Baſe of this laſt be equal to the Diameter of the Sphere, and the Cylinder be moved in the Direction of its Axis, theſe Bodies will ſuffer the ſame Reſiſtance.* To demonſtrate this, ſuppoſe the Bodies at reſt, and that the Liquid moves in the Veſſel with the ſame Velocity that the Bodies had; by this the relative Motion of the Bodies and the Liquid is not changed, therefore the Actions of the Bodies on the Liquid, and of the Liquid on the Bodies, are not changed. The Retardation which the Liquor ſuffers in paſſing by the Body, ariſes only from this, That in that Place it is reduced to a narrower Space, but the Capacity of the Veſſel is equally diminiſhed by each Body; therefore each Body produces an equal Retardation. And becauſe Action and Re-action are equal to one another, the Liquids act equally upon each Body; wherefore alſo each Body is equally retarded, when the Bodies are moved, and the Liquid is at reſt.

326 *This Demonſtration* will alſo obtain, tho' the Veſſel be ſuppoſed much bigger; and it will do in an infinite Liquid compreſſed; therefore it *may be referred to Bodies deeply immerſed.* Here we ſpeak of a continuous Liquid, and whoſe Parts cannot be reduced into a leſſer Space by Preſſure; otherwiſe there will be an Accumulation before the Body, and a Relaxation behind; and ſo much the more, the more blunt the Body
327 is; which alſo cauſes a greater Irregularity in the Motion of the Liquid, and a greater Retardation in the Motion of the Body.

When a Body is moved in any Liquid along the Surface, the Liquid is raiſed before the Body, and depreſſed behind; and theſe Elevations and Depreſſions are greater, *the more blunt the Body is*, and *by that Means it is more retarded*; for there is alſo a greater Irregularity in the Motion of the
328 Liquid in this Caſe, which ſtill more increaſes the Retardation of the Body. *This is alſo true, if the Body be not immerſed deep*; yet in that Caſe the Irregularity of the Motion of the Liquid is the chief Cauſe of the Retardation.

Therefore, to take away theſe Irregularities, we muſt conſider Bodies are deeply immerſed, and give Rules relating to them; by which the Retardations in ſeveral Caſes may be compared together. We ſuppoſe the Bodies ſpherical, tho' the Demonſtrations will ſerve for all ſimilar Bodies moved in the ſame manner.

Here you muſt obſerve, that _the Reſiſtance is to_ 329
be diſtinguiſhed from the Retardation; the Reſiſtance
produces the Retardation. _When we ſpeak of_
the ſame Body, the one may be taken from the other,
becauſe they are in the ſame Proportion; but, ſuppoſing
the Bodies different, the ſame Reſiſtance
often generates different Retardations. _From the_ 330
Reſiſtance ariſes a Motion contrary to the Motion
of the Body; _the Retardation is the Celerity, and_
the Reſiſtance itſelf is the Quantity of Motion.

Let the Bodies be equal, but of different Denſities, 331
and moved thro' the ſame Liquid with equal
Velocity, the Liquid acts in the ſame Manner upon
both; therefore they ſuffer the ſame Reſiſtance,
but different _Retardations_; and they _are_
to one another as the Celerities which may be generated
by the ſame Forces in the Bodies propoſed*;
that is, they are inverſly as the Quantities *330
of Matter in thoſe Bodies*, or _inverſly as the_ * 60
Denſities.

Now, _ſuppoſing Bodies of the ſame Denſity, but_ 331
unequal, moved equally faſt thro' the ſame Fluid,
the Reſiſtances increaſe according to their Superfices,
that is, as the Squares of their Diameters;
the Quantities are increaſed in Proportion to the
Cubes of the Diameter; the Reſiſtances are the
Quantities of Motion, the Retardations are the
Celerities ariſing from them*; dividing the *339
Quantities of Motion by the Quantities of Matter,
you will have the Celerities*; therefore the * 64
Retardations are directly as the Squares of the
Diameters, and inverſly as the Cubes of the Diameters,
that is, _inverſly, as the Diameters themſelves._

If the Bodies are equal, move equally ſwift, 332
and are of the ſame Denſity, but are moved thro'
different Liquids, their Retardations are as the *339
_Denſities of thoſe Liquids_*. 321
322

334 *When Bodies, equally denſe and equal, are carried*
thro' the ſame Liquid with different Velocities, the
*324 *Retardations are as the Squares of the Velocities*.*

329 From what has been ſaid, *the Retardations of*
335 *any Motions* may be compared together, for they
*334 *are firſt, as the Squares of the Velocities*; ſecondly,*
as the Denſities of the Liquids thro' which the Bo-
*333 *dies are moved; thirdly, inverſly, as the Diameters*
*332 *of thoſe Bodies; laſtly, inverſly, as the Denſities of*
*331 *the Bodies themſelves*.*

The Numbers in the Ratio, compounded of thoſe Ratio's, expreſs the Proportion of the Retardations. Multiplying the Square of the Velocity by the Denſity of the Liquid, and dividing the Product by the Product of the Diameter of the Body multiplied into its Denſity, and working thus for ſeveral Motions, the Quotients of the Diviſions will ſtill have the ſame compound Ratio to one another.

Theſe Retardations may alſo be compared together, by comparing the Reſiſtance with the
336 Gravity. It is demonſtrated, that *the Reſiſtance*
of a Cylinder, which moves in the Direction of its
Axis (to which the Reſiſtance of a Sphere of the
325 ſame Diameter is equal) *is equal to the Weight of*
a Cylinder made of that Liquid, thro' which the Body is moved, having its Baſe equal to the Body's Baſe, and its Height equal to half the Height, from which a Body falling in Vacuo *may acquire the Velocity with which the ſaid Cylinder is moved thro' the Liquid.* From the given Celerity of the Body moved, the Height of the Liquid Cylinder is found, as alſo the Weight of it from the known ſpecifick Gravity of the Liquid and Diameter of the Body. Let a Ball, for Example, of 3 Inches Diameter be moved in Water, with that Celerity with which it would go thro' 16 Foot in a Second: From what has been ſaid of falling

Bodies and Pendulums*, as alſo by Experiments* 157
made on Pendulums, it has been found that this 158
is the Celerity which a Body acquires in falling 134
from a Height of 4 Foot; therefore the Weight
of a Cylinder of Water, of 3 Inches Diameter,
and 2 Foot high, that is, a Weight of about 6
Pounds and 3 Ounces, is equal to the Reſiſtance
of the aforeſaid Ball.

Let the Reſiſtance ſo diſcovered *be divided by* 337
the Weight of the Body, which determines its Quantity
of Matter, and *you will have the Retardation**. *79
By which Rule the Proportion of the ſeveral Retardations
is diſcovered*, and found to be the *330
ſame as is given by the foregoing Rule. 335

Having conſidered the Retardation of direct
Motions, we paſs on to the Motion of Pendulums.

The Arc deſcribed by a Pendulum oſcillating *in* 338
Vacuo, with a Celerity that it has acquired by deſcending,
is equal to the Arc which is deſcribed
by the Deſcent*; the ſame does not happen in a *151
Liquid, and there is a greater Difference between
thoſe Arcs, the greater the Reſiſtance is; that is,
if you ſpeak of the ſame Liquid and Pendulum,
the greater the Arc is which is deſcribed in the
Deſcent.

Let the Reſiſtance of the Liquid be in Proportion 339
to the Velocity, and two Pendulums, entirely alike,
oſcillating in a Cycloid, perform unequal Vibrations,
and begin to fall the ſame Moment; they begin to
move by Forces that are as the Arcs to be deſcribed*;
if thoſe Impreſſions alone, which are made *159
the firſt Moment, be conſidered, after a given
Time, the Celerities will be in the ſame Ratio as in
the Beginning; for the Retardations, which are as
the Velocities themſelves*, cannot change their *329
Proportions, for the Ratio between Quantities
is not changed by the Addition and Subſtraction
of the Quantities in the ſame Ratio. Therefore
in

in equal Times, however the Celerities of Bodies are changed in their Motion by the Reſiſtance, Spaces which are gone thro' are as the
*53 Forces in the Beginning; *that is, as the Arcs to be deſcribed by the Deſcent; therefore after any Time the Bodies are in the correſpondent Points of thoſe Arcs. But in theſe Points the Forces are
156 generated in the ſame Ratio as in the Beginning, and the Proportion of the Celerities, which is not varied by the Reſiſtance, ſuffers no Change from the Gravity. In the Aſcent, Gravity retards the Motion of the Body, but in correſpondent Points its Actions are in the ſame Ratio as in Deſcents. And therefore every where in correſpondent Points the Celerities are in the ſame Ratio. But as in the ſame Moments the Bodies are in theſe correſpondent Points, it follows that the Motion of both is deſtroyed in the ſame Moment, that is, *they finiſh their Vibrations in the ſame Time.* The Spaces run thro' in the Time of one Vibration, are as the Forces by which they are run thro';
340 that is, *the Arcs of the whole Vibrations are as the Arcs deſcribed by the Deſcent*, whoſe Double are the Arcs to be deſcribed *in Vacuo. The De-*
341 *fects of the Arcs to be deſcribed in Liquids, from the Arcs to be deſcribed* in Vacuo, are the Differences of Quantities in the ſame Ratio, and *are as the Arcs deſcribed by the Deſcent.*

342 Since there is the ſame Proportion between thoſe different Arcs, it follows, that *the Celerities, in the correſpondent Points of the Arcs deſcribed, are every where as the Arcs deſcribed by the Deſcent*; for theſe correſpondent Points are alſo the correſpondent Points of the Arcs to be deſcribed *in Vacuo*, in which we have demonſtrated that this Proportion holds.

343 Now *let the Reſiſtance increaſe in the Duplicate Ratio of the Velocity, and let the Pendulum perform unequal*

unequal Vibrations, the greateſt will laſt the longeſt, becauſe the Reſiſtance increaſes more than in the Caſe *Numb.* 239.

344 Yet *the Celerities, ſuppoſing the Arcs not very unequal in the correſpondent Points of the Arcs deſcribed, are every where nearly* in the ſame Ratio, and indeed *in the Ratio of the Arcs deſcribed by the Deſcent.* If the Reſiſtance was in the Ratio of the Celerity, this Proportion would obtain; *but now it is diſturbed, by reaſon of* *342 a greater Reſiſtance in a greater Vibration, by which the Motion in this is more diminiſhed. But it is more accelerated by two Cauſes, 1ſt, this greater Vibration laſts longer, * and the *345 Body ſtays longer in a certain Space, than in the correſpondent Space in a leſs Vibration, and is accelerated during a longer Time. 2dly, The Defect of the Arc deſcribed here, from an Arc to be deſcribed *in Vacuo*, is greater in Proportion in a greater Vibration, becauſe in this the Reſiſtance differs more from the Reſiſtance in a leſs Vibration than in *Numb.* 241. Therefore the correſpondent Points, keeping the ſame Proportion, are more diſtant from the loweſt Point in the greater than in the leſſer Arc, as long as the Body deſcends in it: therefore in Proportion it has a greater Acceleration, becauſe the Force which acts continually on the Body, is as its Diſtance from the lower Point; *therefore there is a Compenſation, *156 and the Proportion above-mentioned is reſtored. In the Aſcent of the Body, the Duration of the Retardation concurs with the Reſiſtance to diſturb that Proportion; but now the correſpondent Points are leſs diſtant from the loweſt Point in the greater Arc (the ſame Proportion continuing) than in the leſſer, and the Gravity in Proportion produces a leſs Retardation; and

therefore now (the Proportion continuing) the Difference of the Diſtance of the correſpondent Point from the loweſt Point is increaſed, ſo that a Compenſation is given from this alone.

The Reſiſtances which are as the Squares of the Celerities, and therefore every where in correſpondent Points, as the Squares of the Arcs deſcribed by the Deſcent, in which Ratio alſo the Retardations are*; but, as each of them keep the ſame Proportion in correſponding Points, the Sums of them all will be in the ſame Proportion; that is, the whole Retardations, which are the Defects of the Arcs deſcribed in the Liquid from the Arcs to be deſcribed *in Vacuo*; or, what is the ſame, *the Differences between the Arcs deſcribed in the Deſcent and the next Aſcent*. Therefore theſe Differences, *if the Vibrations are not very unequal, are nearly as the Squares of the Arcs deſcribed by the Deſcent*. Which is alſo confirmed by Experiments in greater Vibrations; for in theſe the Proportion of Reſiſtance, which we treat of here, obtains*. *329 345 *324

346 Fill the wooden Veſſel ABFCD (*Plate* XXII. *Fig.* 6.) 3 Foot long, 1 Foot wide, and 1 Foot high, with *Water*; hang up the Pendulum V*p* by a Hook V hanging over the Middle of the Veſſel. This Pendulum is made of an Iron Wire 7 or 8 Foot long, and a Leaden Ball *p* of the Diameter of an Inch and a Half; when the Pendulum is at reſt, the Ball is diſtant 3 Inches from the Bottom of the Veſſel. At P there is a greater Ball of Lead, of 3 Inches Diameter, joined to the Iron Wire, that the Ball P may be the leſs retarded in Water.

A-croſs the Top of the Veſſel, upon the Brim of it, may be moved a Board about 5 Inches high, to which muſt be applied the divided Braſs Rulers EG, EG, and the Indices M, M, for meaſuring the Angles deſcribed by the Pendulum in the Deſcent, or Aſcent, by the Method given *Numb.* 170. *Page* 70.

Experiment.] Let the Rulers E G, E G, be ſo diſpoſed, that the Ends G, G, may be over-againſt the Pendulum when it is at reſt, and in ſuch Manner, that between their Ends there may be a Diſtance equal to the Diameter of the Wire to which the Bodies P *p* are fixed. Let one Index be applied to the 16th Diviſion of the Ruler, and another to the 14th Diviſion of the other Ruler: Let the Pendulum fall from that Diviſion, and it will riſe almoſt to this. If, inſtead of theſe Diviſions, you take 20 and $26\frac{7}{8}$, the Experiment will ſucceed in the ſame Manner, as alſo when you apply the Indices to the Diviſions 24 and $19\frac{1}{2}$. Take care that the Water be perfectly at reſt.

In this Experiment the Arcs deſcribed in the Deſcent are to one another as 4, 5, and 6, whoſe Squares are 26, 35, 26; the Difference of thoſe Arcs from the Arcs deſcribed in the Aſcent, are 2, $3\frac{1}{8}$, $4\frac{1}{2}$; which Numbers are to one another as the aforeſaid Squares, as appears by multiplying them by 8.

A Body freely deſcending in a Liquid is accelerated by the reſpective Gravity of the Body which continually acts upon it; *yet not equally*, as in a Vacuum*, the Reſiſtance of the Liquid occaſions a Retardation, that is, a Diminution of Acceleration, which Diminution increaſes with the Velocity of the Body. *For there is a certain Velocity which is the greateſt that a Body can acquire by falling*; for *if* its Velocity be ſuch, that *the Reſiſtance ariſing from it becomes equal to the reſpective Weight of the Body*, its Motion can be no longer accelerated; for the Motion which is continually generated 347 *129 348 nerated by the reſpective Gravity, will be deſtroyed by the Reſiſtance, and the Body forced to go on equably: *The Body continually comes nearer and nearer to this greateſt Celerity, but can never attain to it.*

When the Denſities of a Liquid and a Body are given, you have the reſpective Weight of the Body; and by the knowing the Diameter of the Body, you may find out from what Height a Body falling *in Vacuo*, can acquire ſuch a Velocity, that the Reſiſtance in a Liquid ſhall be equal to that reſpective Weight*, which will be that greateſt Velocity above-mentioned. *336

If the Body be a Sphere, it is known that a Sphere is equal to a Cylinder of the ſame Diameter, whoſe Height is two third Parts of that Diameter; which Height is to be increaſed in the Ratio in which the reſpective Weight of the Body exceeds the Weight of the Liquid, in order to have *the Height of the Cylinder of the Liquid, whoſe Weight is equal to the reſpective Weight of the Body*; but, *if you double this Height, you will have a Height, from which a Body, falling* in Vacuo, *acquires ſuch a Velocity as generates a Reſiſtance equal to this reſpective Weight**, *and which therefore is the greateſt Velocity which a Body can acquire, falling in a Liquid from an infinite Height**. 350 *336 *348

349 Lead is eleven times heavier than *Water*, wherefore its reſpective Weight is to the Weight of *Water* as 10 to 1; therefore a leaden Ball, as it appears from what has been ſaid, cannot acquire a greater Velocity, falling in *Water*, than it would acquire in falling *in Vacuo*, from an Height of $13\frac{1}{3}$ of its Diameters.

351 *A Body lighter than a Liquid, and aſcending in it by the Action of the Liquid, is moved exactly by the ſame Laws as a heavier Body falling in the Liquid.* Where-ever you place the Body, it is ſuſtained by the

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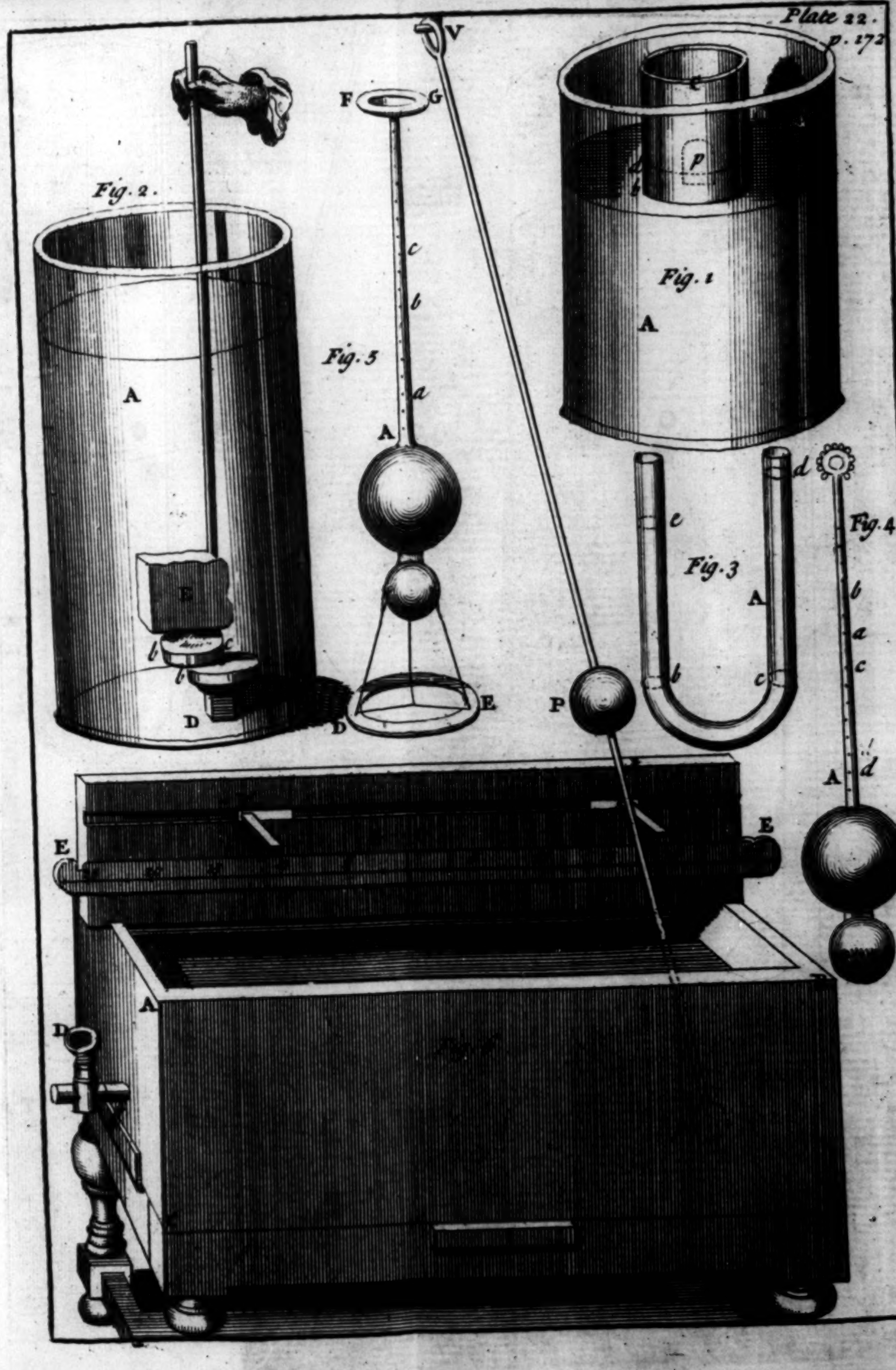

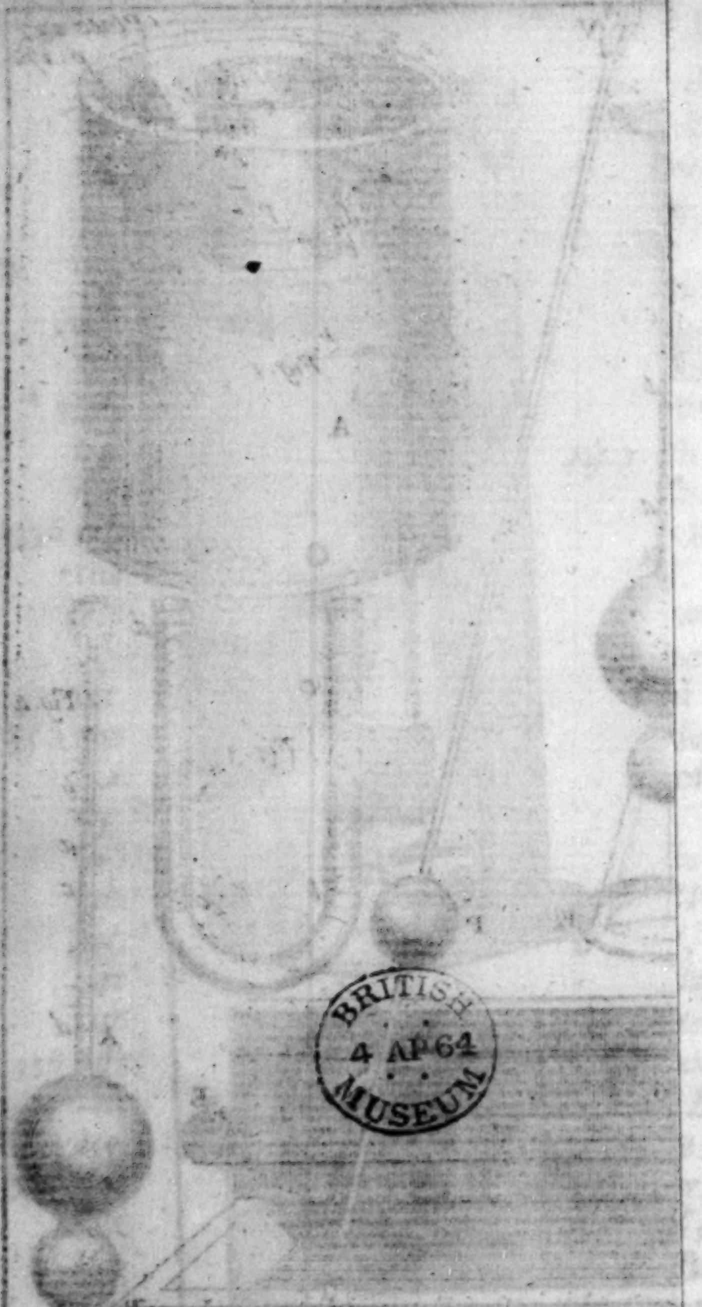

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the Liquid, and carried up with a Force equal to the Difference of the Weight of the Quantity of the Liquid, of the ſame Bulk as the Body, from the Weight of the Body, as appears by comparing *Numb.* 293 with *Numb.* 292; therefore you have the Force that continually acts equably upon the Body, by which, not only the Action of the Gravity of the Body is deſtroyed, ſo that it is not to be conſidered in this Caſe, but by which alſo the Body is carried upwards by a Motion equably accelerated, in the ſame manner as a Body, heavier than a Liquid, deſcends by its reſpective Gravity; but the Equability of the Acceleration is deſtroyed in the ſame manner by the Reſiſtance, in the Aſcent of a Body lighter than the Liquid, and it is deſtroyed in the Deſcent of a Body heavier than the Liquid.

When a Body, ſpecifically heavier than a Liquid, is thrown up in it, it is retarded upon a double Account, on account of the Gravity of the Body, and on account of the Reſiſtance of the Liquid; therefore *a Body riſes to a leſs Height than it would riſe* in Vacuo *with the ſame Celerity. But the Defects of the Height in a Liquid, from the Heights to which a Body would riſe* in Vacuo *with the ſame Celerities*, have greater Proportion to each other than the Heights themſelves; and, *in leſs Heights*, the Defects *are nearly as the Squares of the Heights* in Vacuo. 352 353

Part II. *Of the Motion of Fluids.*

CHAP. VII.

Of the Celerity of a Fluid ariſing from the Preſſure of the ſuperincumbent Fluid.

AN inferior Fluid is preſſed by a ſuperior, *275 and that equally every Way*; becauſe Action is equal to Re-action, it endeavours to recede every Way with equal Force; therefore *if you take off the Preſſure on one Side, the Liquid will* 354 *move towards that Side; and which Way ſoever the Preſſure be taken away, it will move with the ſame Celerity*; which will be confirm'd by the Experiments to be mention'd in the following Chapter.

At the ſame Depth the Celerity is alſo every where the ſame, by reaſon of the Equality of the *273 Preſſure*; but when the Depth is changed, the 272 Celerity is alſo changed.

Yet the Velocity does not follow the ſame Proportion as the Depth; tho' the Preſſure, from which the Velocity ariſes, does increaſe in the *273 ſame Ratio as the Depth*. The Quantity of Motion, which is produced in the Liquid, is the Effect of the whole Preſſure; and this Quantity *59 increaſes as the Preſſure*; but the Ratio of the Quantity of Motion is compounded of the Ratio of the Velocity and the Quantity of the Matter *64 moved*. Here the Matter moved is the Water, which goes out of the Hole, whoſe Quantity, the Time remaining the ſame, increaſes with the

Celerity; it will be double, if the Celerity be doubled, in which Caſe the Quantity of Motion is quadruple; that is, increaſed *as the Square of the Celerity*, which obtains in any Celerity: Therefore that Square *increaſes* as the Preſſure; that is, *is the Height of the Liquid above the Hole from which the Water ſpouts.* 355

356 *Plate* XXIV. *Fig.* 1.] Fill with Water the Parallelopiped A B, which is 15 Inches long, and as wide, and 2 Foot high; it muſt be ſo placed that its Bottom may be raiſed about 8 Inches above the horizontal Bottom of a hallow Trough C D, whoſe Length is almoſt 4 Foot, and Breadth a Foot and a half, and Depth 5 or 6 Inches.

At E, near the Bottom of the Veſſel A B, there is fix'd a Braſs Tube horizontally, above half an Inch in Bore; the fore Part of it is ſhut by a Plate in the Middle of which there is a Hole, whoſe Diameter is equal to $\frac{1}{12}$ Inch: That Hole is ſhut with a Cover that ſcrews on upon the fore Part of the Tube.

The Celerities with which the Water flows out from E, when you have open'd the Hole, are compared together by Help of this Machine. Let it move, for Example, in the Line E L, and at L let it come to the Bottom of the Veſſel C D; this Motion may be reſolved into two Motions; the one horizontal, along E I, in the Direction that the *Water* has in going out of the Hole, and the other vertical, along I L; the firſt is equable, and the *Water*, with the Celerity with which it goes out, runs thro' the Space E I, in the ſame Time that in falling it runs thro' I L*; whatever the Celerity be, I L is not changed, becauſe E I and the Bottom of the Veſſel are horizontal, therefore the Time is not changed, in which ſeveral ſuch Lines as E I may be run thro'

*209

thro', and therefore they are as the Celerities with
53 which the Water goes out: If you meaſure the Diſtance to which the Water ſpouts, you will have the Line E I.

Experiment 1.] Let there be Water in the Veſſel A B, up to the Height of five Inches above the Hole at E; let the Diſtance be meaſured to which the Water ſpouts; if more Water be poured in, to the Height of twenty Inches, it will ſpout to a double Diſtance. The Squares of the Diſtances are here as the Height of the Water, in which Ratio alſo are the Squares of the Celerities.

357 *The Velocity of a Liquid, at any Depth, is the ſame as that which a Body, falling from a Height equal to the Depth, would acquire*; for the Velocity of a Liquid increaſes, when the Depth of the Hole below the Surface of the Liquid increaſes, in the ſame Ratio as the Celerity of the falling Body increaſes, when the Space gone thro' by the Fall
355 increaſes; and in the Beginning theſe Velocities
131 are equal; for in a Liquid the upper Parts, as
130 well as in a Body at the Beginning of the Fall, endeavour to deſcend by Gravity only.

Experiment 2. *Plate* XXIV. *Fig.* 1.] This is performed with the ſame Machine as was uſed in the former Experiment; the Veſſel A B is filled with Water, and a Tube with a Hole, like the Tube E, is placed at F; ſo that the Height of the upper Surface of the Water, above the Bottom of the Veſſel C D, is divided by that Hole into two equal Parts; the Water from that Hole will ſpout to M, ſo that the horizontal Diſtance from the Point M to the Hole will be double the Height of the Hole, above the Bottom of the Veſſel C D; therefore the Water, by an equable Motion, and with the Celerity with which it goes out, runs thro' double the Space of that Height, in the Time in which a Body can fall from F to the Bottom of the Veſſel CD, and therefore it moves with the Celerity which a Body can acquire in falling from that Height*; but this *134 Height is equal to the Height of the Surface of the Water above the Hole.

CHAP. VIII.

Of ſpouting Liquids.

358 A *Liquid, ſpouting vertically out of a Hole*, ariſes up with that Celerity, with which it *would come up to the upper Surface of the Liquid, yet it never comes up to that Height**; and that for *357 137 ſeveral Cauſes. 1. The Celerity, by which the Liquid aſcends, is diminiſhed every Moment, and the Column of the ſpouting Liquid conſiſts of Parts, which are moved to different Heights by different Celerities; all the Parts of a Column, which is every where of the ſame Thickneſs, are neceſſarily moved by the ſame Celerity; the ſaid Column every where will be broader every Moment, as the Celerity of the Liquid is diminiſhed; which ariſes from the Impulſe of the Liquid following, and which from the Nature of a Liquid yields to every Impreſſion, and is eaſily moved every Way: by that Impreſſion the Motion is retarded every where. 2. This Motion is alſo diminiſhed by the Liquid, becauſe, when it hath loſt all its Motion, it hangs in the upper Part of the Column, and is ſuſtained for a Moment by the Liquid that follows, before it flows off on the Sides, which retards the Liquid that follows it, and that Retardation is communicated to the whole Column. 3. By the Friction againſt the

Sides of the Hole, the Celerity of the Liquid is diminiſhed; which Friction is increaſed, when the Liquid is brought through Pipes and Cocks. 4. Laſtly, the Air's Reſiſtance ſtops the Motion of Liquids.

The firſt Cauſe above-mentioned of the Retardation cannot be corrected.

The ſecond is corrected by ſomewhat inclining the Direction of the Liquid, as is ſelf-evident; 359 and this is the Reaſon why *a Liquid riſes higher, if its Direction be a little inclined, than if it ſpouts vertically.*

Experiment 1. *Plate* XXIV. *Fig.* 1.] To the *356 Machine above-deſcribed*, by help of a Screw at N, join the Curve Tube NO, from which the Water, thro' a ſmall Hole, ſpouts up vertically, by turning the Tube a little, which is eaſily done by reaſon of the Screw at N, the Direction of the ſpouting Water will be inclined, and it will aſcend higher. But by this Inclination the Beauty of a Jet is deſtroyed.

As to the third Cauſe of the Retardation, 'tis to be obſerved, that there is a greater Friction, in Proportion, in ſmall Holes, than in great ones; the Celerity being increaſed, the Friction alſo 360 is thereby increaſed; and therefore *the Holes are to be increaſed according to the Heights of the ſpouting Water.*

The Ends of the Pipes, from which the Water ſpouts, have commonly the Figure of a truncated Cone, as is repreſented at P, (*Plate* XXIV. *Fig.* 3.) in which End the Water ſuffers a great deal of Friction, and is moved irregularly, and ſpouts up with that Irregularity. This may be a- 361 mended by *covering the End of the Tube with a flat, ſmooth, and poliſhed Plate, fixed to it, which has a Hole in it*; for then *the Water ſpouts higher*; and becauſe it riſes with a Motion intirely regular, it *is perfectly tranſparent.*

Experiment 2. *Plate* XXIV. *Fig* 3.] Take the Tube above-mentioned P, as alſo the Cylinder Q, ſhut up at one End with a bored Plate; let thoſe be ſcrewed on, one after another, to the End O of the Tube NO (*Fig.* 1.) the Water remaining at the ſame Height in the Veſſel AB; let it ſpout from the Tube P, and the Cylinder Q, and the Experiments will fully confirm what we have ſaid.

The Pipes which bring the Water from a Reſervoir, muſt be very wide, in Proportion to the ſpouting Hole, that the Water may move ſlowly in theſe Pipes, and have no ſenſible Friction. 362

For the ſame Reaſon *the Water-way,* or Paſſage, *of the Cocks muſt be very large.* 363

Experiment 3. *Plate* XXIV. *Fig.* 1.] To the Veſſel AB, at the ſame Height as the Tube F, fix the Cock H, the Pipe of the Cock muſt be ſhut up in the ſame manner as the Tube F, and bored with a Hole of the ſame Size; the Water-way of this Cock is a Quarter of an Inch. The Water which goes thro' this Cock is brought thro' a narrower Space than that which moves thro' the Tube F; therefore this laſt is more tranſparent, and ſpouts to a greater Diſtance.

The Reſiſtance of the Air has a ſenſible Effect upon the Motion of Liquids; (for itſelf may be reckoned amongſt Liquids, as will be ſaid in the Second Part;) therefore we may here apply what has been ſaid of the Aſcent of any Body in a Liquid; *and in ſmall Heights, the Defects of the Heights from the Heights* in Vacuo are in the Ratio of the Squares of thoſe Heights*; that is, abſtracting from the other Cauſes of Retardation, they 364

*353

are *in the Ratio of the Square of the Height of the Liquid above the Hole.* Beſides this Reſiſtance, there is alſo another, not to be over-looked, which is the Action of the Air againſt the ſpouting Liquid. It encloſes the whole Column of the ſpouting Liquid, and reſiſts that Part of its Motion, whereby it ſpreads itſelf ſide-wiſe, as it becomes wider; and there is required a greater Force of the Liquid that comes after, than if this Reſiſtance was taken away; therefore the Air reſiſts by its lateral Preſſure. The Reſiſtance from the Stroke of the Liquid againſt the Air encreaſes with the impingent Surface, that is, if the Celerities remain the ſame, encreaſes with the Hole; in which Ratio alſo, the Quantity of the Matter moved encreaſes, and upon this account it is no matter of what Bigneſs the Hole is.

The lateral Preſſure follows the Proportion of the Surface of the Column; the Matter moved, which (the Celerity being the ſame) is in the ſame
62 Ratio as the Quantity of Motion, follows the Proportion of the whole Column, that is, of the Square of its Surface; and therefore, if the Hole be encreaſed, the Quantity of Motion encreaſes faſter than the Cauſe retarding it; and for that
365 Reaſon, *in the greateſt Heights of ſpouting Liquids*, that the lateral Preſſure (which exerts a greater Action when it acts the longer) may be the better overcome, *greater Holes are required*; which we have alſo ſhewed before to be required in the
360 ſame Caſe from another Cauſe; where, as well as here, we ſuppoſed the greater Holes only neceſſary for the greateſt Heights, though the Demonſtrations proves that theſe Holes, which are very neceſſary in the greateſt Heights, are in general to be preferred to others.

Great Holes alſo hinder the Motion; for then there is a greater Surface, which is preſſed upon by the higheſt Part of the Liquid, which has loſt all its Motion, and hangs on a longer Time, before it runs off down the Sides. From theſe two contrary Effects joined together, *in all* 366 *Heights there is a certain Meaſure of the Hole, thro' which the Liquid will riſe to the greateſt Height poſſible.* Yet one cannot give Rules to determine the Diameter of the Hole, becauſe the Bigneſs of the Pipes of Conduct and their Inflexions require it different, ſo that there may be a Variation to Infinity.

In reſpect to the greateſt Heights, it is to be 367 obſerved, that *the Bigneſs of the Hole, and alſo the Height to which the Liquid can aſcend, have their Limits,* which they cannot exceed: Not only the Liquid which is directly againſt the Hole runs out, but, that there may be a conſtant Supply, the neighbouring Liquid continually comes towards the Hole with an oblique Motion, and in going out, it ſpouts with a compound Motion, and the Motion of the Liquid, ſpouting vertically, is diſturbed; the greater the Hole is, the greater is the Diſturbances ariſing from that Cauſe, and in ſpouting Waters the Holes ſhould never exceed an Inch and a Quarter. When the Celerity of the Liquid is too great, it ſtrikes againſt the Air with ſo much Force, that it is diſperſed into Drops; in which Caſe, by diminiſhing the Celerity, the Height to which the Liquor ſpouts will be increaſed; and there is a Height which is the greateſt to which a Liquor can aſcend, which Height in ſpouting Water ſcarce exceeds 100 Feet.

Liquids, which ſpout obliquely, are not retarded 368 *from ſo many Cauſes, nor ſo much, as thoſe that ſpout vertically.* The ſecond Cauſe of Retardation, abovementioned*, has no Place here, and the Effect of *358 the firſt is leſs. As for the reſt, one may apply here what has been ſaid of Solids oblique pro-

369 jected in *Chap.* XXIV. *Book* I. from *Numb.* 212, quite to the End of the Chapter. And *a Liquid may be conſidered as an innumerable Quantity of Solids, following one another*, and running the ſame Way. In the Motion of the Liquid, the Way gone thro' may be perceived by our Senſes; and what has been ſaid of Solids, obliquely projected, may be reduced to Experiments by the Help of Liquids; for doing which we muſt make uſe of Quickſilver, becauſe of the great Specifick Gravity of this Liquid in reſpect of others: But theſe Experiments are to be made by a particular Machine contrived as follows.

370 *Plate* XXIII. *Fig.* 1.] The wooden Trough A, B, C, D, E, F, H, is four Foot and an half broad, eight or ten Inches long, and ſix or ſeven Inches high; the Bottom is made of a Board hollowed in half an Inch, to contain the Mercury the better.

In the End H, of the Side E, F, H, you have a Board H I ſix Inches wide, and two Foot high, which has in it a Slit *ot*. By this means you may fix to any Height upon the Board the wooden Parallelopiped *s*, which has a Screw fixed in its hinder Part.

The ſecond Figure repreſents this Parallelopiped at S: There is faſtened to it a cylindric Veſſel of Box-Wood, which has a Groove round it to receive two Braſs Plates, one of which may be ſeen at *fe*; their Ends are joined together by the Screw G, ſo as to make the Box-Veſſel immoveable, till it is looſened by unſcrewing *g*, which will allow it to move about its Axis.

In the Bottom of this Veſſel there is a cylindric Cavity *ab*, a Quarter of an Inch Diameter; this communicates with a like Cavity *bc*, which terminates in the Middle of the greater Cavity *cd*, whoſe Diameter is above half an Inch, that it may receive the truncated Cone of Box H, which is joined to the Cylinner I L, *Fig.* 3.

The truncated Cone H exactly fills the Cavity *c d*, and is held faſt in it by help of the Screw R, that goes thro' the Braſs Plate Q O, but ſo that this truncated Cone may turn upon its Axis.

In this Cone, as well as in the Cylinder I L, there is a Cavity *b i l*, of the ſame Diameter as the Cavity *bc*, and anſwering to it. This Cylinder I L has a Glaſs Tube N cemented to it.

The Tube is a Foot and a half long, one End of which is ſeen at N M (*Fig.* 5.) which is cemented alſo to the Box-Cylinder L I, which is hollowed at *l i b*, with a round Hole in the Form of a Gnomon, or the Carpenter's Square; at *bc* the Cavity is greater, to receive the truncated Cone C D, that exactly fills it, and is moveable about its Axis by the Help of the Handle E A.

The Cavity *bi* anſwers to the Cavity *de*, which communicates with *fg*; this Part of the Box has driven upon it an Iron Fernil B Q, in which is drilled a very ſmall Hole *g*, which, when the Parts of the Machine are joined together, communicates with the Cavity of the Box P (*Fig.* 2.)

To prevent the Tube from breaking the Ends L L of the Box-Cylinders (*Fig.* 3. and 5.) together with the Tube, are joined cloſe to a ſtreight and ſtiff Piece of Wood *mn* (*Fig.* 1.) whoſe lower End *m* has an Iron Plate fixed to it (as may be ſeen in *Fig.* 6.) whoſe End L P B Q is bent in the Figure of a double Gnomon; when the End L (*Fig.* 5.) of the Box-Cylinder is applied to the End of the Piece M N I of *Fig.* 5. it fits to I of *Fig.* 6. and the Screw Q, applied to *o*, preſſes the Cylinder

BD of *Fig.* 5. and joins it firmly to the Cylinder L I.

All the Parts of the Machine may be ſeen joined together at *Fig.* 1. Quickſilver being poured into the Veſſel *p*, ſpouts out of the Hole *g*, *Fig.* 5. When the Mercury is at the ſame Height in the Box, and you do not vary the Inclination of the Piece *n m*, the Mercury ſpouts with the ſame Celerity in any Direction; but the Inclination of the Direction may be varied by moving the Handle *e a* (E A, in *Fig.* 5.) the Angle that the Direction, in which the Mercury goes out of the Hole, makes with the Horizon, may be meaſured by Help of the Quadrant *q*, along with the Index *f h* is moveable, which by its Weight is always kept in a vertical Poſition. This Quadrant may be ſeen in *Fig.* 7. with its Index F H: It has two Rings behind, to receive the Handle E A, *Fig.* 5. When this Handle is vertical, the Index hangs againſt the 45th Degree, and the Direction of the Motion of the Mercury, which ſpouts out then, makes a half Right Angle with the Horizon.

In *Fig.* 1. the Jets of Mercury in their ſeveral Directions are repreſented: They become the more viſible by help of a wooden Plane G painted black, along which the Mercury in its Motion does almoſt ſlide: Upon this Plane muſt be drawn the Ways which a Body (according to what is ſaid in *Numb.* 212) runs thro', when it moves with the ſame Celerity according to Directions which make different Angles with the Horizon. Alſo the Semicircle A L of *Plate* XV. *Fig.* 5. muſt be drawn upon this Plane, tho' it could not be repreſented in this Figure.

There are ſeveral other ſuch Planes, in which the ſame Things are drawn, but ſo as to repreſent the Ways of Projectiles, &c. according to different Celerities.

This Plane ſtands upright near the Middle of the Trough, and bears againſt the Side E F H, ſo as to move backwards and forwards according to the Length of the Trough.

The Celerity of the ſpouting Mercury is varied, as you change the Inclination of the Piece *n m*; and, by lowering the Veſſel *p*, the Hole through which the Mercury ſpouts, is ſet to the Direction of the Lines drawn on the Plane.

The Mercury will ſtop its ſpouting, when the Cavity *a b* (*Fig.* 2.) is ſtopped with the Pin D E of *Fig.* 4.

Experiment 4. *Plate* XXIII. *Fig.* 1.] The Parts of this Machine being joined and fixed together, as in the manner above deſcribed, incline the Piece *n m*, till the Height to which the Mercury is to ſpout, when it aſcends to a Direction almoſt vertical, is nearly equal to the Diameter of the Semicircle deſcribed on the Plane G. Let the Veſſel P be fixed at ſuch a Height, and the Plane G be ſo placed, that the Axis of the Circumvolution of the Cylinder B D (*Fig.* 5.) ſhall anſwer to the lower Point of the Semicircle above-mentioned. Which Way ſoever the Inclination of the Direction of the Jet (that is, of the Projection) be, its Amplitude will always be the Quadruple of the Line BM in the Semicircle A B L. (*Plate* XV. *Fig.* 5.) There is indeed a ſmall Difference, which chiefly ariſes from the Reſiſtance of the Air, and muſt be obſerved in the following Experiments. 371

Experiment 5.] The Machine being diſpoſed as in the foregoing Experiment, if the Mercury ſpouts in two Directions, and the Inclination of one of them exceeds a half Right-Angle as much as the other is under it, the Mercury will cut the Horizontal Line which is drawn from the lower Point 372

Point of the Semicircle on the Plane G, juſt in the ſame Place in both Caſes.

373 *Experiment* 6.] Every thing being diſpoſed as before, if the Way for any Direction of Motion be drawn on the Plane, and the Index *f g* agrees with that Diviſion of the Quadrant which denotes that Inclination, the Mercury in its Motion will follow the Line drawn to repreſent its Way. If you draw the Ways for ſeveral Angles, by the Motion of the Handle *a e*, you will bring the Mercury to ſpout in Jets that go along theſe very Lines.

374 *Experiment* 7.] Let there be another Plane, as G, in which all the Lines above-mentioned are drawn from another Celerity of the Mercurial Jet, and the Experiments will ſucceed in the ſame Manner.

By the ſame Method, as we do by a Semicircle determine the Diſtance to which Bodies obliquely projected will fall, one may find the Diſtance to which the Liquor coming out of the Side of a Veſſel ſpouts, when the Veſſel is ſet upon a Horizontal Plane; which Diſtance is different according to the different Height of the Hole, the upper Surface of the Liquid remaining the ſame.

375 *Plate* XXIV. *Fig.* 4.] *Let A B be the Height of a Veſſel filled with a Liquid*; ſuppoſe this Height cut into two equal Parts at C; with the Center C, and Diſtance E A deſcribe a Semicircle; *let there be a Hole at E*; laſtly, draw E D perpendicular to A B, and terminated in the Circumference of the Semicircle at D. *Let the Liquid ſpout from E to F in the Horizontal Plane, and the Diſtance B F will be double the Perpendicular E D.*

Which

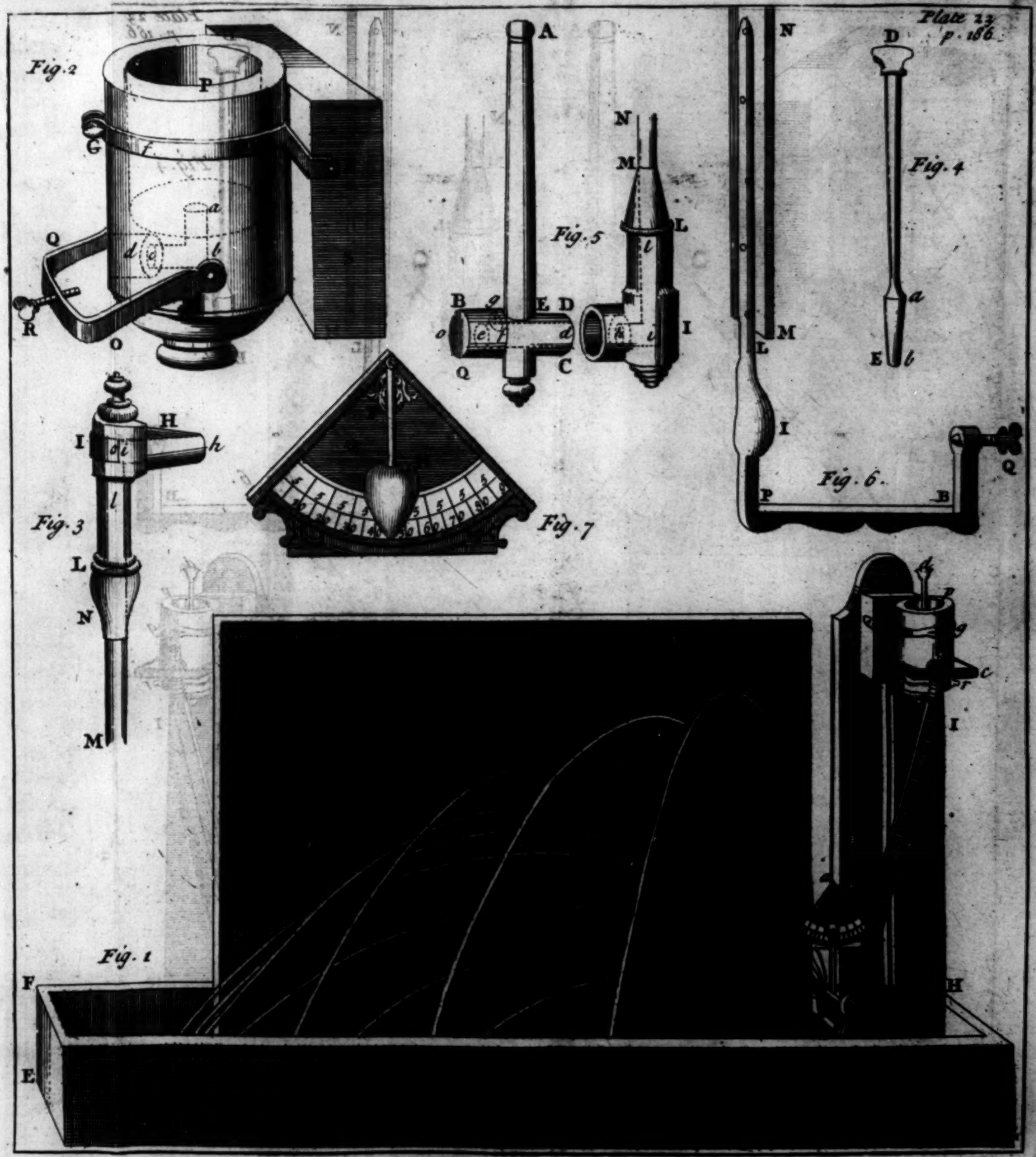
Fig. 2
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Fig. 5
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Fig. 7
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Which will be demonſtrated, if we conſider, that the Liquid, with an equable Motion with the Celerity that it has coming out of the Hole, would (in the Time that a Body can fall from E to B) run thro' the Space BF*. *209

In all Motion, the Time remaining the ſame, the Space gone thro' is as the Celerity*; the* 53 Celerity remaining the ſame, it is as the Time: Therefore if you change the Time and the Celerity, the Space gone thro' will be in a Ratio compounded of the Celerity and the Time; and multiplying the Time by it, you will have the Space gone thro'; that is, if you apply this Operation to different Motions, you will have ſuch Quantities as will expreſs the Proportion of the Spaces gone thro'. If you compute the Squares of the Celerities and the Time, you will have the Proportion of the Squares of the Spaces gone thro'. A E here expreſſes the Square of the Celerity*; E B the Square of the Time*; there- *355 fore the Product of thoſe Lines expreſſes the *131 Square of the Space gone thro' E F. But that Product is the Square of the Line E D, which therefore, changing the Hole, increaſes and diminiſhes in the ſame Ratio as the Diſtance B F. Suppoſe the Hole in the Center C; B G, the Diſtance to which the Liquor ſpouts, is equal to B A*, and it is double the Perpendicular, which *357 from C may be drawn to A B in the Semicircle; 134 which therefore obtains in all Holes, and E D will be the half of BF.

376 Hence it follows, that *a Liquid, ſpouting from a Hole in the Center* C, *will go to the greateſt Diſtance poſſible.*

Experiment 8. *Plate* XXIV. *Fig.* 1.] Here we muſt make uſe of the Machine, deſcribed in the foregoing Chapter*. Let the Water ſpout from *356 the Hole F, as in *Experiment* 2. *Chap.* VII. Let it ſpout at the ſame Time from E, and alſo from G, where there is a Tube like thoſe which are made faſt at F and E; the Hole G is leſs than F, but the Hole E is farther diſtant from the Surface of the Water, the Water comes from neither of them to the Diſtance to which it comes, when ſpouting from F.

Plate XXXIV. *Fig.* 4.] From what has been ſaid, it follows, That *the Water ſpouts to the ſame Diſtance from the Holes* E *e*, *equally diſtant from the Center* C, becauſe in that Caſe the Perpendiculars E D and E *d* are equal.

377 *Experiment* 9. *Plate* XXIV. *Fig.* 1.] From F let there be drawn a horizontal Line, which goes thro' H; if H G and H E are equal, the Water will go from each Hole G and E to L.

CHAP. IX.

Of a Liquid flowing out of Veſſels, and the Irregularities in that Motion.

378 THE Quantity of a Liquid, which *in a given Time* flows *from a given Hole*, increaſes in Proportion to the Velocity of the Liquid going out. This depends upon the Height of the Liquid above the Hole, and it is no matter to what
354 Part the Motion of the Liquor is directed; therefore *the Squares of the Quantities flowing out are in the Ratio of the Heights of the Liquid above*
*355 *the Holes**.

In the Time in which a Body falling freely goes through the Height of the Liquid above the Hole, a Column of the Liquid flows out
357 equal in Length to twice that Height; the
134 Hole itſelf is the Baſe of the Column, and is given;

given; if the Height of the Liquid above the Hole is known, the whole Column is known; the Time alſo is eaſily determined by Experiments of Pendulums*; but having found what Quantity flows out in a known Time, one may know what Quantity will flow out in a given Time. *157

Here you muſt obſerve, that the Reſiſtance of the Air, and the Friction of the Liquid againſt the Sides of the Hole, hinders the Motion of the Liquid, and that the Rule above-mentioned does not exactly obtain, and that there always flow out a leſs Quantity than what there is determined by it. Yet making Experiments with Water, it is plain, that the Quantities which flow from the ſame Hole in equal Times, ſenſibly keep the Proportion of the Squares of the Heights of the Water above the Hole, in Heights not exceeding fifty Foot.

In Veſſels which are not ſupplied by the flowing in of the Liquid, the Celerity of the Liquid flowing out is continually changed; to which Regard muſt be had, when you compare together the Times in which different Veſſels are emptied.

Here we conſider cylindric Veſſels; and what is here ſaid may be applied to any Veſſels that are of the ſame Bigneſs from Top to Bottom; we ſuppoſe the Liquid to flow out from a Hole in the Bottom.

The Times in which cylindric Veſſels of the ſame Diameter and Height are emptied, the Liquid flowing from unequal Holes, are to each other inverſly as thoſe Holes. 379

If we ſuppoſe that theſe Veſſels are divided into very ſmall equal Parts by Planes parallel to their Baſe; and that the Diviſions of each Veſſel do not differ from one another, when we conſider the ſmalleſt Parts, one may conceive that the Celerity is not changed in the Evacuation of one Part. The Quantity of a Liquid which flows from a Hole, if the Celerity is not changed, increaſes with the Hole and with the Time; that is, in a Ratio compounded of the Time and of the Hole. The correſpondent Parts in the Veſſels are emptied with equal Celerities, and the aforeſaid compounded Ratio obtains here: The ſame Parts alſo, that is, the Quantities of the Liquid which flow out, are equal; wherefore the Difference of the Times is recompenſed by the Difference of the Holes; that is, the Times are in the ſame, but inverſe Ratio, as the Holes. Now as this happens in all the correſpondent Parts, it muſt alſo be referred to the Times of the whole Evacuations of the Veſſels.

380 *When the Veſſels are cylindric, unequal, and equally high, they are emptied thro' equal Holes, in Times that are to one another as the Baſes of the Cylinders.*

Let the Veſſels again be ſuppoſed to be divided into very ſmall Parts, and equal in Number in each Veſſel; the Liquid of the correſpondent Parts flows thro' equal Holes, and with equal Celerity: Therefore the Quantities that flow out are as the Times, and conſequently the correſpondent Parts themſelves are in that Ratio of the Times, which are as the Baſes of the Cylinders: But the Times of the whole Evacuations are as the Times in which the correſpondent Parts are evacuated.

381 Laſtly, *Let there be two cylindric Veſſels, whoſe Baſes are equal, but their Heights, for Example, as* 1 *to* 4, *and let them be evacuated thro' equal Holes.*

Let theſe alſo be conceived to be divided into very ſmall Parts, by Planes parallel to the Baſe; and let the Number in thoſe Parts be equal in each Veſſel; thoſe Parts will be one to another as the Veſſels, that is, as 1 to 4. We can conſider every Part as evacuated by an equable Motion, becauſe the Parts are very ſmall; the Celerities in the correſpondent Parts are every where, as 1 to 2*, becauſe the Heights of thoſe Parts above *355 the Baſes are as the Heights of the Veſſels, which are as the Squares of thoſe Numbers. Whence it follows, that the Times, in which correſpondent Parts are evacuated, are to one another, as 1 to 2; becauſe in twice the Time with a double Celerity, a quadruple Quantity is evacuated. But as the Times are in the ſame Ratio for each correſpondent Part, the Times in which the whole Veſſels are evacuated, are alſo as 1 to 2. If the Veſſels are, as 1 to 9, the Times will be, by a like Demonſtration, as 1 to 3; and generally the Times are as the Celerities in which correſpondent Parts are evacuated, the Squares of whoſe Celerities are *as the Heights of the Veſſels**, in which Ratio alſo are *the Squares of the* *355 *Times.*

Experiment 1. *Plate* XXIV. *Fig.* 2.] Let there be three thin cylindric Veſſels of Metal A, C, B, having equal Diameters, and whoſe Heights are, as 1, 3, and 4; let each of them have a Lip in the Top to let the Water run out when it comes to a certain Height, which Lip muſt be reckoned the Top of the Veſſel; in the Bottoms of the Veſſels A and B, which are, as 1 and 4, let there be equal Holes, and let them be filled with Water; let the Holes be opened in the ſame Moment; if the Water running out of B be received in the Veſſel C, it will be filled in the ſame Time that A is evacuated. C contains three Quarters of the Veſſel B; the Quarter which is left will alſo be evacuated in the ſame Time as the Veſſel A, which is evident to Senſe; therefore A is emptied twice, whilſt B is emptied once.

382 *The Times in which any cylindric Veſſels are eva-*
*380 *cuated, are in a Ratio compounded of the Baſes*, of*
*379 *the inverſe Ratio of the Holes*, and of the the ſquare*
*381 *Roots of the Heights*.*

383 *The cylindric Veſſel may be ſo divided, that the Parts intercepted between the Diviſions ſhall be emptied in equal Times, which will happen, if the Diſtances of the Diviſions from the Baſe be as the Squares of the natural Numbers*; for the Times of the Evacuations of the Veſſels, whoſe Heights are in that Proportion, are as the natural Num-
381 bers, and the Differences of the Times are equal.

The Time in which a cylindric Veſſel is emptied, is as the Celerity with which the Liquid be-
381 gins to run out; therefore the Celerity, while
355 the Liquid deſcends in the Veſſel, is diminiſhed in the ſame Ratio as the Time of the Evacuation of the Liquid remaining in the Veſſel, and *the*
384 *Motion of a Liquid, running out of a cylindric Veſſel, is equally retarded in equal Times.*

385 *If thro' equal Holes a Liquid runs out of a Cylinder, and out of another Veſſel of the ſame Height (and in which the Liquid is always ſupplied ſo as to be kept at the ſame Height) in the Time in which the Cylinder is emptied, there runs out twice as much Water from the other Veſſels as from the Cylinder.* For, becauſe of the equal Height of the Veſſels, the Celerities in the Beginning are equal; the Celerity of the Liquid, which comes out of the Veſſel that is always kept full, is equable; the Celerity of the Liquid, which runs out of the Cylinder, is equably retard-
384 ed. Therefore, whilſt the Cylinder is emptying, there will flow twice as much Water out of this Veſſel as out of the Cylinder: For if two Bodies are driven with the ſame Celerity, and the firſt goes

goes with an equable Motion, and the ſecond with a Motion equally retarded, and they move till they have loſt all that Motion, the firſt in that Time will run double the Space of the ſecond; * here the Liquor that runs out may be looked upon as the Space gone through, becauſe the Holes are equal. *135 *136 *134

386 *Beſides the Irregularities from Friction, and the Reſiſtance of the Air, there are ſeveral others ariſing from the Coheſion of Parts, even in Liquors that are not glutinous.* I ſhall here only ſpeak of Water. We obſerve in Relation to it, that tho' it be driven by the ſame Force in any Direction, * the Height of the Water above the Hole remaining the ſame, yet it will deſcend the more ſwiftly in a vertical Direction; the Water in falling, is continually accelerated in its Motion, it coheres with the following, and accelerates that, and increaſes the Velocity of the Water flowing out of the Veſſel. *354

387 *Plate* XXIV. *Fig.* 2.] For this Reaſon *the Motion out of a Veſſel, that has a Tube fixed to its under Side, is alſo accelerated.* Let E be ſuch a Veſſel equal and ſimilar to the Veſſel A, and which, together with the Tube, makes up the Height of the Veſſel B: Let the Tube have the Holes at both Ends equal to the Holes at the Bottoms of the Veſſels A and B: Fill the Water in the Veſſels A, E, and B; in the Beginning of the Motion, the Water flows from the Veſſel E and B with equal Celerity, becauſe the Heights of the Water above the Holes, from which the Water goes out, are equal; but the Celerity in the Veſſel E, is immediately diminiſhed, becauſe there cannot run a greater Quantity of Water out of the Tube than what comes in at the upper Hole of the Tube, into which Hole no more Water

can run in, than what can flow out at the Veſſel A. Since the Parts of the Water cohere, the Water, which runs out, accelerates that which runs into the Tube, and this laſt retards that which runs out; and ſo the Quantity of Water, which in a certain Time runs out of the Veſſel E, is a mean Quantity between the Quantities of Water that can run out at the ſame Time from the Veſſels A and B.

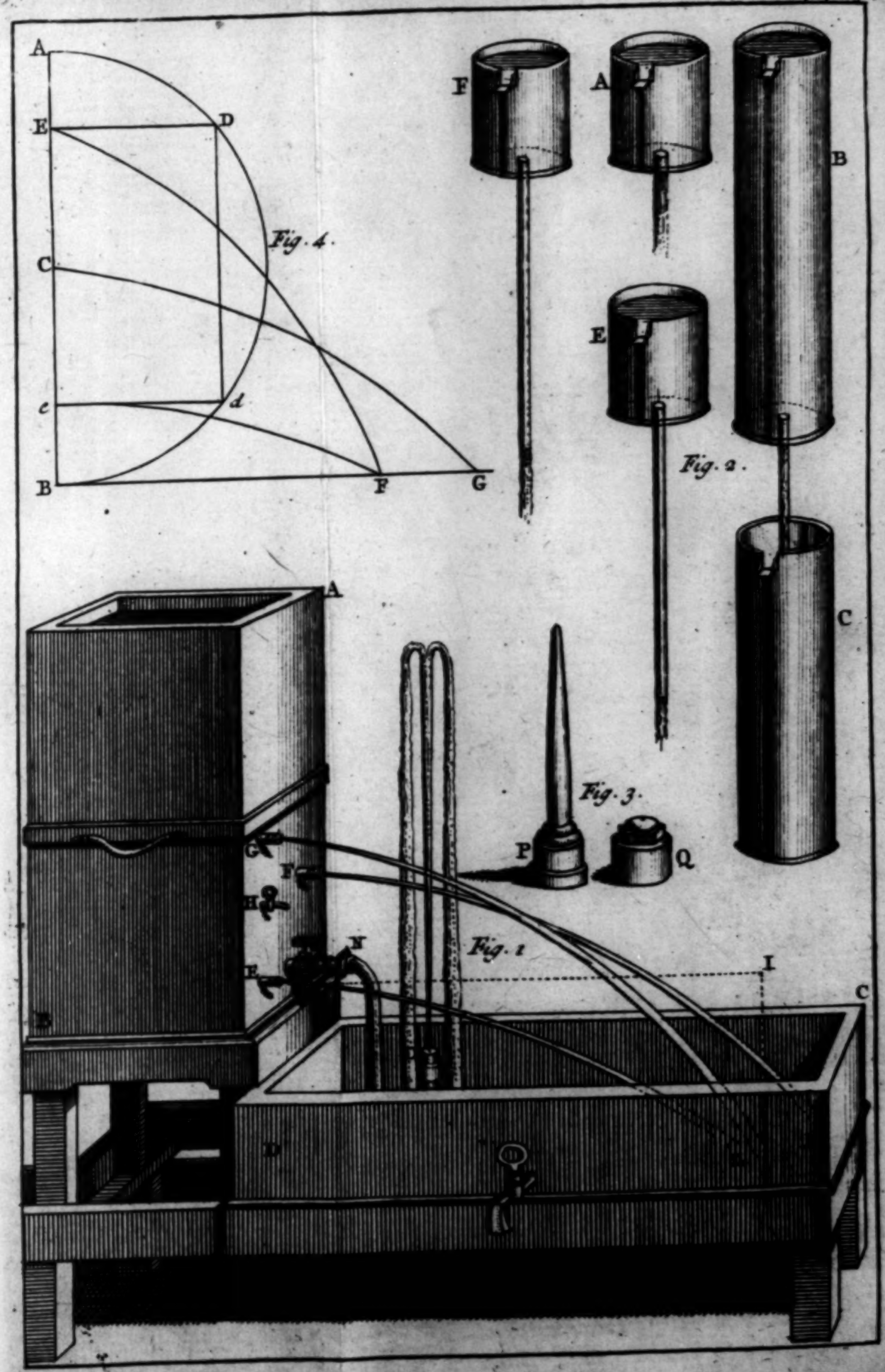
Experiment 2.] The Veſſels AE and B being made of ſome thin Metal, in the Proportions above-mentioned, fill with Water A and E; having opened the Holes at the ſame Inſtant of Time, the Water of the Surface at E will deſcend faſter than that at A: On the contrary, if you make uſe of the Veſſels E and B, it will deſcend faſter in the laſt than in the firſt.

388 *Let the upper Hole of the Tube, by which it communicates with the Veſſel, remain as before; and the lower Hole be opened wider; then a greater Quantity of Water will flow out*, and the Water which goes into the Tube will be more accelerated; this Hole may be made ſufficiently wider without altering the Length of the Tube, inſomuch that a greater Quantity of Water will flow out from it than from the Veſſel B. In that Caſe thro' the upper Hole of the Tube, at a ſmall Depth below the Surface of the Water, there flows out a greater Quantity of Water, than from an equal Hole four times the Depth. The ſame may be done by applying a longer Tube, without widening its lower Hole.

Experiment 3. *Plate* XXIV. *Fig.* 2.] Take the Veſſel F no way different from the Veſſel E, but in having the lower Hole of its Tube bigger; take alſo the above-mentioned Veſſel B: The

Diameters

Plate. 24. p. 194.



Diameters of the Hole in the Bottom of this, and of the upper Hole of the Tube which is joined to F, are of four Lines ($\frac{1}{42}$ of an Inch) the lower Hole of this Tube is of five Lines. Let the Veſſels be filled with Water; and let the Water begin to run out of both at the ſame Moment; the Surface of the Water in F will deſcend faſter than that of B. The Veſſel B is about 16 Inches high.

CHAP. X.

Of the Running of Rivers.

DEFINITION I.

389 *THE Water that runs by its own Gravity, in a Channel open above, as* A E, *is called* a River (*Plate* XXV. *Fig.* 1.)

DEFINITION II.

390 A River is ſaid to remain in the ſame State, or to be in a permanent State, *when it flows uniformly, ſo as to be always at the ſame Height in the ſame Place.*

DEFINITION III.

391 *A Plane, which cutting a River is perpendicular to the Bottom,* as *p o n q, is called* the Section of a River.

When a River is terminated by flat Sides parallel to each other, and perpendicular to the Horizon, and the Bottom alſo is a Plane either horizontal or inclined, the Section of the River with theſe three Planes makes Right Angles, and is a Parallelogram.

392 *In every River that is in a permanent State, the ſame Quantity of Water flows in the ſame Time thro' every Section.* For unleſs there be in every Place

as great a Supply of Water, as what runs from it, the River will not remain in the ſame State. And this Demonſtration will not hold good, whatever be the Irregularity of the Bed or Channel, from which, in another Reſpect, ſeveral Changes in the Motion of the River ariſe; as, for Example, a greater Friction in Proportion to the greater Inequality of the Channel.

The Irregularities in the Motion of a River may be infinitely varied, and Rules cannot be given to ſettle them. Therefore ſetting aſide all Irregularities, we muſt examine the Courſe of Rivers; for, unleſs the Laws of Motion be known in that Caſe, we have no certain Foundation for determining any Thing.

Therefore, we ſuppoſe the Water to run in a regular Channel, without any ſenſible Friction, and that the Channel is terminated with plane Sides, that are parallel to one another, and vertical; and alſo that the Bottom is a Plane, and inclined to the Horizon.

Let A E be the Channel, into which the Water runs from a greater Receptacle or Head; and let the Water always remain in the ſame Height at the Head, ſo that the River may be in a permanent State. The Water deſcends along an inclined
144 Plane, and is accelerated; whereby, becauſe
the ſame Quantity of Water flows through every
392 Section, *The Height of the Water, as you re-*
393 *cede from the Head of the River, is continually diminiſhed*, and the Surface of the Water will acquire the Figure *i q s*.

To determine the Velocity of the Water in different Places, let us ſuppoſe the Hollow of the Channel A D C B to be ſhut up with a Plane; if there be a Hole made in the Plane, the Water will ſpout the faſter through the Hole, as the Hole is more diſtant from the Surface of the

Water *bi*; and the Water will have the ſame Celerity that a Body, falling from the Surface of the Water to the Depth of the Hole below it, would acquire*; which ariſes from the Preſſure *357 of the ſuperincumbent Water. There is the ſame Preſſure, that is, the ſame moving Force, when the Obſtacle at AC is taken away; then every Particle of Water enters into the Channel, with the Celerity that a Body would acquire in falling from the Surface of the Water to the Depth of that Particle. This Particle is moved along, in an inclined Plane in the Channel, with an accelerated Motion; and that in the ſame Manner, as if, in falling vertically, it had continued its Motion to the ſame Depth below the Surface of the Water in the Head of the River*. So, if you *150 draw the horizontal Line *it*, the Particle at *r* will have the ſame Celerity as a Body falling the Length *i*C, and running down C*r*, can acquire; which is the Celerity acquired by the Body in falling down *tr*. Therefore the Celerity of a Particle may be every where meaſured, drawing from it a Perpendicular to the horizontal Plane, which is conceived to run along the Surface of the Water in the Head of the River; and the Velocity, which a Body acquires in falling down that Perpendicular, will be the Celerity of the Particle; which is greater, the longer the Perpendicular is. From any Point, as *r*, draw *rs* perpendicular to the Bottom of the River, which will meaſure the Height or Depth of the River. Since *rs* is inclined to the Horizon, if from the ſeveral Points of that Line you draw Perpendiculars to *it*, they will be the ſhorter, the more diſtant they are from *r*, and the ſhorteſt of them all will be *ſv*: Therefore the Celerities of the Particles in the Line *rs* are ſo much the leſs, the nearer they are to the Surface of the

River,

394 River, *and the lower Water is moved faſter than the upper Water.*

395 *But yet the Celerities of thoſe Waters, as the River runs on, continually approach nearer and nearer to an Equality.* For the Squares of thoſe Celerities are as rt to sv, the Difference of which Lines, as you recede from the Head of the River, *393 is continually leſſened, becauſe of the Height rs*, which is alſo continually diminiſhed as the Lines themſelves are lengthened. Now as this obtains in the Squares, it will much more obtain in the Celerities themſelves, whoſe Difference therefore is diminiſhed as they increaſe.

396 If the Inclination of the Bottom be changed at the Head of the River, ſo as to become yz, *and a greater Quantity of Water flows into the Channel, it will be higher every where in the River, but the Celerity of the Water is no where changed.* For this Celerity does not depend upon the Height of the Water in the River, but, as has been demonſtrated, from the Diſtance of the moved Particle from the horizontal Plane of the Surface at the Head continued over the ſaid Particle; which Diſtance is meaſured by the Perpendicular rt to sv; but theſe Lines are not changed by the Afflux of Water, provided that the Water remains at the ſame Height in the Baſon or Head.

397 *Let the upper Part of the Channel be ſtopped up by an Obſtacle, as* X, *which deſcends a little Way below the Surface of the Water*; the whole Water which comes cannot run through, therefore it muſt riſe up: But the Celerity of the Water below this *396 Cataract is not increaſed*; and the Water that comes on is continually heaped up, ſo that at laſt it muſt riſe ſo as to flow over the Obſtacle or the Banks of the River. But *if the Banks be raiſed, and the Obſtacle be continued, the Height of the Water would riſe above the Line* it; but, before

that, the Celerity of the Water cannot be increaſed: In which Caſe the Height of all the Water in the Head will be increaſed; for as we ſuppoſe the River in a permanent State, there muſt continually be as great a Supply of Water to the Head, as there runs from it down the Channel; but, if leſs Water runs down, the Height muſt neceſſarily be increaſed in the Head, till the Celerity of the Water, flowing under the Obſtacle, be ſo much increaſed, that the ſame Quantity of Water ſhall run under the Obſtacle, as uſed to run in the open Channel before.

All theſe Things, as we have already ſaid, if we abſtract from all the Irregularities, are true; and, the leſs the Irregularities are, the more will the true Motions agree with what we have ſaid: Concerning which, before we can make any Judgment, we muſt be able to compare the Velocities of Water by Experiments, and ſo determine the Velocities themſelves, as to know the Spaces gone through in a certain Time.

Plate XXV. *Fig.* 2.] Let ACB be a Quadrant divided into Degrees*, with a Thread in *398
the Center, that has at the other End a Ball P hanging, which is heavier than the Water.

Let the Ball hang within the running Water, 399
whilſt you hold the Side CA of the Quadrant in a vertical Poſition; the Ball by the Motion of the Water will be ſo far ſuſtained, that the Thread PC will make the Angle PCA with the Side CA, which will ſerve to determine the Celerity of the Water running againſt the Ball.

The Ball, being at reſt in the Water, is drawn by three Powers; by its Gravity it endeavours to deſcend vertically; by the Action of the Liquid it is carried in the Direction of the Motion of the Water; and laſtly, it is drawn by the Thread along P C. Draw the Triangle E F G, in which E F repreſents the vertical Line ; let F G make with that Line the Angle E F G, equal to the Angle which the Direction of the Motion of the River makes with the vertical Line ; laſtly, let the Angle G E F be equal to the Angle P C A. The Sides of the Triangle E F G are parallel to the Directions of the three Powers above-mentioned ; therefore the Powers are to one another, as thoſe *196 Sides*. If therefore E F expreſſes the reſpective Gravity of the Ball, F G will expreſs the Action of the Water on the Ball. If you make ſeveral Experiments in different Places with the ſame Ball, you muſt draw ſuch Triangles, the Side F remaining, (which denotes the reſpective Gravity of the Ball that never changes) the Sides that are as F G, will have the ſame Proportion as the Actions of the Water on the Ball. But theſe are as the Squares of the Velocities of the Waters in the *324 Places in which the Experiments are made*; for there is no Difference in reſpect of the Action of the Water on the Ball, whether the Ball be moved, and the Water at reſt, or, on the contrary, the Water be moved, and the Ball at reſt.

The Action of the Water againſt the Ball may be compared with the Weight ; for it is to the reſpective Gravity of the Ball, as F G to E F.

400 But this Action is equal to the Reſiſtance which a Body ſuffers, when it is moved through quieſcent Water with the ſame Celerity with which the flowing Water does now ſtrike againſt the Body which is at reſt : By knowing the Weight, which is equal to the Reſiſtance, we know what Space could be run through in a given Time, with the Celerity with which the *336 Body moves*; therefore we ſhall alſo here know what Space the Water can go through in a known Time, and ſo likewiſe what Quantity of

Water flows in a given Time through a Place given in the Section of the River.

Here it is to be obſerved, that the Determination of the Velocity of the Water will not be exactly ſettled, if the Experiment be made towards the Surface of the Water, becauſe there the Action of the Water upon the Globe is irregular*. *328

401 This Celerity may be determined by immerging in Water a Body which is but a little lighter than Water, and which, ſwimming at the Surface, does not float ſo high above it, as to be affected by the Motion of the Wind; as the ſpecifick Gravities of the Water and the Body ſcarce differ at all, and this Body may be looked upon as wholly immerſed, it will move with the ſame Celerity as the Water; and you may, by Help of a Pendulum, meaſure the Time, in which a Body runs through a certain Space that was meaſured before. When the Surface of the Water is agitated by the Wind, the Experiment will not ſucceed well, becauſe of the Motion of the Waves, which cauſe an Irregularity in the Motion of the Body.

CHAP. II.

Of the Motion of the Waves.

402 THE Surface of the ſtagnant Water is plain, and parallel to the Horizon*; if it becomes hollow at A (*Plate* XXV. *Fig.* 3.) upon any Account whatever, this Cavity is ſurrounded with the Elevation B B; this raiſed Water deſends by its Gravity, and, with the Celerity acquired in deſcending, it forms a new Cavity, by which Motions the Water aſcends at the Sides *272

of this Cavity, and fills the Cavity A, whilſt there is a new Elevation towards C; and, when this laſt is depreſſed, the Water riſes anew towards the ſame Part; whence there ariſes a Motion in the Surface of the Water, and a Cavity, which carries an Elevation before it, is moved from A towards C.

DEFINITION I.

403 *This Cavity, with the Elevation next to it, is called* a Wave.

DEFINITION II.

404 The Breadth of a Wave *is the Space taken up by a Wave in the Surface of the Water, and meaſured according to the Direction of the Wave's Motion.*

The Cavity, as A, is encompaſſed every Way with an Elevation, and the Motion above-mentioned expands itſelf every Way; therefore the
405 *Waves are moved circularly.*

Plate XXV. *Fig.* 4.] Let AB be an Obſtacle, againſt which the Wave, whoſe Beginning is at C, does run; we muſt examine, what Change the Wave ſuffers in any Point, as E, when it is come to the Obſtacle in that Point. In all Places through which the Wave runs, whilſt it goes forward its whole Breadth, the Wave is raiſed; then a Cavity is formed, which is again filled up; which Change while the Surface of the Water undergoes, its Particles go and come through a ſmall Space. The Direction of this Motion is along CE, and the Celerity may be repreſented by that Line; let this Motion be conceived to be reſolved in two other Motions along GE and DE, whoſe Celerities are reſpectively repreſented by thoſe Lines*. By the Motion along
*192

long D E the Particles do not act againſt the Obſtacle, and after the Stroke continue the Motion in that Direction, with the ſame Celerity; and this Motion is here repreſented by E F, ſuppoſing E F and E D to be equal to one another, by the Motion along G E, the Particles ſtrike directly againſt the Obſtacle, and this Motion is deſtroyed*; for though theſe Particles are elaſtick, *168 yet, as in the Motion of the Waves they run thro' but in a ſmall Space, going backward and forward, they are moved ſo ſlowly, that the Figure of the Particles cannot be changed by the Blow, and ſo they are ſubject to the Laws of Bodies perfectly hard. But there is a Reflexion of the Particles from another Cauſe; the Water which cannot go forward beyond the Obſtacle, and is puſhed on by that which follows it, yields that Way where there is the leaſt Reſiſtance, that is, aſcends: And this Elevation greater than in other Places, is cauſed by the Motion along G E, becauſe 'tis by that Motion alone that the Particles come againſt the Obſtacle. The Water, by its Deſcent, acquires the ſame Velocity with which it was raiſed; and the Particles of Water are repelled from the Obſtacle with the ſame Force in the Direction E G, as that with which they came againſt the Obſtacle. From this Motion and the Motion above-mentioned along E F, ariſes a Motion along E H, whoſe Celerity is expreſſed by the Line E H, which is equal to the Line C E; and by the Reflexion the Celerity of the Wave is not changed, but it returns along E H in the ſame Manner, as if, taking away the Obſtacle, it had moved along E *b*. If from the Point C, C D be drawn perpendicular to the Obſtacle, and then produced, ſo that D *c* ſhall be equal to C D, the Line H E continued will

go through *c*; and as this Demonſtration holds good in all the Points of the Obſtacle, it follows,
406 that *the reflected Wave has the ſame Figure on that Side of the Obſtacle, as it would have had beyond the Line* A B, *if it had not run againſt the Obſtacle.*
407 *If the Obſtacle be inclined to the Horizon,* the Water riſes and deſcends upon it, and ſuffers a Friction, whereby *the Reflexion of the Wave is diſturbed, and often wholly deſtroyed.* This is the Reaſon why very often the Banks of Rivers do not reflect the Waves.

408 *When there is a Hole, as* I, *in an Obſtacle, as* B L, the Part of the Wave which goes through the Hole, continues its Motion directly, and expands itſelf towards Q Q, and *there is a new Wave formed, which moves in a Semicircle, whoſe Center is the Hole.* For the raiſed Part of the Wave, which firſt goes through the Hole, immediately flows down a little at the Sides, and then, by deſcending, makes a Cavity, which is ſurrounded with an Elevation on every Part beyond the Hole, which moves every Way in the ſame manner as was ſaid concerning the Generation of the firſt
402 Wave.

409 In the ſame manner a Wave, to which an Obſtacle, as A O, is oppoſed, continues to move between O N; but expands itſelf towards R in a Part of a Circle, whoſe Center is not very far from O.

410 Hence we may eaſily deduce what muſt be the Motion of a Wave behind an Obſtacle, as M N.

411 *Waves are often produced by the Motion of a tremulous Body, which alſo expand themſelves circularly,* tho' the Body goes and comes in a Right Line; for the Water, which is raiſed by the Agitation, deſcending, forms a Cavity, which is every where ſurrounded with a Riſing.

Different Waves do not diſturb one another, when they move according to different Directions. The Reaſon of which Effect is, that whatever Figure the Surface of the Water has acquired by the Motion of the Waves, there may in that be an Elevation and a Depreſſion, as alſo ſuch a Motion as is required in the Motion of a Wave. 412

Whoever has attentively conſidered the Motion of the Waves, will find that all theſe Things agree with Experiments.

To determine the Celerity of the Waves, another Motion, analogous to their Motion, is to be examined. Let there be a Liquid *in the recurve cylindric Tube* EH (*Plate* XXV. *Fig.* 5.) and let the Liquid in the Leg EF be higher than in the other Leg by the Diſtance *l*E; which Difference is to be divided into equal Parts at *i*. The Liquid by its Gravity deſcends in the Leg EF, whilſt it aſcends equally in the Leg EH; and ſo, when the Surface of the Liquid is come to *i*, it is at the ſame Height in both Legs, and that is the only Poſition in which the Liquid can be at reſt: But, by the Celerity acquired by deſcending, it continues its Motion, and aſcends higher in the Tube GH, and in EF it is depreſſed quite to *l*, except ſo much as it is hindered by the Friction againſt the Sides of the Tube. The Liquid in the Tube GH, which is higher, alſo deſcends by its Gravity; and ſo the Liquid in the Tube riſes and falls, till it has loſt all its Motion by the Friction. 413

The Quantity of the Matter to be moved is the whole Liquid in the Tube; the moving Force is the Weight of the Pillar *l*E, whoſe Height is always double the Diſtance E*i*; which Diſtance therefore increaſes and diminiſhes in the ſame Ratio with the moving Force. But the Diſtance E*i* is the Space to be run through by the Liquid, that from the Poſition E H it may come to the Poſition of Reſt; which Space therefore is always as the Force which continually acts upon the Liquid: But we have demonſtrated, that it is upon this Account, that all the Vibrations of a Pendulum, oſcillating into a Cycloid, are performed in the ſame Time; and * therefore here alſo, *whatever be the Inequality of the Agitations, the Liquid always goes, or comes, in the ſame Time.* *156

414 *The Time in which a Liquid, thus agitated, aſcends, or deſcends, is the Time in which a Pendulum vibrates, whoſe Length*, that is, the Diſtance between the Center of Oſcillation and Suſpenſion, *is equal to Half the Length of the Liquid in the Tube*, or to Half the Sum of the Lines E F, F G, G H. This Length is to be meaſured in the Axis of the Tube.

Plate XXV. *Fig.* 6.] Let ſuch a Pendulum vibrate in a Cycloid, in the manner explained above (*Page* 62 *and* 63.) Let the Pendulum B C and the Arc A D be of the ſame Length; for the Arc C A is equal to the Arc A D; and the Thread, by which the Pendulum is ſuſpended, applies to it, when the Body ſuſpended is at A; in that Point the Direction of the Curve is perpendicular to the Horizon, and the Body endeavours to deſcend with all its Weight along the Curve: But this Weight is to the Force acting upon the Body, when it is at P, as A D or P C to P D*. Now let the Liquid be in ſuch a Poſition, that *i*E (*Fig.* 5.) be equal to P, D; the Weight of the whole Matter to be moved, that is, of the whole Liquid, is to the Weight *l*E (which is the Force acting upon the Liquid in that Poſition) as the Length of the Liquid in the Tube to the Line *l*E, in which Ratio alſo the Halves of thoſe *156

Quantities are, that is, PC to PD (*Fig. 6.*) Therefore in the Pendulum the Weight of the Matter to be moved is to the Force acting upon it, at P, as in the Tube, the Weight of the Matter to be moved is to the Force acting upon it in the Poſition EH. Therefore the pendulous Body and the Liquid, in this Caſe are acted upon by equal Forces; and this always obtains where the Spaces to be run through by the Liquid in Agitation, and by a Body in Vibration, are equal; therefore in this Caſe the Agitation and the Vibration are performed in the ſame Time, and not only in this Caſe, but always*. *413 But, as the ſmall Vibrations in a Circle do not differ from the Vibrations in a Cycloid, the Demonſtration will agree with them.

Experiment.] Take a cylindric recurve Tube, as EFGH; let the Length of the Legs be one Foot, and the Bore of the Cylinder half an Inch; pour Mercury into this Tube; and having made a Pendulum, whoſe Length is equal to Half the Length of the Cylinder of Mercury in the Tube; if the Mercury be agitated in the Tube, it will aſcend and deſcend in the ſame Time as the Pendulum will go and come.

Plate XXV. *Fig.* 7.] To determine the Celerity of the Waves from what has been ſaid, we muſt conſider ſeveral equal Waves that follow one another immediately, as A, B, C, D, E, F, which move from A towards F, the Wave A runs its Breadth, when the Cavity A is come to C; which cannot be, unleſs the Water at C aſcends to the Height of the Top of the Waves, and again deſcends to the Depth C; in which Moment the Water is not agitated ſenſibly below the Line *b i*; therefore this Motion agrees with the Motion

tion in the Tube above-mentioned, and the Water aſcends and deſcends; that is, the Wave goes through its Breadth, whilſt a Pendulum, of the Length of half B C, performs two Oſcillations,
*414 * or whilſt a Pendulum of the Length B C D, that is four times as long as the firſt, performs
158 one Vibration.

Therefore the Celerity of a Wave depends upon the Length of a Line B, C, D, which is greater, according as the Breadth of the Waves is greater, and as the Water deſcends deeper in the Motion of the Waves.

415 In the broadeſt Waves, which do not riſe high, ſuch a Line as B C D does not much differ from the Breadth of the Wave; and in that Caſe *a Wave runs through its Breadth, whilſt a Pendulum, equal to that Breadth, oſcillates once*. In every equable Motion, the Space gone through increaſes with the Time and the Celerity; wherefore multiplying the Time by the Celerity, you have the Space gone thro': Whence it follows, that the *Celerities*
416 *of the Waves are as the Square Roots of their Breadths*: For as the Times in which they go
158 through their Breadths are in that Ratio, the
415 ſame Ratio is required in their Celerities, that the Products of the Times, by their Celerities, may be as the Breadth of the Waves, which are the Spaces gone through.

All theſe Things muſt be only looked upon as nearly true, becauſe the Motion of the Waves differs ſomething from the Motion in the Tube; which Error is in part taken off, becauſe the Length of the Pendulum is meaſured along the inclined Lines B C and C D.

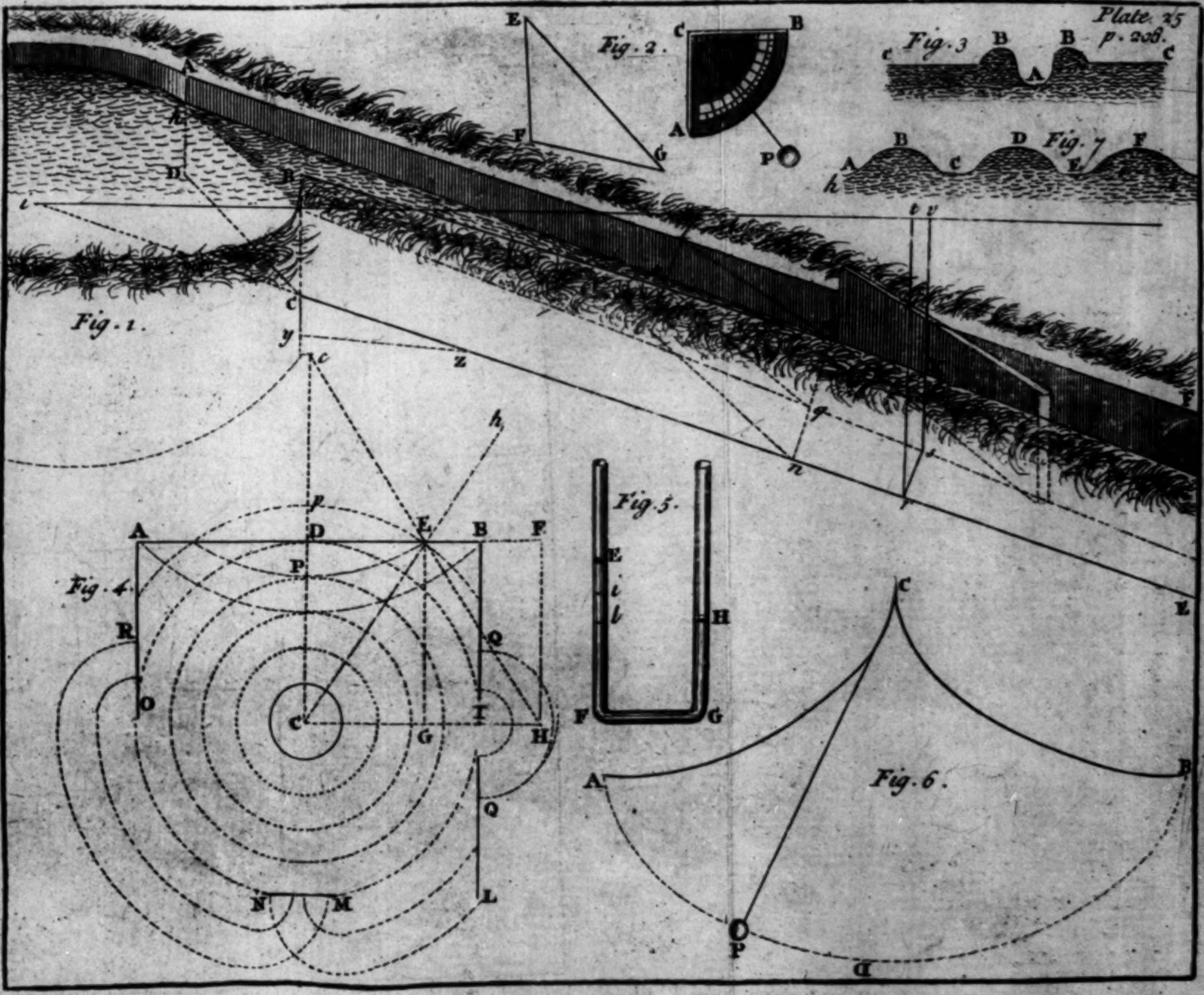


Plate 25 p. 208.

PART III. *Of the Air, as an Elaſtic Fluid.*

CHAP. XII.

That Air has the Properties of Fluids.

WE have often ſpoken of the Air; and as we live in, and are always encompaſſed by it, we muſt have regard to its Effect in ſeveral Experiments, as we have ſaid in other Parts of this Treatiſe: But now we ſhall conſider its Properties ſingly.

The Air is corporeal, heavy, its Parts yield to any Force impreſſed, and are very eaſily moved one amongſt another; it preſſes in Proportion to its Height, and the Preſſure every way is equal: It is plain therefore, that *it ought to be reckoned amongſt Fluids.* 417

DEFINITION I.

All the Air which the Earth is encompaſſed with, conſidered together, is called the Atmoſphere of the Earth, *or ſimply,* the Atmoſphere. 418

DEFINITION II.

The Height of the Air above the Surface of the Earth, is called the Height of the Atmoſphere. 419

That the Air is a Body, appears from its excluding all other Bodies from the Place where it is*. 420

421 *That it yields to any Impreſſion, and has its Parts eaſily moved*, is not doubted by any one.

422 *That it is heavy*, is proved by its preſſing upon the Surface of other Fluids, and ſuſtaining them in Tubes.

Experiment 1. *Plate* XXVI. *Fig.* 1.] Take a Glaſs Tube A B, about three Foot long, of about ¼ Inch Bore; if you ſtop up the End A, and let the Tube be filled with Mercury, and let the other End be immerſed in a Veſſel full of Mercury, the Mercury will be ſuſtained at the Height of about 29 Inches. This is occaſioned by the Preſſure of the Air on the Surface of the Mercury in the Veſſel, which cannot preſs equally in every Part of it, unleſs in the Tube where no Air is there be a Column of Mercury which preſſes equally with outward Air*. *274

Experiment 2. *Plate* XXVI. *Fig.* 1.] That this Preſſure may not be changed when the Tube is inclined, it is required that the Mercury ſhould keep the ſame perpendicular Height* *277: If therefore there be two Veſſels containing Mercury, in which Tubes in the Manner above-mentioned are immerſed, of which E D is inclined to the Horizon, the Mercury is ſuſtained at the Heights *b f* and *i g*, ſo that *f* and *g* are in the ſame Horizontal Lines, ſuppoſing the Surfaces of the Mercury in the Veſſels to lie in the ſame Plane.

Experiment 3. *Plate* XXVI. *Fig.* 2.] The ſame Preſſure of Air ſuſtains the Water of the Glaſs U, which is yet immerged in Water, and filled with it, and then is pulled out all but the Orifice, which ſtill remains immerſed.

423 Water would be ſuſtained in the ſame Manner, tho' the Height ſhould be 32 Foot; for Quick-

ſilver is 14 times heavier than Water, and a Pillar of Water, a little more than 32 Foot high, preſſes equally with a Column of Mercury 29 Inches high; which Preſſure is equal to the Preſſure of the Atmoſphere.

That the Preſſure of the Air depends upon its Height, may be eaſily deduced from what has been ſaid; but it is immediately proved by carrying the Tube with the Mercury above-mentioned to a higher Place; for when you carry this Machine up a Hill, for 100 Foot that you riſe perpendicularly, the Mercury deſcends a Quarter of an Inch. 424

That Air preſſes equally every way, appears from this, that the Preſſure is ſuſtained by ſoft Bodies, without any Change of Figure; and brittle Bodies, without their breaking, though this Preſſure be equal to the Preſſure of a Pillar of Mercury 29 Inches high, or a Height of Water of 32 Foot *; any Body may ſee that nothing can preſerve theſe Bodies unchanged, but the equal Preſſure on all Parts; but it is plain that the Air does preſs in that Manner. * If you take away the Air on one Side, the Preſſure is ſenſible on the oppoſite Side. 425 * 423 * 291

Exper. 4. *Plate* XXVI. *Fig*. 3.] Hang a Glaſs Tube to one of the Scales of a Ballance A B, which is ſhut at D, and 3 Foot long; fill this Tube with Mercury, and let the End E be immerſed in the Mercury that is contained in the Veſſel U. The Mercury, by the Air's Preſſure, is ſuſtained at the Height *f* in the Tube, and the upper Part of the Tube *f* D is left void of Air: To make an Æquilibrium, you muſt put into the oppoſite Scale a Weight equal to the Weight of the Tube and the Mercury contained in it. The Mercury in the Tube cannot preſs the Ballance;

for its Action againſt the Sides of the Tube is horizontal; but the Air acts upon the upper Part of the Tube, and the Column that is ſuſtained by the Tube is æquiponderate with the Column of Mercury that is contained in the Tube: If letting the Mercury run out, you ſuffer the Air to come in, then nothing but the Tube weighs down the Scale; which proves, that the Action againſt the inferior Surface of the upper Part of the Tube deſtroys the Action on the exterior Surface, and that the Air preſſes upwards and downwards with the ſame Force.

By this Experiment alſo is confirmed what has been ſaid of the Air's Gravity.

CHAP. XIII.

Of the Air's Elaſticity, or Spring.

WE have ſhewn, the Air has the Properties of other Liquids; but beſides, it has another Property, which is, that it can take up a greater or leſſer Space, according as it is compreſſed with a different Force; and, as ſoon as that Force is diminiſhed, it expands itſelf.

426 By reaſon of the Analogy of this Effect with the Elaſticity of Bodies, this Property of the Air is called its *Elaſticity*.

427 That *the Air may be compreſſed*, appears from
14 an Experiment already mentioned.

428 That *it may be dilated*, may appear from the following

Experiment 1. *Plate* XXVI. *Fig.* 4.] Take the Tube AB cloſe at the End A, and pour Mercury into it, ſo that there may be ſome Air left in the Tube, which, when in the State of the external Air, will take up the Space A *l*; if the End B of the Tube be immerſed into Mercury in a Veſſel, the Mercury in the Tube will deſcend to *g*, and there remain. The Height *ig* differs very much from the Height of the Mercury in the firſt Experiment of the foregoing Chapter, which does not ariſe from the Weight of the Air in the Tube; for its Weight is too little to produce any ſenſible Difference in the Height of the Mercury: The Expanſion of the Air cauſes this Effect.

From this Experiment we deduce this Rule, That *the Air dilates itſelf in ſuch a manner, that the Space taken up by it is always inverſely as the Force by which it is compreſſed.* 429

The Force, by which the common or external Air is compreſſed, is the Weight of the whole Atmoſphere, which is equal to a Pillar of Mercury of the Height *bf*, *Fig.* 1. therefore the compreſſing Force may be expreſſed by that Height; the Space taken up by the Air in the Tube, when it is compreſſed with ſuch a Force, is A *l*.

But in the laſt Experiment, the Preſſure of the Atmoſphere exerts two Effects; it ſuſtains the Pillar of the Mercury *ig*, and it reduces the Air in the Tube to the Space *g* A; if the Force, by which the Mercury is ſuſtained at the Height *gi*, be ſubſtracted from the Preſſure of the whole Atmoſphere; that is, if the Height *gi* be taken from the Height *bf*, (*Fig.* 1.) there remains the Force by which the Air is compreſſed in the upper Part of the Tube; but this Difference of the Heights of the Mercury *bf*, and *gi*, is always to *bf*, as A *l* to A *g*, that is, their Forces are inverſely as the Spaces.

This Rule alſo obtains in compreſſed Air.

Experiment 2. *Plate* XXVI. *Fig.* 5.] Take a curve Tube A B C D, open at A, and ſhut at D; let the Part B C be filled with Mercury, ſo that the Part C D may contain Air of the ſame State or Tenor as the external Air; therefore, the compreſſing Force is the Column of Mercury, whoſe Height is *b f*, *Fig.* 1. and by this Height muſt this Force be expreſſed, as in the foregoing Experiment; but the Space taken up by the Air is C D. Pour Mercury into the Tube A B, that it may riſe up to *g*, the Air will be reduced to the Space *e* D: Now the compreſſing Force acts as ſtrongly as a Column of Mercury of the Height *f g*, and alſo the Preſſure of the external Air upon the Surface *g* of the Mercury; this Force is expreſſed by the Sum of the Heights *f g* in this Figure, and *b f*, in *Fig.* 1. This Sum is always to *b f* (*Fig.* 1.) as C D to *e* D; and again the Forces are inverſely as the Spaces.

430 *The Elaſticity of the Air is as its Denſity*; for this laſt is inverſely as the Space taken up by the
* 489 Air*; therefore, as the Force compreſſed the
* 429 Air*, which is equal to that by which the Air endeavours to expand itſelf; but this Force is its Elaſticity.

Hence it follows, that the Air in which we live is reduced to the Denſity which it has near the Earth, by the Preſſure of the ſuperincumbent Air, and that it is more or leſs compreſſed, according to the greater or leſs Weight of the Atmoſphere; for which Reaſon alſo the Air is leſs denſe at the Top of a Mountain than a Valley, as being compreſſed by a leſs Weight.

How far this Property of expanding itſelf is extended, we do not certainly know; and it is very probable that it can be determined by no Ex-

Experiments. Nevertheleſs, if you compare the following Experiments with the Experiment of the Air compreſſed in a Pump *, it will appear * 14 that the Air may take up twenty thouſand times more Space in one Caſe than in the other.

Experiment 3. *Plate* XXVI. *Fig.* 6.] Let the Glaſs AB, about fourteen Inches high, be exactly filled with Water: It has a braſs Cap fixed to it at the End B, by which it is to be ſcrewed to the Pump that is repreſented in *Plate* XXIX. *Fig.* 6. by drawing out the Piſton of the Pump, the Water deſcends into it by its Gravity; and the Place in the upper Part of the Veſſel is void both of Air and Water. The Air-bubbles in the Water, which are now compreſſed, becauſe the Air does not act upon the Surface of the Water, expand themſelves, and riſe up to the Surface of the Water; in that Motion the Bubbles are accelerated, ſo as not to be ſeen diſtinctly near the Surface, upon account of their very ſwift Motion; they alſo grow bigger as they aſcend, and if you compare the Diameter of a Bubble at B with its Diameter when it is come almoſt up to the Surface of the Water, but ſo far from it as to be ſeen diſtinctly, its Diameter is at leaſt four times as great as before.

The upper Part of the Glaſs, as was ſaid before, is entirely void of Air; for the ſmall Quantity of Air, which is continually going out of the Water, is not to be taken notice of here; therefore the Air-bubbles near B, which is about a Foot below the Surface of the Water, are compreſſed only by the ſuperincumbent Water; which Preſſure is to the Preſſure of the Atmoſphere, nearly as one to thirty-two; * in which Ratio * 423 alſo is the Space taken up by the Air, when it is compreſſed by the whole Atmoſphere, to the

Space taken up in the Bubbles above-mention-
429 ed; their Diameter in their Aſcent, as has been ſaid before, becomes quadruple; that is, the Bubble becomes 64 times bigger than it was; and ſo the Space taken up by the Air, in this laſt Caſe, is to the Space taken up by the Air, when compreſſed by the Atmoſphere, as 64 times 32 (that is 2048) to 1. The Air compreſſed by the Atmoſphere is reduced to a Space 10 times leſs in a Forcing-Pump; and ſo the Denſity of the Air above-mentioned is to the Denſity of this Air, as 1 to 20480. Extracting the Cube Roots of theſe Numbers, we ſhall find that the Diſtances between the Center of the Particles, in theſe two Caſes, are as 1 to 27.

Hence we conclude, that the Particles of Air are not of the ſame Nature with other elaſtic Bodies; for the ſingle Particles cannot expand themſelves every way into 27 times the Space, and ſo be increaſed 2000 times, preſerving their Surface free from every Inequality or Angle; for in every Expanſion or Compreſſion, the Parts are eaſily moved one amongſt another; but as the
431 Air may be dilated much more than in this Experiment, it follows, that *the Air conſiſts of Particles which do not touch one another, and that repel each other.* We have ſhewn, that in ſeveral Caſes there are Particles endowed with ſuch a Proper-
* 40 ty*; and it is plain enough, that it obtains here; but we are entirely ignorant of the Cauſe of this Force, and it muſt be looked upon as a Law of Nature, as is plain from what has been ſaid between *Numb.* 4 and *Numb.* 5.

432 *The Force, by which the Particles of the Air fly from each other, increaſes in the ſame Ratio as the Diſtance in which the Centers of the Particles are diminiſhed*; that is, that Force is inverſely as this Diſtance. To demonſtrate which, let us conſider two

equal Cubes A and B (*Plate* XXVI. *Fig.* 7.) containing unequal Quantities of Air; let the Diſtances between the Center of the Particles be as 2 to 1, the Numbers of the Particles will be in the ſame, but inverſe Ratio, in the Lines *d e* and *b i*; the Numbers of the Particles acting upon the Surfaces *d g* and *h m*, are as 1 to 4, namely, as the Squares of the Numbers of the Particles in equal Lines, and as the Cubes of thoſe Numbers, that is, as 1 to 8, ſo are the Quantities of Air contained in the Cubes; in which Ratio alſo are the Forces compreſſing the Air in the Cubes*. *429
The Forces acting upon the equal Surfaces *dg* and *h m*, are as the Forces by which the Air is compreſſed*; they are alſo in a Ratio compounded of the Numbers of the Particles acting, and the Action of the ſingle Particles; therefore this compound Ratio is the Ratio of 1 to 8: The firſt of the compounding Ratio's, as has been ſaid, is that of 1 to 4; wherefore neceſſarily the ſecond is that of 1 to 2, which is the inverſe Ratio of the Diſtances of the Particles. And this Demonſtration is general; for by 1 and 8 we expreſs any Cubes whatever; by 1 and 4, the Squares of the Cube Roots; and laſtly, by 1 and 2, the Roots of thoſe Cubes. This Demonſtration proves that the Action, which the Particles continually ſuffer from all Sides, is increaſed between the Ratio in which the Diſtance of the Centers of the Particles is diminiſhed, whether the Action is to be referred only to neighbouring Particles, or alſo to thoſe which are more diſtant. In the firſt Caſe the repellant Force itſelf, which every Particle is endowed with, is as the Action above-mentioned, that is, inverſely as the Diſtance between the Centers of the Particles. *126

In the ſecond Caſe, the repellant Force is equal at all Diſtances; for then the Action againſt each Particle depends upon their Number in the ſame Line, which Number is inverſely as the Diſtance between the Number of the Particles. Then alſo, ſuppoſing the Air of the ſame Denſity, the Elaſticity will be the greater, where the Quantity of the Air will be the greater; but, as this does not agree with Experiments, therefore the firſt Cauſe muſt be true.

433 *The Effects of the Elaſticity of the Air are like thoſe of its Gravity*, and included Air acts by its Elaſticity, juſt as Air not included does by its Weight.

The Air which is loaded by the Weight of the whole Atmoſphere, preſſing every way from the very Nature of Liquids, and the Force which it exerts, does no way depend upon the Elaſticity; becauſe, whether you ſuppoſe Elaſticity or not, that Force which ariſes from the Weight of the Atmoſphere, and is equal to it, can be no way changed; but, as the Air is elaſtic, it is reduced to ſuch a Space by the Weight of the Atmoſphere, that the Elaſticity, which re-acts againſt the
126 compreſſing Weight, is equal to that Weight. But the Elaſticity increaſes and diminiſhes as the
432 Diſtance of the Particles diminiſhes or increaſes; and it is no matter whether the Air be retained in a certain Space by the Weight of the Atmoſphere, or any other way; for in either Caſe it will endeavour to expand itſelf with the ſame Force, and preſs every way. Therefore, if the Air near the Earth be included in any Veſſel, without altering its Denſity, the Preſſure of the included Air will be equal to the Weight of the whole Atmoſphere.

Experiment 4. *Plate* XXVII. *Fig*. 3.] Take the Tube mentioned in the firſt Experiment of the laſt Chapter, immerge it in Mercury included in the

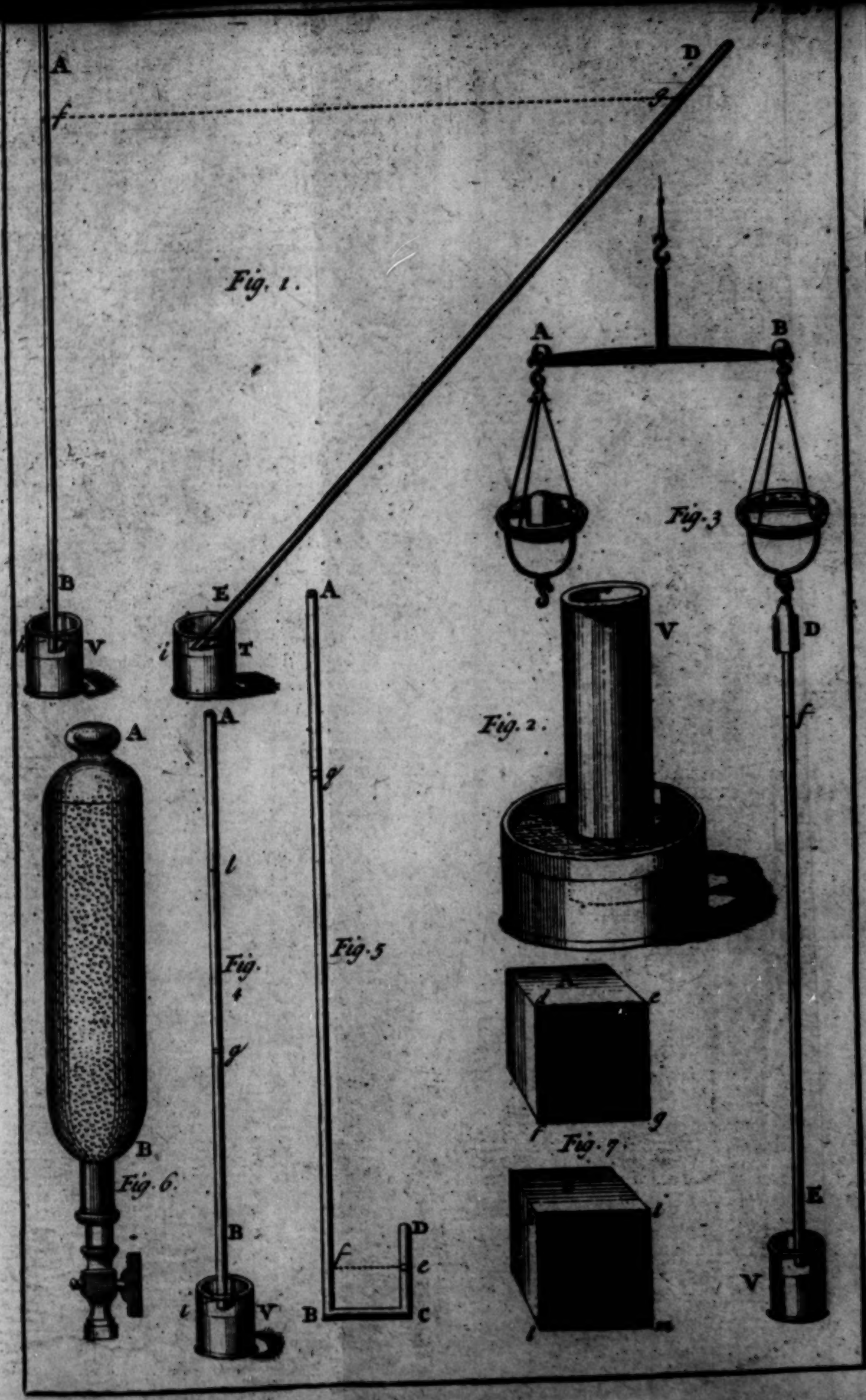


Fig. 1.
A
f
D
g
B
V
i
E
T
i
Fig. 3
A
B
D
f
E
V
Fig. 2.
V
Fig. 5
A
g
f
e
D
B
C
Fig. 4
A
l
g
B
i
V
Fig. 6.
A
B
Fig. 7.
A
e
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l
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AP 64
MUSEUM

the Glaſs D C, ſo that the Air, preſſing upon the Surface of the Mercury contained in the Veſſel U, may have no Communication with the external Air; the Mercury in the Tube is ſuſtained at the ſame Height by the Elaſticity of the Air, as it was ſuſtained in the open Air.

The Tenor of the Air continuing the ſame, 434 what we have ſaid will always obtain; but this Tenor of Temper of the Air is not always the ſame; *the repellent Force of the Particles is often increaſed or diminiſhed, though the Diſtance between their Centers is not changed.* I ſhall ſpeak of this Alteration in the following Book: *The Elaſticity increaſes by Heat, and diminiſhes by Cold.*

CHAP. XIV.

Of the Air-Pump.

THE Elaſticity of the Air is the Founda- 435 tion of the Conſtitution of a Machine, by which the Air may be drawn out of any Veſſel. This Machine is called an Air-pump, which is made ſeveral Ways: The chief Part in all of them is a Barrel, or hollow Cylinder of Metal, bored ſmooth, and poliſhed in the Inſide; in this Barrel muſt move a Piſton, that fills its Bore ſo exactly, as to let no Air ſlip by. This Piſton is thruſt down cloſe to the Bottom of the Barrel, and then raiſed up in ſuch a manner as to exclude all the Air from the Cavity of the Cylinder or Barrel: If this Cavity communicates with any Veſſel, by means of a Pipe at the Bottom of the Barrel, the Air in the Veſſel will expand it ſelf, and Part of it will enter into the Barrel, ſo that the Air in the Barrel, and in the Veſſel, will have the ſame Denſity. Shut up the Communication between the Veſſel and Barrel, and letting the Air out of the

Barrel, apply the Piſton cloſe to the Bottom. If you raiſe the Piſton a ſecond time, and open the Communication between the Barrel and Veſſel above-mentioned, the Denſity of the Air in the Veſſel will again be diminiſhed; and repeating the Motion of the Piſton, the Air in the Veſſel will be reduced to the leaſt Denſity. Yet all the Air can never be exhauſted by this Method; for at every Stroke the Air does ſo expand itſelf, as to have the ſame Denſity in the Barrel as in the Veſſel; in which laſt therefore there is always a little Air left.

436 All Air-Pumps have in common the Parts above deſcribed, but they differ in ſeveral other Things. *Firſt*, the Communication between the Receiver to be exhauſted, and the Cylinder or Barrel, is opened and ſhut different Ways. *Secondly*, there are different Ways of getting the Air out of the Cylinder or Barrel, when the Piſton is brought to the Bottom. *Thirdly*, the Piſtons differ in different Pumps. *Fourthly*, the Poſition of the Cylinder is not the ſame in all Pumps. *Fifthly*, there are different Contrivances for moving the Piſton.

There are often two Barrels, in one of which the Piſton is raiſed, when it is depreſſed in the other.

Our Pump is here repreſented in *Plate* XXVII. *Fig.* 1. the other Side of it is repreſented in *Plate* XXX. *Fig.* 2. I ſhall defer the particular Deſcription of it to another Place, and only here mention ſome Things in general. This Pump has two Braſs Barrels C, C, of 2 Inches Diameter, and about 5 Inches high.

In theſe Barrels the Piſtons move, one of which deſcends, while the other riſes, which Motion is communicated to them by the Wheel R, which is moved by the Handle MM, *Fig.* 2.

fixed to the Axis *a*. The angular Motion of the Wheel is the eighth Part of a Circle, by which in a leſs Wheel there is produced an angular Motion of 120 Degrees. This leſſer Wheel is fixed to a third Wheel, by means of which the Piſtons are immediately moved; they make a Stroke of 3 Inches and a half.

The Contrivance of the Piſton is much the ſame as in the Pumps which they uſe in *England*; tho' we think that we have made ours more perfect, by ſome Alterations in them.

The Glaſs is to be exhauſted, or ſet upon the round Plate L L; they communicate with the Barrels, by means of a Pipe, one End of which is at D, and which ſoldered to the lower Side of the Plate, the Continuation of this Tube is ſeen at E E; there are two Cocks in it, E, E; between the Cocks is fixed the Pipe *l*, *l*, which communicates with the Cylinders C, C.

When the Air is exhauſted, one of the Cocks above-mentioned ſerves to ſhut the Communication between the Receiver (ſo the Glaſſes are called from which the Air is to be pumped out) and the Barrels; the other Cock ſerves to let the Air in again, and to cut off the Communication with the mercurial Gage.

438 The mercurial Gage could not be conveniently repreſented in this Figure; it ſerves to determine what Quantity of Air is drawn out of the Receiver, as alſo what Quantity of Air remains in it; it is likewiſe of uſe for meaſuring the ſolid Contents of the Receivers, which ought to be exactly known in ſeveral Experiments; our Gage differs from the common Gages in ſeveral Reſpects.

A little Cylinder, with a Screw upon it, is often ſcrewed into the Plate at D, for applying a Globe to be exhauſted to the Pump.

In the Middle of the Plate L, L there is a Hole, which is ſhut up with a Screw; but ſometimes it ſerves for joining ſeveral Machines to the Plate.

439 By this Means alſo there is often applied to the Pump a cylindrick Box, full of Leathers ſoaked in Wax, thro' the Center of which a braſs Wire paſſes, which may be moved by the Help of a Handle, ſo as to communicate Motion into a Place void of Air; the Box has a Cover, which enters into it with a Screw, for preſſing the Leathers together; and to prevent the entring in, or eſcaping
440 out of the Air, ſuch a Box, or Collar of Leathers, is often joined to the Cover which is laid over the Recipients, as may be ſeen in *Fig.* 2. *Plate* XXVIII. and in *Fig.* 2. *Plate* XXIII.

441 When the Receivers are laid upon the Plate L, L, or when the Receivers are ſtoped with Covers, or when the Screws are joined to the Machine, and in general when the Air is to be hindered from running in, we make uſe of Wax, which is ſoftened by mixing as much Oil and Water to it as is found neceſſary.

CHAP. XV.

Several Experiments concerning the Air's Gravity and its Spring.

442 WE have ſhewn that Air is heavy, it may be weighed like other Bodies, and ſo its Denſity may be compared with that of other Bo-
288 dies. If the Veſſel that contains the Air be weighed, when it is full of Air; and again, when the Air is exhauſted, the Difference between their Weights is the Weight of the Air; which Method has this Inconvenience, that ſuch a ſmall Difference of Weight cannot eaſily be diſcovered, when a Ballance, tho' ever ſo nice, is loaden with a great Weight; therefore we muſt make uſe of the following Method.

Experiment 1. *Plate* XXVIII. *Fig.* 2.] Having 442 exhauſted the Air out of the Glaſs-Ball, whoſe ſolid Contents are 283 Inches; and having tied ſuch a Weight to it, that it may be almoſt equal in ſpecifick Gravity to Water, let it be immerſed into the Water contained in the Veſſel D E, and let it be faſtened by a Thread to the Hook of the Scale of the Ballance A B, above deſcribed*: *271 Raiſe the Ballance till you make an Æquilibrium with a very ſmall Weight; if by opening the Cock you let the Air into the Globe, a Weight I, of about 100 Grains, will be required in the oppoſite Scale to reſtore the Æquilibrium, ſometimes more, or ſometimes leſs, according to the different Tenor of the Air, which here, near the Earth, is varied according to the different Weight of the Atmoſphere, and according to the Difference of Heat and Cold.

Bodies immerſed in Liquids are ſuſtained by them, and the more or leſs, according to the greater or leſs Bulk of the Body*; and the Weight *299 loſt in that Caſe is determined from the known Denſity of the Liquid*. By the foregoing Expe- *297 riment, therefore, it may be known, how much Bodies gravitate leſs in Air than in a Vacuum.

Hence alſo may be deduced, that *Bodies that* 443 *are in Æquilibrio in the Air, if their Bulks are unequal, will loſe their Æquilibrium in a Vacuum:* Which is confirmed by the following Experiment.

Experiment 2. *Plate* XXVIII. *Fig.* 3.] In the Scales of the Ballance *a b*, lay a Piece of Wax *c*, and a Weight of Metal *p*, and you will have an Æquilibrium. Hang up the Ballance in a Glaſs Receiver, and, having exhauſted the Air, the

Wax will preponderate, its Bulk being greater than the Bulk of the Body *p*; it muſt be more ſuſtained by the Air; and, therefore, when you let the Air into the Receiver again, the Æquilibrium is reſtored.

The Elaſticity or Spring of the Air, which has been proved in *Chap.* XIII. becomes more ſenſible by the following Experiment.

444 *Experiment* 3.] Tie up a Bladder very cloſe, with a ſmall Quantity of Air in it; put a Receiver over it, and pump out the Air, whereby the Preſſure upon the external Surface of the Bladder is diminiſhed, and immediately the Air included in the Bladder will expand itſelf, and ſwell it out. We have proved, that the Spring of the Air is equal to the Weight of the whole Atmoſphere; the following Experiment will make it viſible.

445 *Experiment* 4. *Plate* XXVIII. *Fig.* 1.] Take a Bladder tied up very cloſe, and not quite full of Air, and put it in a Braſs Box A, whoſe Diameter is three Inches and an half; ſo that the Cover, which is of Wood, and does not exactly fit the Box, may be ſuſtained by the Bladder; you muſt put the Lead Weights P, P, upon the Cover: They have a Hole in the Middle for a wooden Cylinder E, which is fixed to the Cover, to go thro': When you pump out the Air, the Bladder is ſwelled, as in the foregoing Experiment, and by that means the Weights are raiſed. You may uſe ſeveral Weights according to the Bigneſs of your Glaſs Receiver; and tho' they ſhould amount to 60 or 70 Pounds, they would be eaſily raiſed. The Gravity of the Air, its Preſſure that ariſes from the Gravity, as alſo its Elaſticity, produce

very different Effects; ſome of which I ſhall ſelect, and confirm by Experiments.

Experiment 5. *Plate* XXVII. *Fig.* 3.] To the 446
Hole in the Middle of the under Side of the Air-Pump L L, ſcrew on a ſmall hollow braſs Cylinder which has a Hole through it, and an open Glaſs Tube A B cemented to it at Bottom, whoſe lower End B muſt be immerged into Mercury. Let Mercury be ſuſtained in the Tube *e g*, that is cloſe at *e*, and void of Air in the Manner beforeſaid. * Set the Veſſel U with the Tube upon * 422
the Plate L L, and cover it with a tall Glaſs D C, ſo as to cut off all Communication between the external Air and the Veſſel U, as alſo the Cavity of the Tube A B. The Air in this Tube does, by its Elaſticity, hinder the Mercury from riſing up in the Tube, by the Preſſure of the external Air. The Air alſo, that is included in the Receiver D C, does by its Spring ſuſtain the Mercury in the Tube *g e* *. Pump the Air out * 433
of D C; as the Denſity diminiſhes, the Elaſticity does alſo decreaſe, * and the Force by which the * 430
Mercury is ſuſtained in the Tube *g e* becomes leſs; therefore, the Mercury deſcends. At the ſame time the Preſſure of the external Air overcomes the Reſiſtance in the Tube A B, and the Mercury aſcends in the Tube. The Diminution of the Spring in the Tube A B, and in the Veſſel B C, is the ſame, and the Effect of the Diminution the ſame in both Caſes; therefore, the Mercury deſcends as much in the Tube *e g* as it riſes in the Tube A B, which agrees with the Experiment. By this Method the Mercury is raiſed up to *f*, while the Tube *g e* becomes almoſt wholly empty; when you let in the Air again, the Mercury riſes in the Tube *g e*, as it is depreſſed in the Tube A B.

447 *Experiment* 6. *Plate* XXVIII. *Fig.* 4.] Take the little Pump or Syringe A, and its Piſton being thruſt cloſe to the Bottom, let the Tube which is joined to the Syringe be immerged in Water; when you raiſe up the Piſton, the Water will follow it, and fill up the Cavity between the Bottom of the Pump and the Piſton; which Effect ariſes from the Preſſure of the external Air.

For this Reaſon Water does not riſe *in vacuo*.

448 *Experiment* 7. *Plate* XXVIII. *Fig.* 5.] Join the Glaſs Tube *b c* to the Syringe A, which is ſcrewed to the Cover of a Glaſs Receiver, ſo that the End *c* of the Tube may deſcend below the Surface of the Water in the Veſſel U; thruſt down the Piſton to the Bottom of the Syringe, and let all the Air be pumped out of the Receiver. If then you pull up the Piſton, the Water will not riſe.

449 *Experiment* 8. *Plate* XXVIII. *Fig.* 6.] The Force, by which the Air preſſes upon Bodies, often breaks them, when the Preſſure is not equal every Way. Let the Braſs Cylinder A be covered with a flat Piece of Glaſs; when you pump the Air out of this Cylinder, the Plate of Glaſs will be broken into a great many little Pieces by the Preſſure of the external Air.

450 *Experiment* 9. *Plate* XXIX. *Fig.* 1.] Take a Syringe A of an Inch Diameter; puſh down the Piſton to the Bottom of it, and ſhut up the Hole at the Bottom of the Syringe, and hang on a Weight P of 10 Pounds to the lower End of the Syringe; if you hold the Handle B of the Piſton in your hand, the Syringe will not deſcend; for it cannot deſcend, unleſs the Weight hanging at

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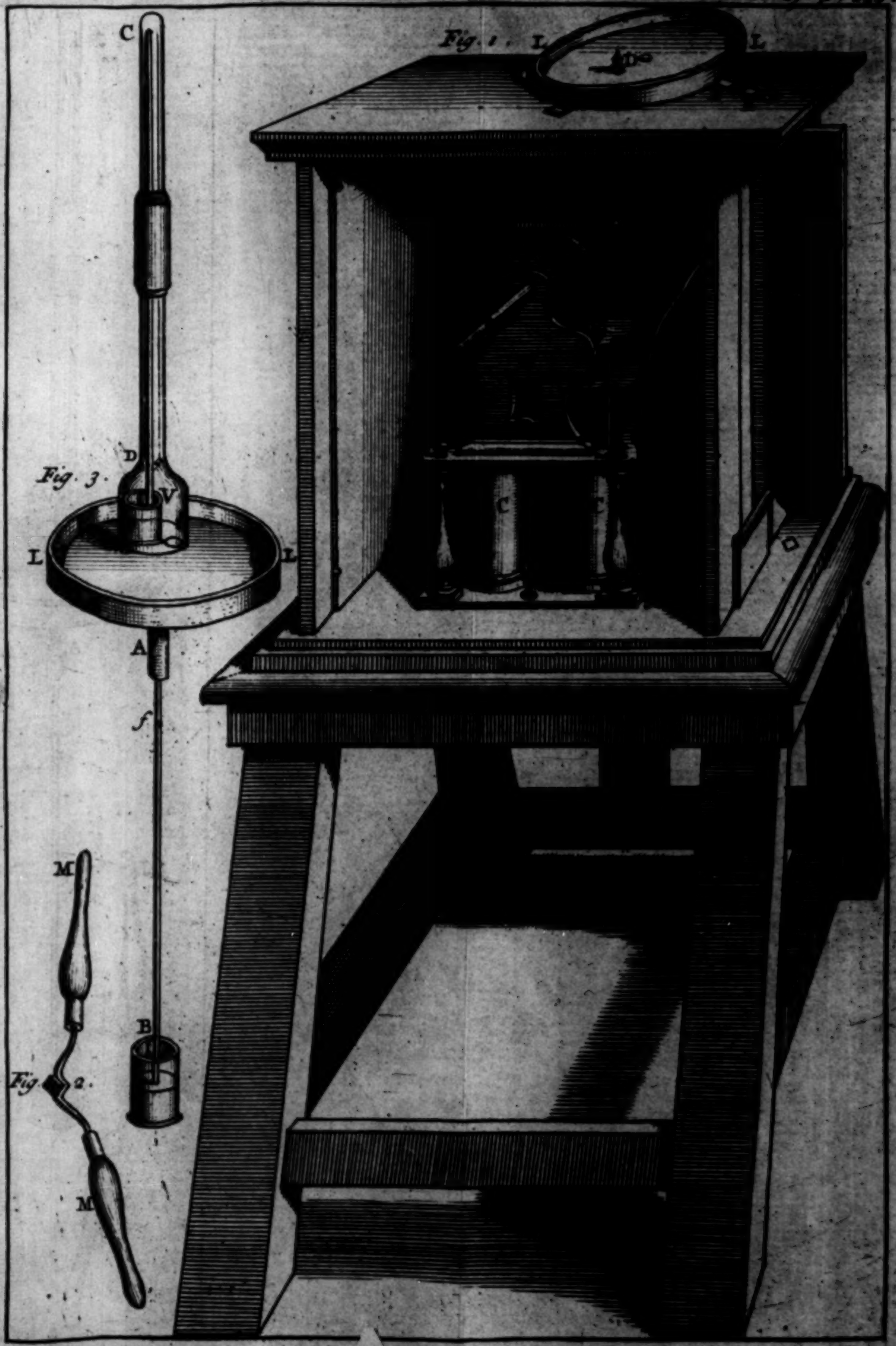

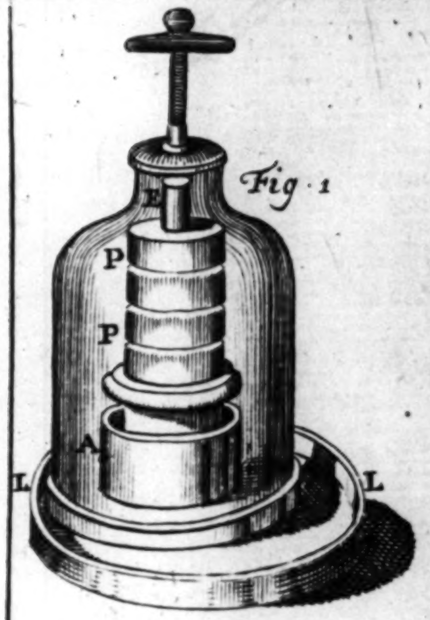
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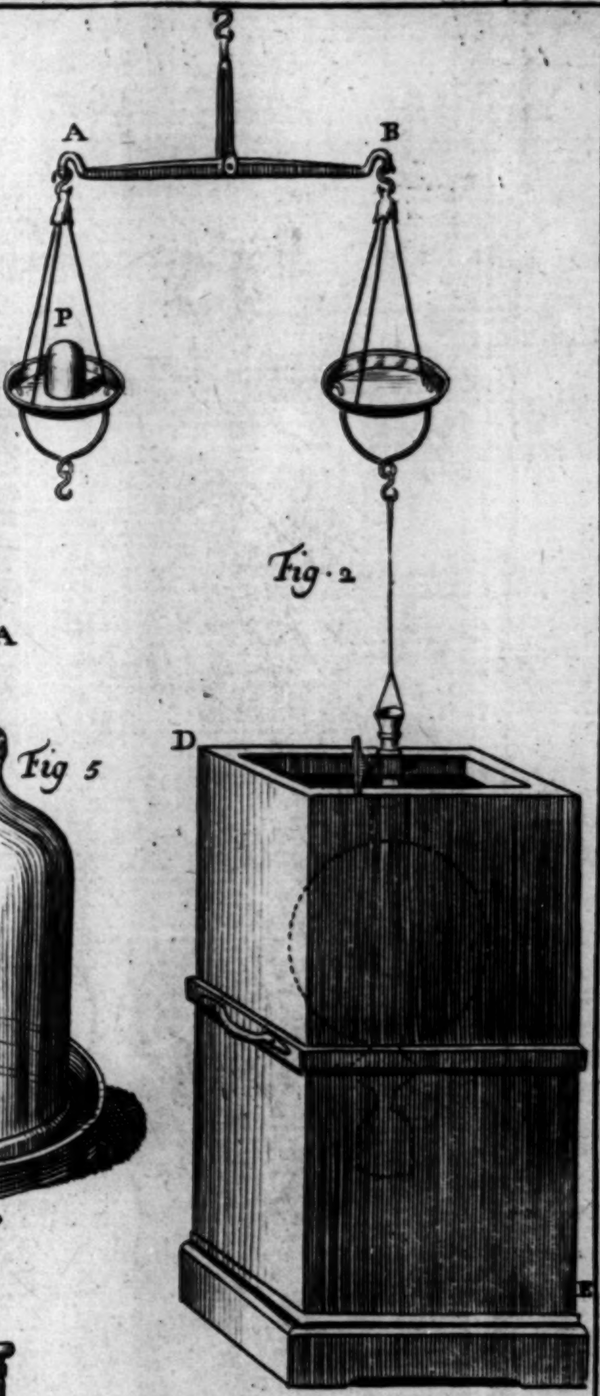
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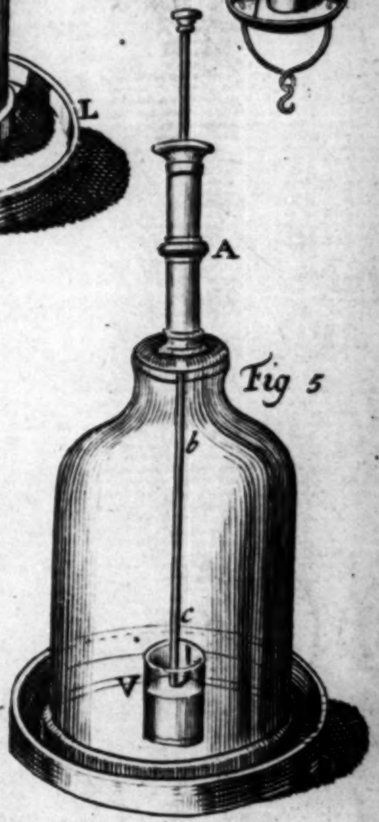
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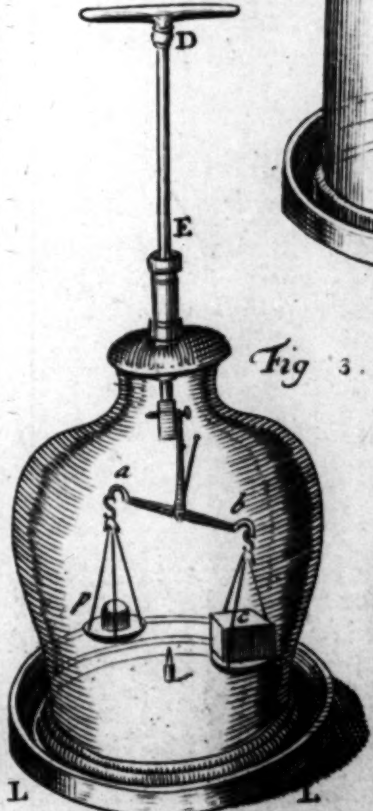
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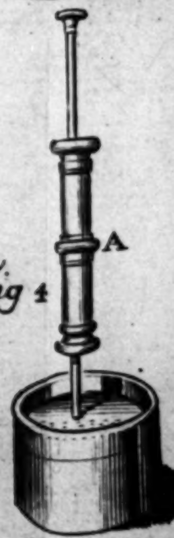
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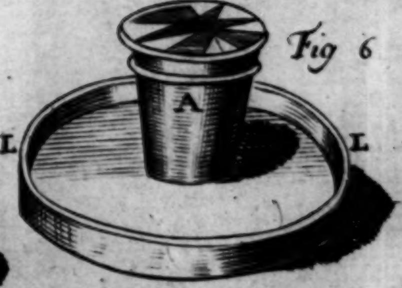


Fig 6

it overcomes the Preſſure of the Air and Friction of the Piſton; but the Preſſure of the Air alone does here exceed 10 Pounds.

Experiment 10. *Plate* XXIX. *Fig.* 2.] The Syringe deſcends by the Weight *p* alone in a Vacuum, which is but juſt ſufficient to overcome the Friction of the Piſton. 451

Experiment 11. *Plate* XXIX. *Fig.* 3.] We ſee a more ſenſible Effect of the Preſſure of the Air, when the two Segments of a Sphere, H and I, are joined together. Let the Brim or Edge of each of them be well poliſhed, ſo that they may fit together, and when they are applied cloſe, put a little Wax between, to exclude the Air. There is a Cock in the Segment H, by which the two Segments, when joined together, may be applied to the Air-Pump, and which muſt be ſhut, when you have exhauſted the Air. The Segments are ſuſpended by the Ring A, and, by the Help of the Ring Q, you may hang to them the Weights that are laid upon the great wooden Scale T. If the Diameter of the Segments be three Inches and an Half, a Weight of about 140 Pounds will be required to pull them aſunder. 452

Experiment 12. *Plate* XXX. *Fig.* 1.] Let the Segments be joined together and exhauſted, as in the former Experiment; if they be ſuſpended in a Vacuum, with a little Weight P hanging on, which is juſt able to overcome the Coheſion of the Wax, they will be ſeparated in this Experiment. There muſt be faſtened to the Plate L L, the little Braſs Box, or Collar of Leathers,*, thro' which a Braſs Wire, that has the Weight hanging to it, ſlips. Leſt the Receiver ſhould be broke by the Fall of the lower Segment, you muſt put un- 455 *439 der it the hollow wooden Cylinder M, to let it fall into. In this Figure the Segments are ſuſpended to the Cover of the Receiver which is to be exhauſted; they may alſo be ſuſpended from a Pillar faſtened to the Cylinder M. That the Hemiſpheres may not be ſeparated without Difficulty, it is not required that they ſhould be empty of Air; as great a Force will be required to ſeparate them as in the 11th Experiment; when having included them in a Veſſel, and applied them cloſe together (ſo as to leave them full of common Air, and ſhut the Cock, that the Air between them may not be changed;) the Air on the Outſide of them in the Veſſel, that contains them, is reduced to a double Denſity; which to confirm by an Experiment, we muſt firſt deſcribe the Machine with which we make Experiments in compreſſed Air.

454 *Plate* XXXI. *Fig.* 5.] A round braſs Plate N is laid upon a Board *a a*, about 15 Inches long, and 10 wide; the Diameter of this Plate is about 5 or 6 Inches, as you may ſee by its ſeparated Figure at N, in *Plate* XXIX. *Fig.* 4.] and has fixed to it here a Cylinder P, which is not perforated, and goes through the Board *a a*. Upon this Plate you muſt put a Glaſs U U, about 10 Inches high, that is terminated at each End in a cylindric Form, and the cylindric Parts have braſs Rings, or Ferrels, upon them.

The Veſſel muſt have upon it the Cover D. The Pillars CS, CS, are faſtened to the Board *a a*, and go thro' the Wood *de*, by which the Cover D is firmly joined to the Glaſs, as alſo the Glaſs to the Plate N, by the Force of the Screws *f f*. It is very neceſſary to preſs all theſe Parts cloſe together, having firſt ſpread Wax upon the upper and lower Edges of the Glaſs.

The Cover is repreſented in the ſeparated Figure D, *Plate* XXIX. *Fig.* 4.] There is fixed to it the Collar of Leathers ; * and, leſt the Piece of * Wood *d e* ſhould be applied to too ſmall a Surface, the Cover is made in the Shape of a round open Box.

There is a perforated braſs Wire C, which goes through the Collar of Leathers, to which a Cock B is joined.

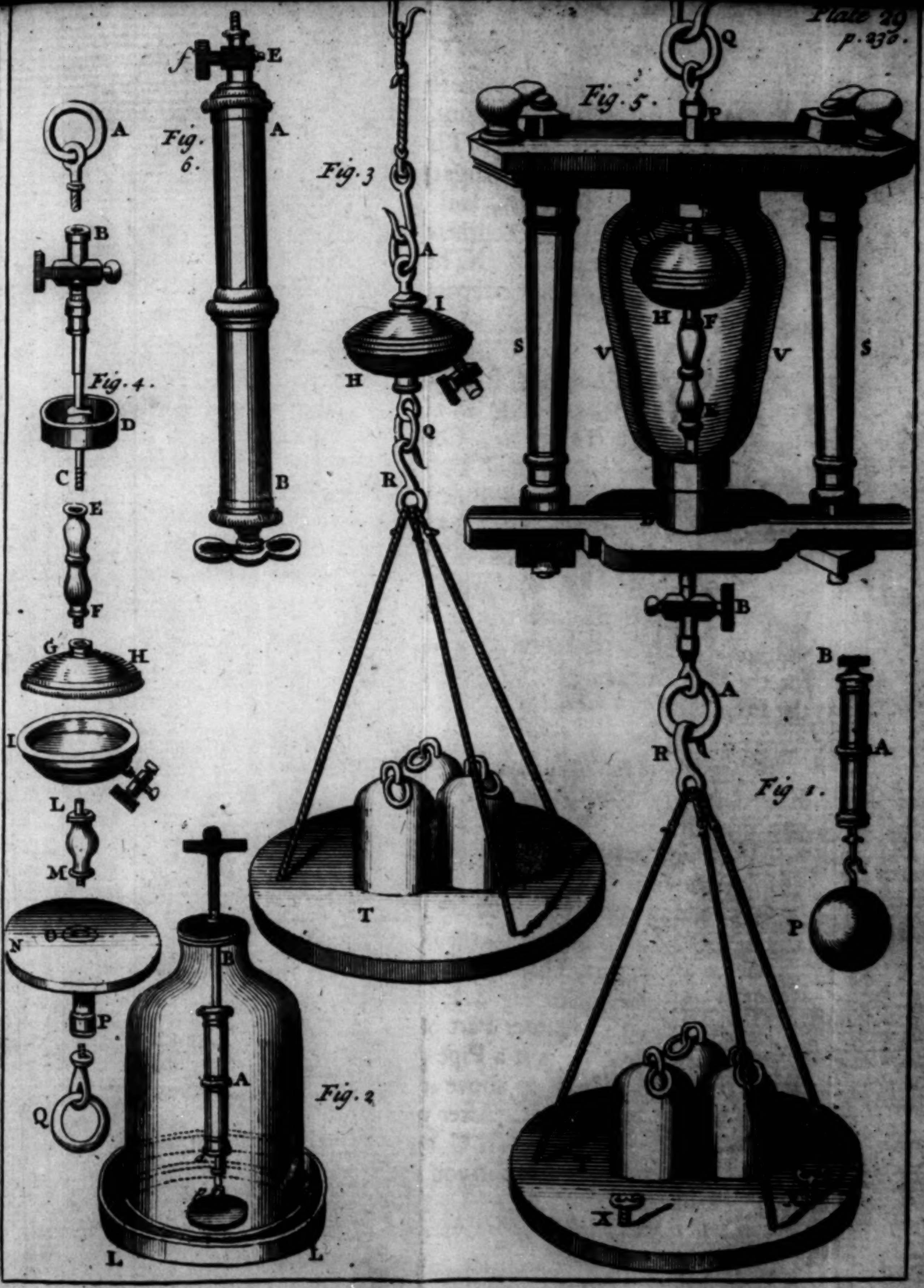
Plate XXIX. *Fig.* 6.] In order to compreſs the 455 Air in this Veſſel, ſcrewing on the Syringe A B to the Cock B laſt mentioned, which Syringe has joined to it another Cock, in the Key of which, beſides the uſual Hole, there is another oblique Hole which goes to *f*, and by which when you ſhut the Communication between the Glaſs and the Syringe, the Syringe has a Communication with the common Air, and is filled with it, when you raiſe up the Piſton. When you open the Communication between the Glaſs and the Syringe, by puſhing down the Piſton, you force the Air which was contained in the Syringe into the Glaſs ; and, by often repeating this Operation, you at laſt bring it to the Denſity required.

Experiment 13. *Plate* XXIX. *Fig.* 4.] Now, to 456 ſeparate theſe Segments or Cups in compreſſed Air, the Segment I is joined to the Plate N, by Means of the Pillar M L, which has Screws at M and L. The other Segment H, by Screws at F and E, is joined to the Wire C. The Segments muſt be applied to each other: As for the reſt, you muſt obſerve what has been ſaid in the Deſcription of the Machine ; and the Air muſt be compreſſed in the Veſſel, ſo as to have twice the Denſity of that which is compreſſed only by the Atmoſphere. At P the Ring Q is joined to the Plate N, as alſo the Ring A to the Cock B. Invert the Machine, as in *Fig.* 5. and ſuſpend it by the Ring Q. The Scale T, on which the Weights are laid, hangs upon the Ring A; and, until the Weights, laid on, come to be about 140 Pounds, the Segments will not be ſeparated. Three Screws X, X, hinder the Scale T from deſcending too low in ſeparating the Segments.

457 *Experiment* 14. *Plate* XXX. *Fig.* 2.] Apply the Tube A B to the under Side of the Air-Pump Plate L L, which Tube has a Cock in its upper Part, and is joined to the ſmall Tube which ſtands above the Plate. Put on the Receiver R, which covers the Prominent Tube. The End B of the Tube A B muſt be immerſed in the Water contained in the Veſſel U, and having exhauſted the Veſſel R, you muſt open the Cock; the Water will ſpout up into the Receiver with a great Force, for the ſame Reaſons as the Water is ſuſtained at
423 the Height of 32 Foot in a Pipe void of Air.

458 *Experiment* 15. *Plate* XXX. *Fig.* 3.] The Air's Elaſticity produces the ſame Effect. Let there be a braſs Cylinder U exactly ſhut. There muſt be a Hole in the Bottom to pour in Water, which afterwards you ſhut up with a Screw. To the upper Part of the Veſſel there is ſoldered a Pipe, which goes down almoſt to the Bottom; and to the other End of it, that ſtands above the Veſſel, a Cock is joined (See *Fig.* 4.) This Veſſel is to be ſcrewed on to the lower Part of L L, the Air-Pump Plate, and from it a Pipe goes quite thro' the Plate, and ſtands up above it, which is covered by the Receiver R. After you have pumped out the Air, the Veſſel U being about two Thirds full of Water, when you open the Cock, the

Plate 29
p. 230.



the Water will violently ſpout up into the Receiver, by the Force of the Spring of the Air contained in the upper Part of the Veſſel U. Here the Air preſſes upon the Surface of the Water; when you open the Cock, the Preſſure in the Tube becomes leſs, therefore the Water muſt go into the Tube.

Experiment 16. *Plate* XXX. *Fig.* 4.] Even in 459 the open Air, the Water will violently ſpout out of the Veſſel U, if, having filled it two Thirds full of Water, the Air be compreſſed in the upper Part of it; which is done by help of the Syringe above-mentioned*. *455

Experiment 17. *Plate* XXXI. *Fig.* 1.] Invert the 460 Glaſs R, and immerge it in the Water contained in U, the Air keeps out the Water at whatever Depth it be immerged; yet the deeper the Glaſs is put down, the leſs Space the Air is reduced into.

Upon this Principle are made the Machines in 461 which Divers go down into the Sea. They are made like Bells, and deſcend by their own Gravity; the Water does not riſe up to the Diver in the Bell; freſh Air is ſent down continually by Bladders tied to a Rope, which he draws down to him; the Air, heated by his Reſpiration, riſes to the upper Part of the Bell, and is there driven out through a Cock, by the Preſſure of the Water, that puſhes up, and compreſſes the Air in the lower Part of the Bell; which Preſſure overcomes the Force with which the Water endeavours to deſcend thro' the Cock; for the Preſſure of Liquids is increaſed in Proportion to their Depths*. *290

Experiment 18. *Plate* XXXI. *Fig.* 2.] Take lit- 462 tle Figures of Glaſs that are made hollow, of an Inch and half long, repreſenting Men, which may be had at the Glaſs-Blowers; theſe little

Images have a ſmall Hole in one of their Feet, and are lighter than Water. Immerge them into the Water contained in the Glaſs AB. This Glaſs is about a Foot, or 15 Inches high, and covered with a Bladder, which is tied faſt over the Top. A ſmall Quantity of Air is to be be left between the Bladder and Surface of the Water. If the Veſſel be preſſed with the Finger, the Air above-mentioned is reduced to a leſs Space, and the Surface of the Water is more compreſſed; the Water which is more compreſſed, enters in the little Men through the Hole at their Feet, and compreſſes the Air in their Bodies more than it was. The little Images, becoming heavier, by this Means deſcend towards the Bottom of the Veſſel, and that faſter or ſlower, according to the Bigneſs of the Hole, and alſo according as the ſpecifick Gravity of the Images comes nearer to the ſpecifick Gravity of the Water. Taking away your Finger, the Air in the little Men, being leſs compreſſed, expands itſelf, and drives out the Water; ſo the Images riſe up again to the Surface of the Water.

463 *Experiment* 19. *Plate* XXXI. *Fig.* 3.] Animals cannot live without Air. If any Animal be included in the Receiver U, and the Air be drawn out, the Animal will immediately be in Convulſions, and will fall down dead, unleſs the Air be ſuddenly re-admitted. Some Animals will live in a Vacuum longer than others.

464 *Experiment* 20. *Plate* XXXI. *Fig.* 4.] Some Fiſhes alſo cannot live without Air; but in others you ſee no ſuch Change, but the Swelling of their Eyes. What Experiments you make upon Fiſhes, muſt be made in the Glaſs Receiver U, which is ſet upon the Plate of the Air-Pump, and to the Hole,

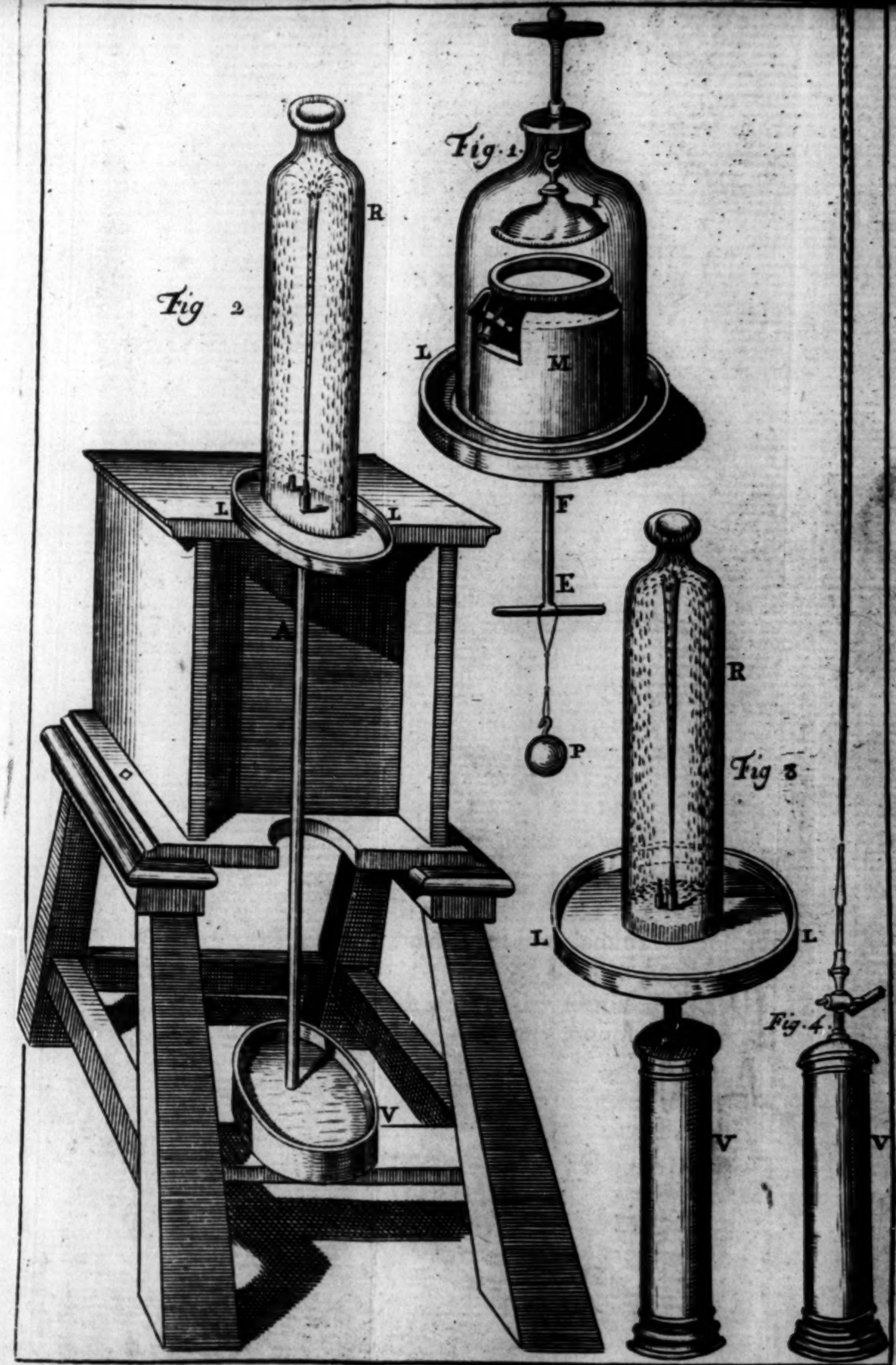


Fig. 1.
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V

Hole, through which the Air is drawn out, you muſt ſcrew on a Pipe, which comes up almoſt to the upper Part of the Glaſs U; pour in Water, and then put a Cover over the Glaſs Receiver U, and exhauſt the Air out of the upper Part of it. Having taken away the Preſſure of the Air from the Surface of the Water, the Air in the Fiſh's Body expands itſelf, by which Means the Fiſh, becoming lighter, cannot deſcend in the Water.

Experiment 21. *Plate* XXXI. *Fig.* 5.] Experiments are made upon Animals in compreſſed Air, by Help of the Machine above deſcribed.* In that Caſe Animals do not ſoon die, becauſe the Veſſels in the Body are not broken; yet if they continue long in that condenſed Air, it muſt be hurtful to them; nay, and in a greater Compreſſion of the Air (for which a Veſſel of Metal is required) they will die in a little Time. 465 *454

Experiment 22.] Several Liquors contain Air. If you put them under a Glaſs Receiver, and draw out the Air, then the Air contained in the Liquors will expand itſelf and go out. In that Caſe very often the Action of the Particles of the Liquid upon one another, is changed, and a Fermentation ariſes. 466

CHAP. XVI.

The Deſcription of ſeveral Machines, and the Explanation of their Effects.

[*Experiment* 1. *Plate* XXXII. *Fig.* 1.]

LET one End *a*, of the Curve Tube *a* S *b*, be immerged in Water, whilſt the other End *b* deſcends below the Surface of the Water. If by ſucking, or any other Way, the Air be taken out 467

out of this Tube, the Water will run through *b*. This Inſtrument is called a *Syphon*.

468 This Effect ariſes from the Preſſure of the Air, which drives on the Water in the Syphon, by its Preſſure upon the Surface of the Water in the Veſſel. The Air does alſo preſs againſt the Water that goes out of the Orifice *b*, and ſuſtains it. Theſe Preſſures are equal, and act contrariwiſe in the upper Part of the Syphon, with a Force equal to the Weight of the Atmoſphere, taking away the Weight of the Pillars of Water which are ſuſtained by the Preſſure. The Pillar of the Water in the Leg S*b*, is longer than the oppoſite Pillar of Water; therefore, the Preſſure of the Air is more diminiſhed on the Side *b*S, and the oppoſite Preſſure overcoming it, the Water flows towards *b*.

469 *Experiment* 2. *Plate* XXXI. *Fig.* 6.] The Syphon above-mentioned has this Inconveniency, that, if once it ceaſeth to work, the Water will not run again, unleſs the Air be drawn out of the Tube afreſh. This may be corrected by making a Syphon in the Figure *a*S*b*, whoſe Legs are equal, and turned up again: For if the Syphon be filled with, and one Leg be immerſed in Water, ſo that the Surface of the Water may be above the Orifice, then the Water will run out through the other Leg, for the Reaſon given in the Explication of the former Experiment. Since the Legs are returned upwards, the Syphon will not be emptied, when the running out of the Water ceaſes; and ſo the Syphon, being once filled, is always ready to work its Effect. The Water runs backward or forward through it, according as it is higher on one Side or the other.

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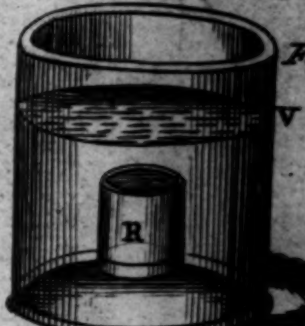
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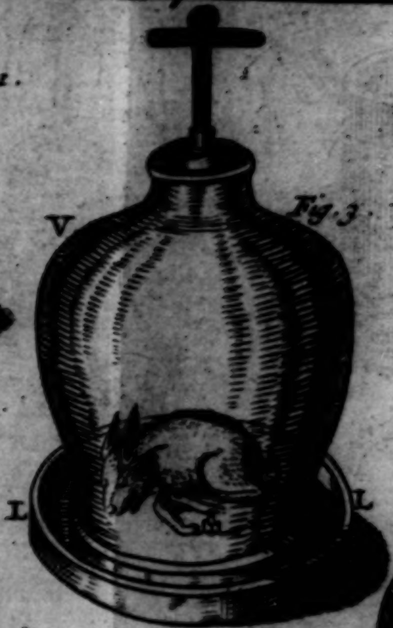
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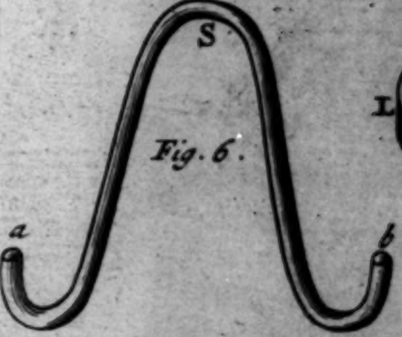
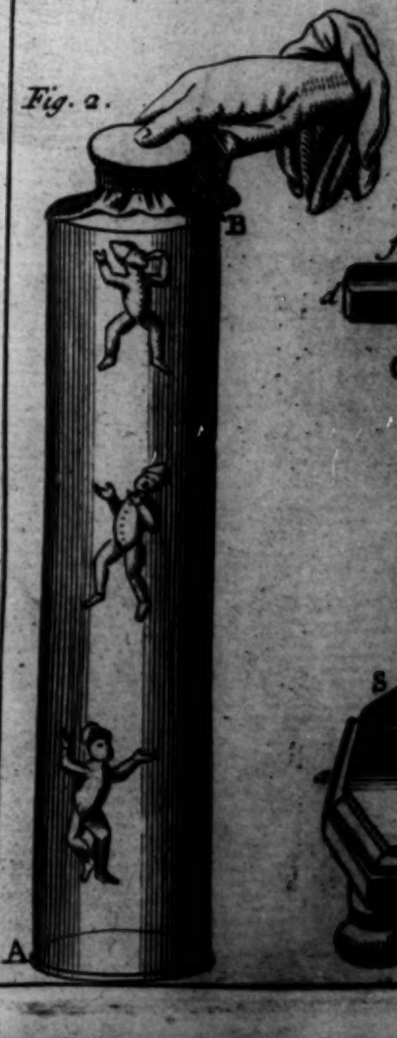


Fig. 6.

Fig. 4.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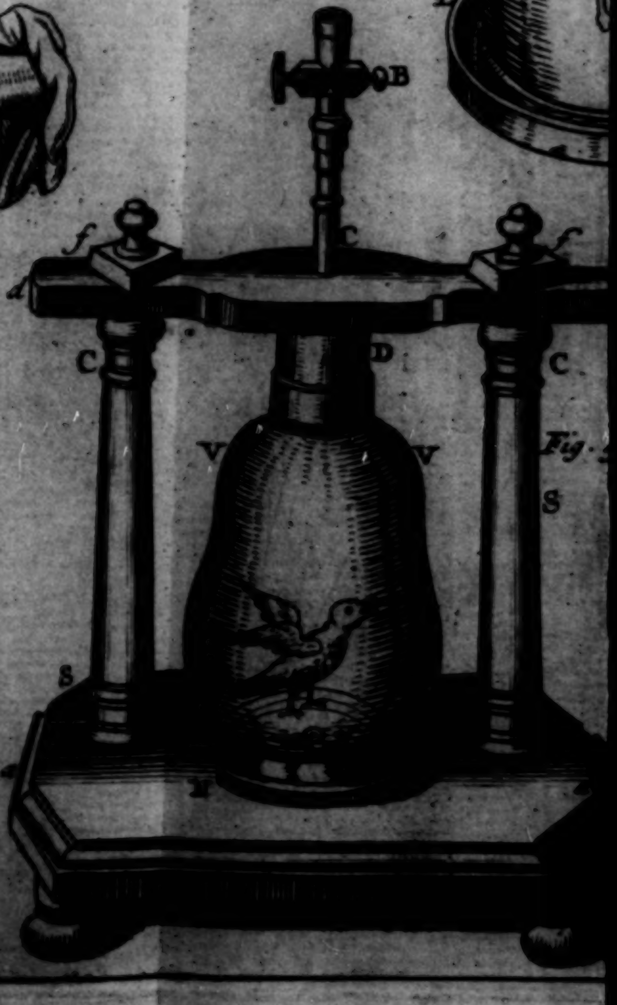
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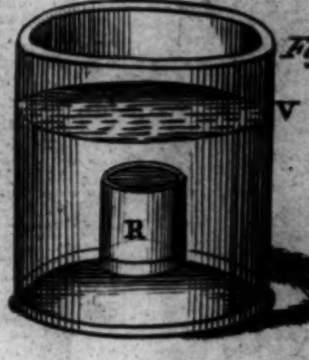

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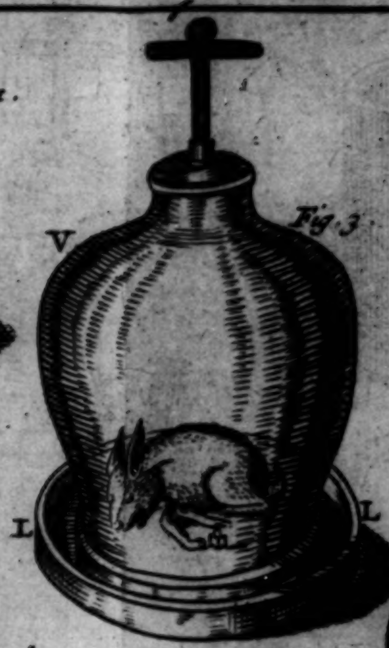
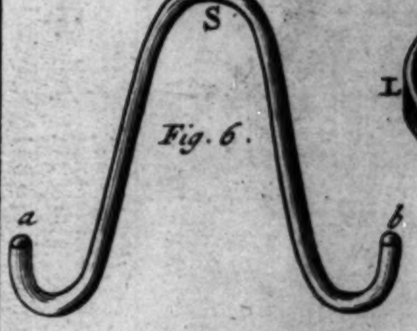


Fig. 3.



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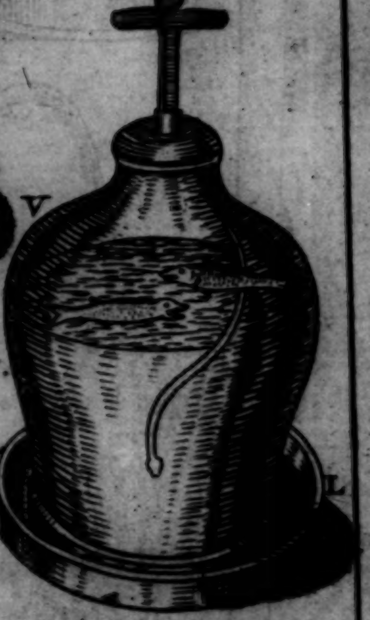

Fig. 4.

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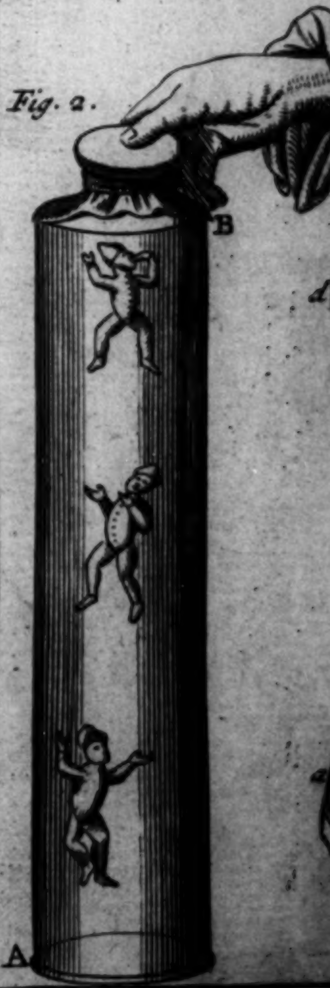

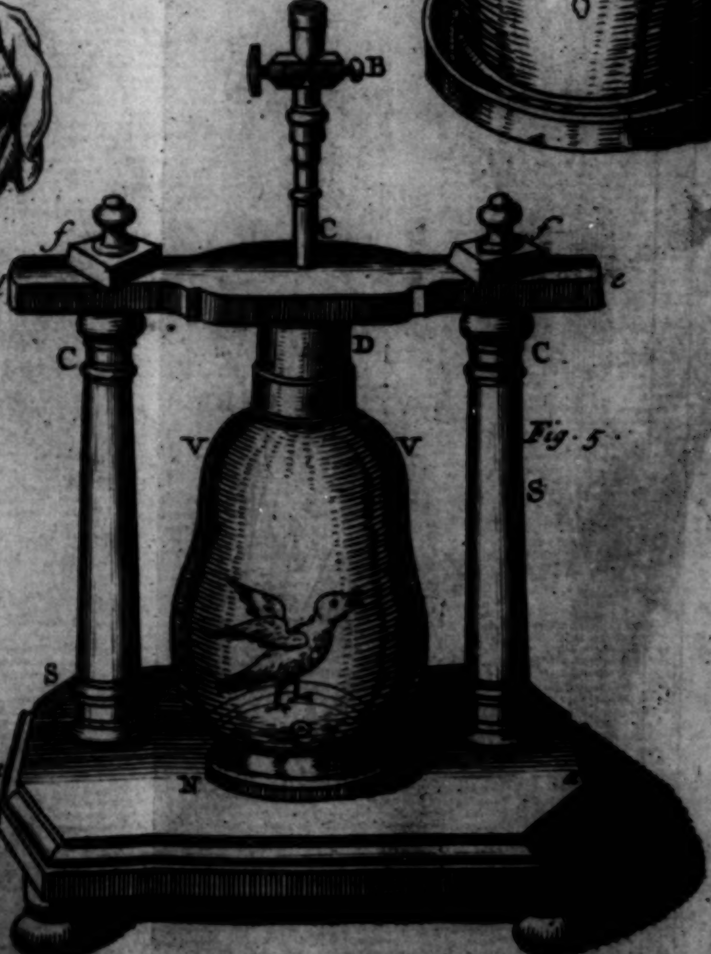

Fig. 5.

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Plate XXXII. *Fig.* 2.] Upon the ſame Principle as the foregoing Machines, is contrived the Syphon for raiſing Water into a Ciſtern. The Effect of this Syphon becomes viſible by the Help of a Machine made up of two hollow Glaſs Balls H and I, which are joined together by the Tube CDE, the Ball I communicates with the Water to be raiſed by means of the Tube AB, which comes up almoſt to the Top of the Ball; to the Ball H at the lower Part is joined the Tube FG, as long as the whole Tube AB. 470

The Ball H muſt be filled with Water through a Hole by a Funnel, and then the Hole muſt be ſhut up cloſe.

In ſuch Machines as are applied to uſe, for raiſing Water out of Reſervoir that contains it, the Water is brought away in the Veſſel H and the Communication between the Veſſel and the Reſervoir is ſhut up with a Cock.

Experiment 3.] Opening the Cock G, the Water will run out that Way, and the Water will aſcend through the Tube AB up into the Veſſel I; which being filled, the Water is ſuffered to run away to the Place where you would have it; and by repeating the Operation, the Elevation of the Water continues.

Opening the Cock G, the Air preſſes againſt the Water going out of the Tube FG, the Air alſo preſſes upon the Water in the Reſervoir, and ſuſtains that which is in the Tube AB. Theſe Preſſures are equal, and if you take from them the Columns of Water which they ſuſtain, you will have the Forces by which they act upon the Air, contained in the upper Part of the Veſſels and the Tube CDE. The Pillar FG, becauſe there is ſuperadded to it the Height of the Water in the Veſſel H, does 471 always overcome the Column in the Tube A B, as being longer; therefore the Preſſure at G is leſs diminiſhed than the other, and ſo overcome by it; and therefore the Water muſt riſe in the Tube A B, and deſcend down F G.

472 To render the Effect of common Pumps viſible, let there be a little Pump made of Glaſs in the following Manner: A B (*Plate* XXXII. *Fig.* 3.) muſt be a Cylinder of Glaſs, and about an Inch and a half Diameter. In the Bottom of it join a Tube of any Length, as C D. Let the upper Part of it be ſhut with a leaden Ball, ſo that the Water may not be able to deſcend out of the Cylinder, but may eaſily riſe into it, by raiſing up the Ball, which we make uſe of here inſtead of a Valve. The Piſton is moved in the Cylinder A B, which, being ſurrounded with Leather, exactly fills its Cavity: There is a Hole in the Piſton, which likewiſe is ſtopped with a Ball of Lead inſtead of a Valve; ſo that the Water may riſe, but not deſcend through the Piſton.

Experiment 4.] Puſh down the Piſton to the Bottom; pour Water upon it to hinder the Paſſage of the Air; if the End of the Tube C D be immerſed into Water, and the Piſton be raiſed, the Water will aſcend up into the Cylinder A B* *447 from which it cannot deſcend; wherefore, it comes up through the Piſton, when it is puſhed down. If you raiſe the Piſton again, the Cylinder is again filled with other Water, and the firſt Water is raiſed up into the wooden Cylinder which is joined into the glaſs one, from which it runs out thro' the Tube G.

473 Since the Effects of all the Machines, deſcribed in this Chapter, depend upon the Preſſure of the Atmoſphere, the Water will not riſe in theſe Machines much higher than 32 Foot.* *423

There are ſeveral little artificial Fountains, that are called the Fountains of *Hero*; I ſhall here give the Conſtruction of one of them.

Plate XXXII. *Fig.* 4.] Let there be two equal elliptical Veſſels, A B and C D, exactly ſhut on all Sides, made of one Sort of Metal. 474

In each of them there is a Separation paſſing through the Center of the Ellipſe, which divides the whole Veſſel into two equal Parts.

The Separation *m ni*, in the Veſſel D C, is perpendicular to the Axis of the Ellipſe, the Separation *c f g h* of the other Veſſel muſt be inclined to that Axis.

There is a Brim raiſed round about the upper Part of the Veſſel A C B, to make a Baſon.

Four Tubes are joined to theſe Veſſels. The firſt *o p* goes through the Cavity B of the Veſſel A B, without having any Communication with it, and deſcends almoſt to the Bottom of the Cavity D; the ſecond *s t* is ſoldered to the upper Part of the Cavity D, and aſcends to the upper Part of the Cavity B, but not quite ſo high as to touch the upper Plate of it. The third *q r* reaches from the lower Part of the Cavity B, almoſt to the Bottom of the Cavity C; the 4th, *x u*, is made faſt to the upper Part of the Cavity C, and reaches almoſt to the upper Part of the Cavity A.

Laſtly, there is a Tube *z y*, which, going thro' the upper Plate, is ſoldered to it, and reaches down ſo deep in the Cavity A, that its End *z* is but a little Way off of the Bottom.

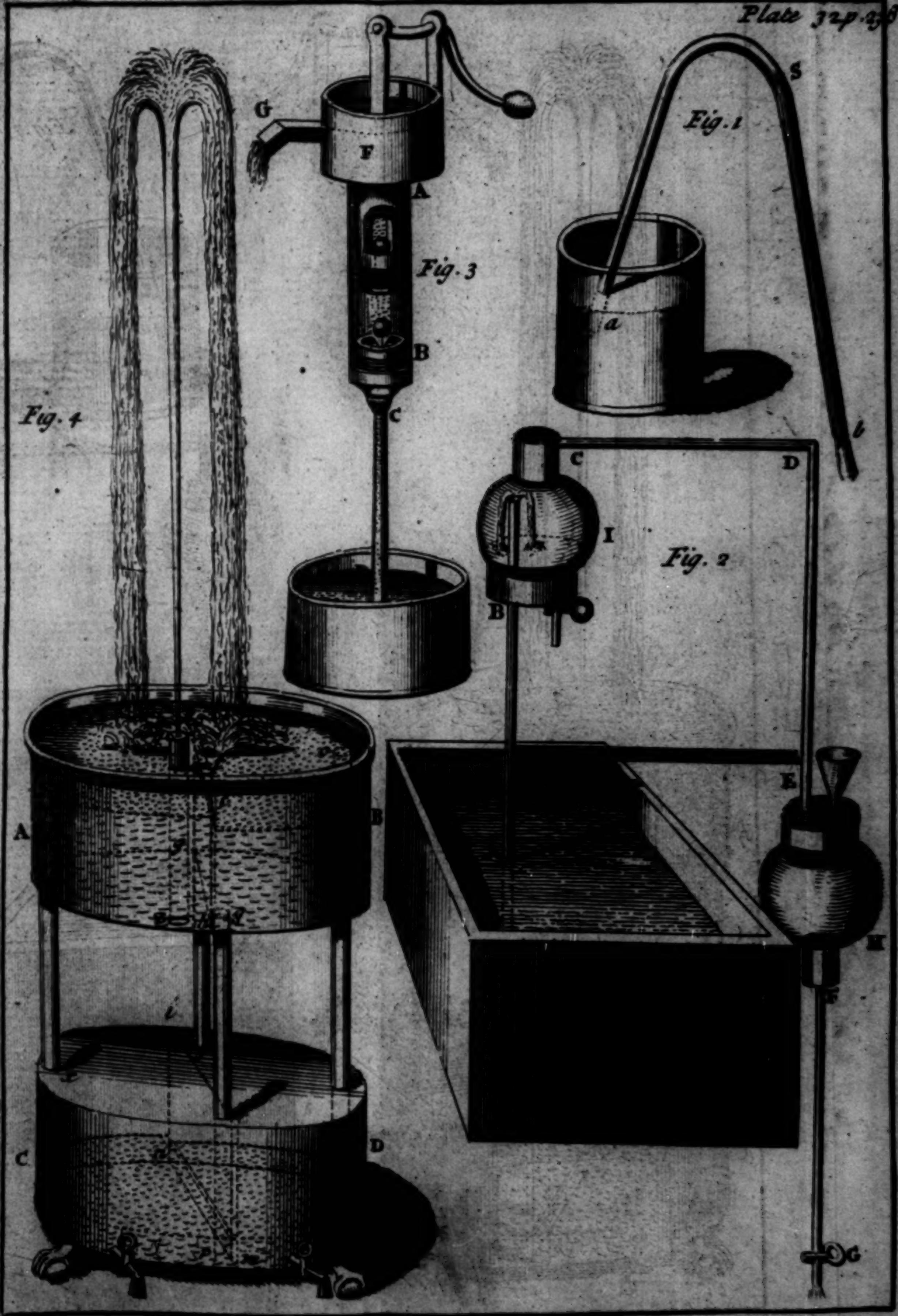
There are Cocks joined to every one of the Cavities; or elſe they have other Holes that are ſhut up with Screws that have Leathers on them; the chief Uſe of them is to let out the Water very clean from the Cavities, leſt they ſhould grow ruſty, when the Machine is not in Uſe.

Experiment 5.] Pour in Water through the Tube *op*, ſo as to fill the Cavity D; if you continue to pour in Water, it will riſe up through the Tube *st*, and then deſcend through *qr* into the Cavity C, which is alſo filled, the Air aſcending up through *xu*, and going out through *zy*.

Turn the Machine upſide down, opening the Cocks of the Gavity C and D, the Water will deſcend into the Cavities B and A. Having again ſhut the Cocks, as alſo the Hole *y* of the Tube *zy*, ſet the Machine again the right Side upwards, and pour in Water again through the Tube *op*, till the upper Surface of the Machine be covered with Water. Now, if the Hole *y* be opened, the Water will ſpout up to almoſt twice the Height of the Machine, and the Motion of the Water will continue till the Cavity A be emptied of its Water. The Height of the ſpouting Water will continually diminiſh, and at laſt it will not be double the Diſtance of the Veſſels.

475 The Effect of this Machine is to be attributed to the Compreſſion of Air in the Veſſels. The Preſſure of the Atmoſphere at *o* and *y*, as alſo in the Veſſels, is equal, and theſe Preſſures deſtroy one another; therefore, they are not to be conſidered in the Examination of the Machine. When at laſt the Water is poured into the Tube *op*, it is ſuſtained in it by the Preſſure of the Air contained in the Cavity D, and acting upon the Surface of the Water which ſtands at a ſmall Height in that Cavity; which Air, therefore, is compreſſed by the Weight of the Water, whoſe Height is *po*. We ſpeak of the Preſſure, by which the Preſſure of the Atmoſphere is overcome. The Air in the upper Part of the Cavity B communicates with the Air above-mentioned by the Tube *st*, and is equally compreſſed, and acts with the ſame Force upon

Plate 32 p. 238



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upon the Surface of the Water in that Cavity. This Preſſure is to be added to the Preſſure ariſing from the Height of the Water, in order to have the Force by which the Air is compreſſed in the Cavity C, as alſo in the upper Part of the Cavity A, by reaſon of the Communication through the Tube *x u*. The Preſſure, therefore, upon the Surface of the Water in that Cavity A, is equal to a Pillar of Water, whoſe Height is almoſt double the Weight of the whole Machine. And therefore it ſpouts as if it was preſſed by ſuch a whole Column, that is, to a Height not much wanting from the Height of that whole Column*. *358

The Height is continually diminiſhed, for the Columns of Water, which compreſs the Air, continually become ſhorter, becauſe the Water aſcends in the Cavities C and D, and its Height is diminiſhed in the Cavity B. In the ſame Time the Cavity A is continually evacuated, and the Water aſcends through a greater Space, before it comes to *y*; therefore, it is driven to a leſs Height above *y*.

CHAP. XVII.

Of the undulatory Motion of the Air, where we ſhall treat of Sound.

If the Air be agitated in any Manner, the Particles moved recede from their Place, and drive the neighbouring Particles in a leſs Space; and as the Air is dilated in one Place, it is compreſſed in the Place next to it; the compreſſed Air, by the Reſtitution of the Spring, not only returns to its firſt State, but is alſo dilated by the Motion acquired by the Particles. 476

The Air, being firſt dilated by that Motion, is reſtored to its firſt State, and the Air is compreſ-

ſed towards other Parts. This again obtains,
when the Air laſt compreſſed expands itſelf, by
which a Compreſſion of Air is again produced;
therefore, *from any Agitation there ariſes a Motion*
analogous to the Motion of a Wave on the Surface of
*402 *Water**. The Air is compreſſed in that Manner
with a Dilation following, ſo as to make what is
*703 *called a Wave of Air*:* Compreſſed Air always
dilates itſelf every Way, and *the Motion of theſe*
477 ~~778~~ *Waves is the Motion of a Sphere expanding itſelf*
in the ſame manner as the Waves move circularly
405 upon the Surface of the Water.

478 *Whilſt a Wave moves in the Air, wherever it paſſes, the Particles are removed from their Place, and return to it, running through a very ſhort Space in going and coming.*

Plate XXXIII. *Fig.* 1.] Now, to explain the Laws of this Motion, let us conceive Particles of Air to be placed at equal Diſtances, and to be in a right Line, as *a*, *b*, *c*, *d*, &c. and f. Let the Wave be ſuppoſed to move along that Line. Now let us ſuppoſe it to be come forward along that Line, as far as between *b* and *p*, and that the Air is dilated between *b* and *b*, but compreſſed between *h* and *p*, as all this is repreſented in *Line* 1.

479 *The greateſt Denſity is at m, which is the Middle between h and p; and the greateſt Dilation between b and b, is in the Middle e.*

480 *Wherever the neighbouring Particles are not equally diſtant, the Motion, ariſing from Elaſticity, cauſes the leſs diſtant Particles to move towards*
*432 *thoſe that are moſt diſtant**; and this Motion alone, abſtracting from all other Motion acquired, is to be examined.

Between b and e there is a Motion from *b* towards *e*, that is, *conſpiring with the Motion of the Waves*; *there is alſo ſuch a Motion between m and p.*

But *there is a* contrary *Motion between e and m, and it is directed from m towards e.* 482

At m and e, where the Directions of the Motions are changed, *no Action ariſes from the Elaſticity,* becauſe the neighbouring Parts are placed at equal Diſtances among themſelves. 483

In the Places b, b, and p, the Difference of the neighbouring Parts is the greateſt of all; and therefore there *is the greateſt Action of the Elaſticity.* 484

From this it follows, that a Particle, according to its different Place in a Wave, ſuffers a different Action from the Elaſticity by which its Motion is generated, accelerated, diminiſhed or deſtroyed; therefore, the Direction of the Motion of a Particle cannot be determined from the Action of the above-mentioned Direction only, and does not always agree with that Direction, and the Motion of the ſingle Particles is changed every Moment.

All the Particles between *b* and *p* are removed according to the Order of the Letters. The Particles between *b* and *p* continue their Motion, and the reſt between *b* and *b* return towards *b*, as will be ſaid hereafter.

Theſe continue in the Motion by which they return, until, by the Action of the Elaſticity, whoſe Direction is changed in the Point *e*, the Motion acquired anew be deſtroyed; in which Caſe a Particle, as *b*, returns to reſt, and its firſt State. In the following Moment the Particle *c* comes to reſt in its firſt State, but *p* comes forward to *q*, as in the Line 2, and ſucceſſively in equal Moments, the Wave has all the Poſitions which are here repreſented in the Lines 1, 2, 3, *&c.* and 13; and, *whilſt the Wave,* from the Poſition in the Line 1, comes to the Poſition in the Line 13, *it runs thro' its whole Breadth. The Particle p,* in that Motion, *goes and returns, and* the Mo- 485 tion of it is made ſenſible in the Figure, and, as it plainly appears, this Particle *goes ſucceſſively through all the Situations of the Particles*; in the Waves all the Particles ſingly are agitated by the like Motion.

486 *The Motion of any Particle*, as *p*, *in its going backward and forward, is analogous to the Motion of a vibrating Pendulum, whilſt it* performs two Oſcillations, that is, *does* once *go forward and backward*. A Pendulum deſcends in its Oſcillation, and the Motion acquired conſpires with the Motion of Gravity, and is accelerated by it, until it comes down to the loweſt Part of the Arc to be deſcribed, that is, the Middle of the Way to be run through; the Pendulum goes on by the Motion acquired, which is deſtroyed by the Action of Gravity, whoſe Direction changes in this Point, whilſt the Body aſcends up the other Part of the Arc to be deſcribed: This Body returns by the ſame Laws.

The Particle *p* is moved by the Elaſticity, and this Motion is accelerated by the Action of the Elaſticity, until it comes to the Situation of the Particle *m*, the Line 1*, which Situation is ſeen in Line 4, in which the Particle *p* is, in the Middle Point of the Space, to be run through by the Motion backward and forward. By the Motion acquired, though Gravity acts againſt it*, it perſeveres in its Motion, until, by the Action of the ſaid Elaſticity, the Motion be wholly deſtroyed, which happens, when it has gone through a Space equal to that in which it was generated; then the Particle is in the Poſition which is ſeen in Line 7, which anſwers to the Situation of the Particle *b* in Line 1. Then, by the Elaſticity, the Particle returns, and is accelerated, until it has acquired the Situation of the Particle *e*, in Line 1*, as in Line 10, that is, until, as in Line

*481

*482

*482

Line 4, it comes again to the Point that is in the Middle of the Way to be run through. The Particle continues in its Return, until by the Action of the Elaſticity, whoſe Direction is again changed*, the whole Motion is deſtroyed; and then *483 the Particle returns to its firſt Poſition, as in the Line 13, and there, not being agitated by any new Motion, it remains at reſt. Therefore, *the* 487 *Motion of the tremulous Body, by which the Air is agitated, ceaſing, there are no new Waves generated*, and the Number of the Waves is the ſame as the Number of the Agitations of that Body.

If, after two Vibrations of a Pendulum, the Action of Gravity ſhould ceaſe, as in the Air, after the going and returning of a Particle, the Action of the Elaſticity on that Particle ceaſes, the Motion of a Particle of Air would wholly agree with the Action of a Pendulum. In the middle Point of the Arc, which is to be run thro' in the Oſcillation, there is no Action of Gravity, and its Direction is changed; in the middle Point of the Space to be gone through by the Particle *p* in its going and coming, in which it is in the 4th and 10th Line, the Situation of this Particle agrees with the Situation of the Particles *m* and *e* in Line 1, in which Points there is no Action of Elaſticity, and its Direction is changed*. In a Pendulum, the more a Body oſcillating is diſtant from the loweſt Point or Middle of *483 the Arc to be deſcribed, by ſo much greater is the Force of Gravity acting upon it; the more alſo the Particle *p* is diſtant from the Space to be run through, the more is the Action of the Elaſticity upon it; and in the Lines 1, 7, and 13, the Particle is moſt diſtant from the Point above-mentioned, and its Situation there agrees with the Points, *b*, *b* and *p* in the Line 1, in which the Action of the Elaſticity is greateſt of all*. *484

According to which Law, ſince this Action of Elaſticity increaſes with the increaſed Diſtance of the often-mentioned middle Point, it is determined from the very Law of the Elaſticity of the Air, whoſe Particles drive one another away with a Force which is inverſly as the Diſtance between *432 the Centers of the Particles*: and it is demonſtrated, that the Action of Elaſticity upon ſuch a Particle, as *p*, is increaſed or diminiſhed in Proportion to the Diſtance of the middle Point of the Space to be run through: And, therefore, alſo in that Part there is an Analogy between the Motion of a Particle and the Motion of a Pendulum *156 oſcillating in a Cycloid*.

If the Breadth of a Wave remaining, the Particles run out thro' a greater Space, the compreſſion and Dilatation of the Air in the Wave will be greater, and there will be a greater Action of Elaſticity, and that greater in the ſame Ratio in which the Space gone through in the going and coming is increaſed: And the Motion of a Particle, as *p* in this Caſe, differs from the Motion in the foregoing Caſe, as the unequal Oſcillations of different Pendulums differ; which, as they are performed in *156 equal Times*, the ſame will alſo obtain here.

Therefore, a Particle, as *p*, if the Breadth of the Wave continues, the ſame goes and comes in the ſame Time, through whatever Space it be carried out of its Place; that is, the Wave will go 488 its Breadth in the ſame Time; therefore, *all equal Waves, whether the Air be more or leſs agitated, are equally ſwift*.

489 Now *let* us examine unequal *Waves*; let them *be as* A to B, and let the Space gone through by the Particles in the Motion of each of them, in going and coming, be in the ſame Ratio; in that Caſe the Compreſſions and Dilatations in corre-

ſpondent Places will be equal; *the* Actions, therefore, from the Elaſticity, don't differ in correſpondent Diſtances from the middle Point of the Spaces to be run thro' by the Particles, in their going and coming. Therefore, thoſe *Motions are analogous to the Motions of two Pendulums, whoſe Lengths are as* A *and* B, and which run thro' ſimilar Arcs; for, in the correſpondent Points of thoſe Arcs, the Action of Gravity is the ſame.

In Pendulums the Action of Gravity increaſes
as the increaſed Quantity of Matter; and whatever be this Quantity, the Motion is equally ſwift,
when the Gravity is not changed: On the contrary, the Action of Elaſticity is determined in
the Motion of Waves, and depends upon the Diſtance between the Particles and the Velocity,
which is generated from it; the Elaſticity remaining the ſame, is inverſly as the Quantity of Matter to be moved*. *In the Waves above-mentioned,* *65
the Quantities of Matter are as the Breadth of the 490
Waves *a* and *b*, and the Velocities generated by
the Elaſticity are, therefore, in correſpondent
Points, as *b* to *a*. Therefore, theſe *Motions are analogous to the Motions of Pendulums* deſcribing ſimilar Arcs, and *moved with different Forces of Gravity, which are to one another as* B *to* A; for, in correſpondent Points of ſimilar Arcs, the Celerities ariſing from different Gravities are as thoſe Gravities.

Now to compare the Motion of Waves with
the Motion of Pendulums, we muſt conſider Pendulums differing in Length, and on which different Forces of Gravity act*, and we have ſhewn *489
what theſe Cauſes produce ſingly in the Dura- 490
tion of the Vibrations*. Both theſe are to be *158
joined together, and the Squares of the Times 165
of the Oſcillation of Pendulums, whoſe Motions

are analogous to the Motion of the above-men-
480 tioned Waves, are as the Length A and B, and
158 inverſely, as the Gravities B and A*; that is,
*190 again directly, as A and B; the Ratio of which
165 Ratio's is a Ratio, compounded of the Squares of the Quantities A and B. Therefore, the Times of the Oſcillations are as A and B, and the Times are in the ſame Ratio in which the Particles of the Waves go and come; that is, the Waves run through their Breadths, which are as A to B; which Times are, therefore, as the Spaces gone thro' by the Waves, and, therefore, the Motions are equally ſwift. If the Space be changed thro' which the Particles go and come, the Velocity
488 of the Waves is not changed; wherefore, the Proportion which we have put down for a Demonſtration, between the Spaces gone thro' by the Particles in their going and coming, may be neglected, and the Propoſition will be generally
491 true, that *Waves, whether equal or any way unequal, move with the ſame Velocity.*

492 This Rule will hold good, *if* the State of the Air is not changed; but *the Elaſticity remaining the ſame, the Denſity of the Air often varies; and* the Elaſticity *may be changed*, the Denſity remaining the ſame; laſtly, both are often liable to be changed.

In the firſt Caſe, ſuppoſing both the Waves to be equal, and alſo the Spaces thro' which the Particles go and come, the Celerities ariſing from the Elaſticity, which is always the ſame, are in-
65 verſely as the Denſities; but this Variation of
288 Celerity anſwers in the Motions of equal Pendu-
490 lums, with the Variation of the Gravity; in which Caſes the Squares of the Celerities of the
165 Vibrations are as the Gravities themſelves; therefore, *in Waves their Squares of the Celerities are inverſely*

inverſely as the Denſities. The Suppoſition of the Equality of the Waves, and the Spaces gone thro' by the Particles, does not hinder this Demonſtration from being univerſal*. *488

491 *When the Denſity remains the ſame, but the Elaſticity is changed,* the Celerity ariſing from it varies in the ſame Ratio as the Elaſticity; wherefore, from the Demonſtration of the foregoing Propoſition in this Caſe, *the Squares of the Celerities of the Waves are as the Degrees of the Elaſticity.* 493

494 *If the Elaſticity and the Denſity differ, the Squares of the Velocities of the Waves will be in a Ratio compounded of the direct Ratio of the Elaſticity*, *493 *and the inverſe Ratio of the Denſity**. *492

495 *If the Denſity and the Elaſticity increaſe or decreaſe in the ſame Ratio,* the inverſe Ratio of the Denſity will deſtroy the direct Ratio of the Elaſticity, and *the Celerity of the Waves will not be changed.*

496 This laſt Caſe happens in the Compreſſion of the Air*. *430 Therefore, *from the changed Height of the Pillar of Mercury, which is ſuſtained in a Tube void of Air by the Preſſure of the Atmoſphere*, *422 which ſhews, that the Weight, by which the Air is compreſſed near the Earth, is changed, 497 *we muſt not judge the Celerity of the Waves to be changed.* For the ſame Reaſon, *the Waves are moved with the ſame Celerity in the Top of a Mountain as in a Valley,* unleſs there be a Change of the Elaſticity itſelf, by reaſon of the Cold, which is almoſt always more intenſe on the Top of a Mountain than in a Valley; and this would occaſion the Waves to move ſlower*. *493

498 It is plain alſo, that *the Waves move faſter in Summer than in Winter**. *434

499 The Celerity of the Waves is compared to the Celerity which a Body acquires in falling, *by determining,* from the known Height of the

Mercury, which weighs equally with the Preſ-
*422 ſure of the Atmoſphere *, and alſo the Denſity
*442 of the Air * ; *the Height of the Atmoſphere, ſuppoſing it every where equally denſe with the Air near the Earth* ; *the Velocity of the Waves will be the ſame as a Body could acquire in falling from half that Height.* Which Velocity, from what has been ſaid, may be eaſily diſcovered by Expe-
*157 riments made upon Pendulums *.

158 If the Weight, by which the Air is compreſſed, be diminiſhed, the Air expands itſelf in the ſame
*42 Ratio * ; and ſuppoſing the Atmoſphere every where of the ſame Denſity, its Height does not vary, which agrees with what has been ſaid, that the Velocity of the Waves is the ſame in different
*496 Compreſſions of the Atmoſphere *.

500 The Motion of the Air, which we conſider in this Computation, ariſes from Elaſticity alone ; and the Computation would be exact, if the Particles themſelves had not a ſenſible Proportion to the Interſtices between them ; but if we ſuppoſe here that they bear a ſenſible Proportion to them, the Motion of the Waves will be ſwifter ; for it is propagated through ſolid Bodies in an Inſtant, which muſt alſo be referred to heterogeneous Corpuſcles ſwimming in the Air.

501 *The Motion of Waves in the Air produces Sound* ; of which, before we ſpeak, we muſt lay down ſomething in general relating to Senſation.

502 So ſtrict is the Union of the Body and the Mind, that ſome Motions in the Body do, as it were, cohere with certain Ideas in the Mind, and they cannot be ſeparated from each other. From the Motion of the Body are new Ideas every Moment excited in the Mind, and ſuch are the Ideas of all ſenſible Objects ; yet we find nothing common between the Motion in the Body and the

Idea in the Mind. We cannot perceive what Connexion is here, nor that any Connexion is poſſible. There are an infinite Number of Things hidden from us, of which we have not ſo much as an Idea.

The undulatory Motion of the Air agitates the *Tympanum*, or Drum of the Ear, by which Means a Motion is communicated to the Air contained in that Organ, which being carried to the auditory Nerve, excites in the Mind the Idea of Sound. 503

The Structure of the Ear, both internal and external, is wonderful; but here we treat of the Motion of the *Air*; that it *is the Vehicle of Sound*, is proved by the following Experiment.

Experiment 1. *Plate* XXXIII. *Fig.* 2.] Take the Leaden Plate O, which has two cylindric Pillars of the ſame Metal C, C, fixed to it; join a little Bell A to the braſs Wire B D, and let it be tied with Strings to the Pillars C, C; lay the Plate O upon the braſs Plate of the Air-Pump, putting between a little Cuſhion of Cotton, or Raw-Silk; ſet a Receiver on over all this Apparatus: Cover the Receiver with a Plate that has the Collar of Leathers ſcrewed to it, through which the braſs Wire D E can ſlip up and down*; to the braſs Wire you muſt faſten the Plate *e f*, ſo that, by turning the Wire round, the Bell A may be agitated. Pump out the Air from the Receiver, and ſhaking the Bell in the Manner before deſcribed, you will not hear the Sound. By turning the Wire D E, the Bell will move backward and forward ſeveral times; but we are only to obſerve that Motion in which the Plate *e f* doth not touch the Wire *b d*. Letting in the Air, the Sound will be heard as before.

*440

From

505 From this alone, that the Air is the Vehicle of Sound, and that Sound is moved thro' it without the Air's being carried from one Place to another, it evidently follows, that in Sound there is an undulatory Motion of the Air, and that Sound ariſes from the tremulous Motion of Bodies. That this obtains in Chords, or Strings of muſical Inſtruments, no body doubts, ſince, by giving them a tremulous Agitation, they produce a Sound. In great Bells, and ſeveral other Bodies, this tremulous Motion is very ſenſible; but it will become viſible by the following Experiment made upon a ſounding Glaſs Bell.

Experiment 2.] Let the Glaſs Bell C C be fixed with Plaiſter, or Cement, to a wooden Screw, by Means of which it may be made very faſt to the tranſverſe Piece of Wood A B; this Wood muſt be ſuſtained by two wooden Pillars S S, to which it is firmly join'd with Screws and Nuts. There is a Pin with a Screw upon it, that goes through one of the Pillars, juſt even with the Mouth of the Bell; ſo, by ſcrewing it forwards or backwards, you may ſet it nearer to, or farther from, the Edge of the Bell. If this Diſtance be very ſmall, and the Bell be ſtruck, it will, by its tremulous Motion, ſtrike ſeveral times againſt the Pin with its Edge.

506 Hence we deduce, that *a Body that is ſtruck, continues to give a Sound ſome Time after the Blow*; the agitated Fibre will continue his Vibration ſome Time, on Account of the Elaſticity*; *215 we often ſee, as in *Experiment* 1, that a Body gives a Sound, tho' the Air, agitated by it, has no Communication with the outward Air; whence it 507 follows, that *by the Agitation of the Air, the Fibres of*

of which Bodies conſiſt, are moved; which Motion is transferred into the external Air.

This Tranſlation of the Sound, by the tremulous Motion of the Fibres, is very remarkable; and how the Communication of this Motion extends itſelf, will appear by a ſingle Experiment.

Experiment 3. *Plate* XXXIII. *Fig.* 2.] This Experiment differs from the firſt only in this; that if, inſtead of tying the Bell to the leaden Machine COC, it be faſtened to the Ends of a braſs Plate bent in the Figure of a double Gnomon, which is made faſt by a Screw to the Plate of the Air-Pump, and the Air be pumped out, and the Bell ſhak'd in the ſame Manner as in the firſt Experiment; you will find but very little Difference between the Sound that is made, when the Air is exhauſted, and when the Air is re-admitted.

The tremulous Motion of the Parts of the Bell is communicated to the braſs Wire *b d*, ſo as to move the Strings by which the Bell is ſuſpended, and this Motion is transferred to the bent braſs Plate; the Screw, with which this Plate is joined to the braſs Plate of the Air-Pump, touches the Plate, and communicates a tremulous Motion to it, by which the Air is agitated, and the Sound of the Bell is heard.

The Celerity of the Sound is the ſame as the Ce- 508
lerity of the Waves, which ſtrike the Ear; and to *491
this muſt be referred what has been ſaid of their 492,
Celerity*. In reſpect to *Numb.* 499, it is to be 493,
obſerved, that the Celerity of Sound can no way 494,
be determined by Calculation; for the Propor- 495,
tion between the Diameters of the Particles and 496,
in the Interſtices between them, is not known, 497, 498,
neither 499,

neither how large a Space the heterogeneous Particles take up in the Air.

The Celerity of Sound may be immediately determined by an Experiment.

509 If a Flaſh of Fire goes off at Night with a
Noiſe, and a Spectator ſtands at any known Diſtance
from the Fire, who, with a ſhort Pendulum,
meaſures the Time between ſeeing the Light,
and hearing the Sound, he will have the Celerity
of the Sound; for the Motion of Light, at leaſt,
thro' the Space as ſuch an Experiment can be
made in, is inſtantaneous.

510 By ſuch an Experiment made in *France*, it appear'd,
that Sound run 1800 *French* Feet in a Second
Minute of Time; but this Celerity is not
498 conſtant.

511 If at the ſame Time in which the Velocity of
the Sound is determined by this Method, there
422 be made the two Experiments above-nam'd,
499 one may, by Calculation, determine the Motion
499 of Sound by the Elaſticity of the Air, and by
comparing it with the Velocity immediately mention'd,
you will have the Acceleration of the
Sound, from the Thickneſs of the Particles, and
512 the heterogeneous Matter.

*491 *The Celerity of the Sound is equable*; yet in
going through a greater Space, it is ſometimes ac-*
*493 *celerated or retarded**, from the different Degree
of Elaſticity in different Places, in which there
434 are different Degrees of Heat or Cold.

513 *The Celerity of the Sound does not much differ,
whether it goes with the Wind, or againſt the Wind.*
By the Wind a certain Quantity of Air is carried
from one Place to another; the Sound is accelerated
as long as it moves through that Part of
the Air, if the Direction of the Sound be the
ſame with the Direction of the Wind; but as
Sound moves very ſwift, in a very ſhort Time it
will

will run through the Air which is agitated by the Wind, and the Acceleration does not laſt long, which, indeed, is not very great; for the moſt violent Winds, which are ſtrong enough to root up Trees, and blow down Houſes, have their Celerity to the Celerity of the Sound, but about as one to 33; by the ſame Argument it is proved, that no ſenſible Retardation is occaſioned by the Wind, when the Sound moves againſt it.

The Space which the Particles run through, as they come and go, may be increaſed and diminiſhed by the Wind; *therefore, the Sound may be heard at a greater or ſmaller Diſtance, according to the Direction of the Wind.*

The Intenſity of the Sound depends upon the Strokes of the Air on the auditory Nerve, and theſe Strokes are as the Quantities of Motion in the Air.

Whence it follows, that, *cæteris paribus, the* 515
Intenſity of the Sound is as the Space run through by
*the Particles in their going and coming**. *488

All Things remaining as before, if the Weight 53,
by which the Air is compreſſed be changed, the 63.
Celerity does not vary*, but the Denſity is chan- *496,
ged in the ſame Ratio as the Weight*. *429, 229

Therefore, *cæteris paribus, the Intenſity of the* 516
Sound is as the Weight by which the Air is com-
*preſſed**; that is, this Intenſity increaſes and de- *62
creaſes, as the Pillar of Mercury, which is *in Æquilibrio* with the Weight of the Atmoſphere.

Experiment 4. *Plate* XXXIII. *Fig.* 3.] Shake
the Bell A in compreſſed Air*, exactly in the *454
Manner as it was ſhaked *in Vacuo*, in *Experiment* 1, and the Sound will be increaſed; which will again be diminiſhed, if opening the Bell you let the Air return to its firſt State.

As the Intenſity of the Sound in compreſſed Air, included in a Veſſel, is greater, ſo the Fibres, of which the Glaſs V V is made, are agitated, and a greater Agitation is communicated to the external Air.

517 *If all Things remain as before, but the Elaſticity*
be increaſed, the Denſity is diminiſhed in the ſame
430 Ratio as the Elaſticity is increaſed; but the Celerity
increaſes as the Square Root of the Elaſticity*;
*493 therefore, *the Intenſity of the Sound is* directly
as the Square Root of the Elaſticity, and inverſely
*64 *as the Elaſticity itſelf**; but the Ratio, compounded
of theſe, is the inverſe Ratio of the above-
518 mentioned Square Root of the Elaſticity. The
Intenſity of the Sound is diminiſhed; therefore,
519 as its Velocity is increaſed, and *in Summer*, cæteris
paribus, *the Intenſity of Sound is leſs than in Winter*;
yet, in Summer Bodies do more eaſily tranſmit
Sound, becauſe their Parts cohere leſs ſtrictly, as
will be explained at a proper Time, and they do
more eaſily acquire a tremulous Motion.

Experiment 5. *Plate* XXXIII. *Fig.* 5.] Hang up the Bell A in a Glaſs, and opening the Cock, that the Air in the Glaſs may have Communication with the external Air, let the Glaſs be ſhaked, and the Diſtance be determined when the Sound can be heard; warm the Glaſs, and repeat the Experiment, and the Sound will be heard at a greater Diſtance.

520 *The Intenſity of Sound, conſidered in general, is*
in a compound Ratio of the Space run thro' by the
*515 *Particles, in their going backward and forward**, *of*
*516 *the Weight compreſſing the Air**; *and laſtly, of the*
*517 *inverſe Ratio of the Square Root of the Elaſticity**.
521 There is alſo a Difference in Sound, from the
Number of the Vibrations of the Fibres of the

Body which produce the Sound, that is, of the Number of the Waves produced in a certain Time, according to the different Number of the Percuſſions in the Ear, the Mind receiving a different Senſation.

A Muſical Tone depends upon this Number of 522 Vibrations, which is ſaid to be the more *acute*, according as the Returns in the Air are more frequent; and more *grave*, the leſs the Number of the Waves is; *and the Degrees of the Sharpneſs of the different Sounds are to one another, as the Number of the Waves which are produced in the Air at the ſame Time.* 523

A Tone does not depend upon the Intenſity of the 524 *Sound, and an agitated Cord gives the ſame Sound, whether it vibrates through a greater or a leſs Space**. *257

Concords ariſe from the Agreement between the 525 *different Motions of the Air, which affect the Auditory Nerves at the ſame Time.*

If two tremulous Bodies perform their Vibra- 526 tions in the ſame Time, there will be no Difference between their Tones and this Agreement, which is the moſt perfect of all, is called *Uniſon*.

If the Vibrations are as 1 to 2, this Conſonance, 527 or Agreement, is called *Octave*, or *Diapaſon*.

Suppoſing the Vibrations as 2 to 3, that is, if 528 the ſecond Vibration of one Body always agrees with the third of another, ſuch a Conſonance is called a *Fifth*, or *Diapente*.

Vibrations, which are as 3 to 4, give a Con- 529 ſonance, which is called a *Fourth*, or *Diateſſaron*.

Ditonus is, when the Returns of the Air are 530 as 4 to 5.

And *Seſquiditonus* is a Conſonance, from a Con- 531 courſe of the fifth Vibration of one Body with the ſixth of another.

A Conſonance from the Agitation of Cords, if they be of the ſame Kind, is eaſily determined, by knowing their Dimenſions and Tenſion.

532 Cæteris paribus, *if the Lengths of two Cords are as the Number of Returns in a Conſonance, you will have the Conſonance between the Sounds which the Strings produce**. *259

533 *The ſame* obtains, *if*, cæteris paribus, *the Diameters have the aforeſaid Proportion**. *260

534 And alſo, *if*, cæteris paribus, *the Proportion of the Vibration in a Conſonance be given between the Square Roots of the Tenſions**. *258

535 *And generally, ſuppoſing any Chord of the ſame Kind, if the Ratio be compounded of the Direct Ratio of the Lengths and of the Diameters, and the inverſe Ratio of the Square Roots of the Tenſions be the Ratio between the Numbers of Vibrations performed in the ſame Time in any Conſonance whatever, you will have that Conſonance by the Agitations of thoſe Cords**. *261

All theſe have been experimentally tried by Muſicians; they have obſerved a very remarkable Phænomenon relating to theſe Cords, whoſe different Caſes very well deſerved to be explained.

536 *Let any Muſical Strings be ſo extended, as to perform their Vibration in equal Times; if you give Motion to the one, the other will alſo move.* Every Wave of the Air, ariſing from the tremulous Motion of the firſt String, ſtrikes the ſecond String, and gives it a little Motion; the String from the leaſt Motion, goes backward and forward ſeveral Times*, and is moved by the Stroke of the firſt Wave, whilſt the ſecond Wave comes forward, whoſe Motion conſpires with the Motion of the String*, and accelerates it. What is ſaid of the ſecond Wave muſt alſo be referred to the other Waves that follow, and there will be an Acce- *257 *257

Acceleration, 'till the Motion of both Strings be almoſt equal.

From the ſame Demonſtration it follows, that *an agitated String will communicate Motion to another, which performs two or three Vibrations, whilſt the firſt performs but one.* 537

Now, if the agitated String performs ſeveral Vibrations, whilſt the String that is to be moved by the Air can perform but one, from the foregoing Demonſtration it would follow, that it muſt communicate a particular Motion to it. To diſcover which, it is to be obſerved, that the Duration of the Vibration and the Length of the String are reciprocal; ſo that every thing elſe continuing as before, the determined Length can no way be ſeparated from the unchanged Duration of the Vibration. If therefore any String be ſtruck with ſeveral Strokes, by which Motion is communicated to it, and the Strokes are more frequent than what is agreeable to the Length of the String; that Part of it, whoſe Length agrees with the Time of the communicated Vibration, will be agitated as much, and there will be, as it were, an undulatory Motion communicated to the String; the Length of the Waves in the String will depend upon the Duration of the communicated Vibration, that is, upon the Time between the Strokes.

Take two Strings, in ſuch Proportion, that one may vibrate twice whilſt the other vibrates but once, and let the firſt String be put in Motion; the Duration of the Vibrations, which are communicated to the laſt String with the Motion of the Air, agrees with a String of half its Length *; and ſuch is the Length of the Waves in it: Therefore, *by the communicative Motion, the String is divided into two equal Parts*, and the Middle 538

*259

Point is at reſt. This is confirmed by an Experiment, if you lay a Piece of Paper upon the String to which the Motion is communicated; for it will remain at reſt, if you lay it upon the Middle of the String, but any other Part of it will be affected with a tremulous Motion.

539 *If the String which is put into Motion, in order to cauſe Motion in another, performs three Vibrations, whilſt the String, to be moved, performs but one, the laſt will be divided into three Parts by the communicated Motion, and there will be two Points of reſt*; which may be confirmed by the ſame Experiment above-mentioned. All other Caſes that have communicated Motion, which are obſerved by Muſicians, are eaſily deduced from what has been ſaid.

540 What has been ſaid of the Reflexion and In-
406 flexion of the Waves in Water, may be referr'd
408 to their Reflexion in Air, the Elaſticity in this
469 Cauſe producing the ſame Effect as the Preſſure of
410 the raiſed Water in that.

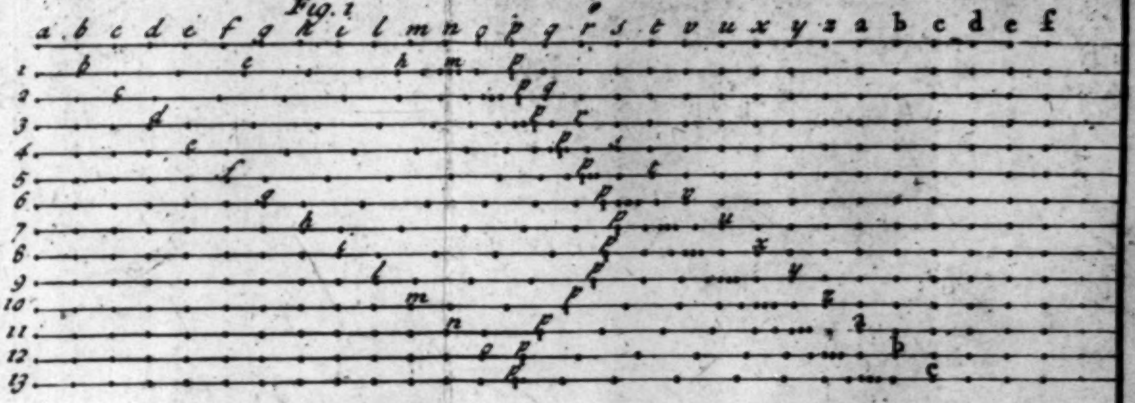
541 *From the Reflexion of the Sound there often ariſes a Repetition of it, which is called an Eccho.* If different Parts of the ſame Wave, expanding itſelf
477 into a Sphere, ſtrike upon different Surfaces, ſo that being reflected they concur together, the Motion of the Air will be ſtronger there, and the Sound will be heard. *The ſame Sound is often repeated different times* from the different Parts of the ſame Way reflected to different Diſtances, and ſome of which alſo ſucceſſively concur at the ſame Place.

542 Such a Repetition ſometimes happens from the Reflexion being repeated.

541 *The Sound is often increaſed by Reflexion in a Tube:* The moſt perfect Figure of all that can be given to ſuch a Tube, is that of a Parabola, revolving about a Line a Quarter of an Inch diſtant from the

at the end

Fig. 1



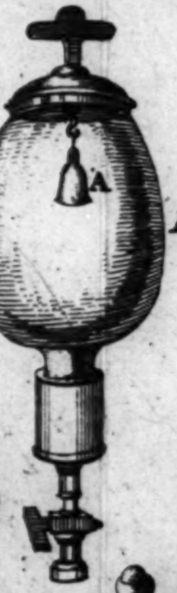


Fig. 5

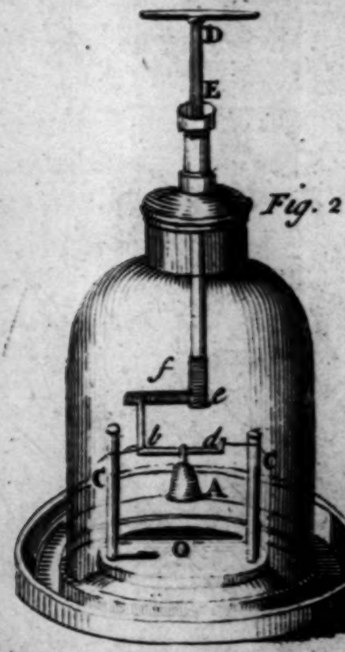
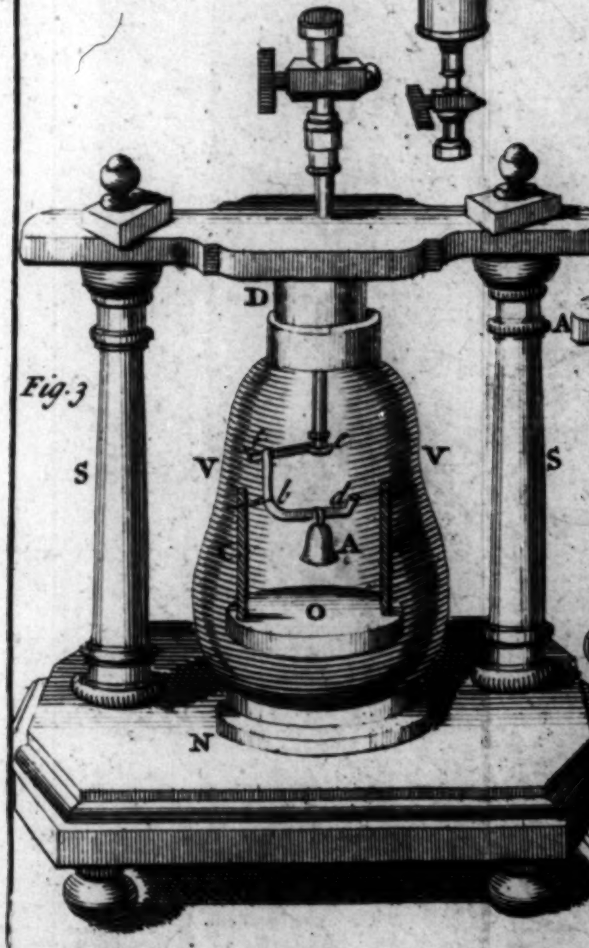


Fig. 2

Fig. 3



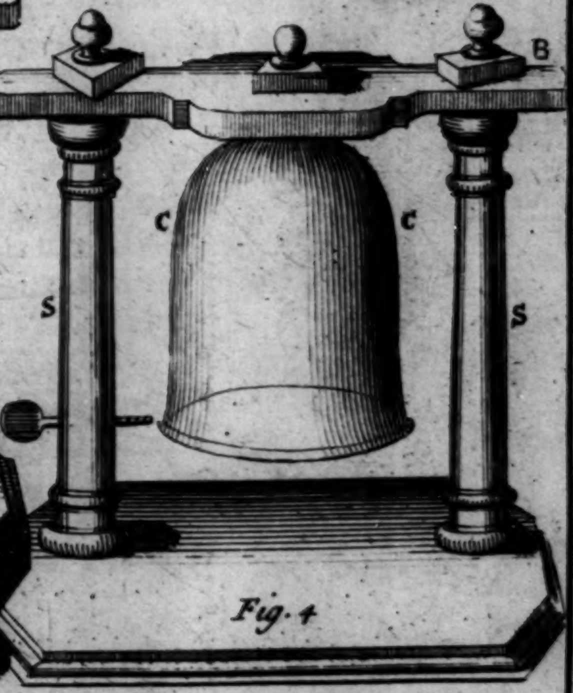


Fig. 4

the Axis. For if any one ſpeaks in ſuch a Tube, ſetting his Mouth in the Axis of the Machine, and in the Focus of the Parabola, the Waves will be ſo reflected, that every one of their Parts will acquire a Motion parallel to the Axis of the Machine, whereby the Force of the Wave, and alſo of the Sound, will be very much increaſed. There muſt be a Mouth-Piece, to fit the Lips, fixed to the End of the Tube.

FINIS.

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◆

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